Corporate Liquidity and Debt Buybacks*

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preliminary; comments welcome

Abstract

Firms often retire their bonds before maturity at market prices, even though the leverage ratchet effect implies that buybacks at fair prices cannot benefit shareholders. We show that corporate liquidity restores the gain: a buyback is a fair bet with creditors in par-valued dollars that moves inside cash out of the default states, where it is worth par, into the states where it carries a liquidity premium. We solve in closed form a continuous-time model with cash and smoothly adjusted consol debt, in which the firm issues debt when cash is scarce and buys it back when cash is abundant but not yet enough to pay out. In a verified sample of 4,347 corporate bond tender offers over 2000–2024, cash-rich firms tender more, and the TCJA repatriation shock leads moderately treated firms to retire more bonds while the most treated shift to equity payouts, as the model's payout boundary predicts.

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1 Introduction

Firms retire their bonds before maturity, and they pay the market price to do so. Over 2000–2024, U.S. nonfinancial firms completed 4,347 verified cash tender offers that retired \$1.08 trillion of corporate bonds, on top of a further \$136 billion bought in the open market. Yet the leading theory of leverage dynamics says this should not happen. When a firm repurchases debt at its fair price, the reduction in leverage lowers the default boundary and raises the value of the bonds that remain, and shareholders who must pay the higher post-repurchase price cannot capture that gain: it accrues to the creditors they did not buy out. This is the leverage ratchet effect of Admati et al. (2018), the corporate analogue of the sovereign "buyback boondoggle" of Bulow et al. (1988), and in the dynamic no-commitment setting of DeMarzo and He (2021) it becomes a sharp prediction: equity holders issue debt continuously and never buy it back, no matter how levered the firm is. Debt buybacks at equilibrium prices, on this view, are hard to sustain.

This paper argues that the missing ingredient is corporate liquidity. Firms hold cash because outside finance is costly or unavailable when it is needed most, and a firm that runs out of cash is liquidated even when its going-concern value is positive. Cash is thus valuable as insurance but costly to carry, and retiring debt at the market price becomes the natural use of cash that is abundant, but not yet abundant enough to pay out: buybacks occupy the middle ground between hoarding and distributing.

In the data, corporate bond buybacks track corporate cash. Panel (a) of Figure I plots aggregate tender offer activity for the U.S. nonfinancial firms in our sample over 2000–2024 alongside their aggregate cash and short-term investments: activity climbs from fewer than a hundred transactions retiring some \$10 billion a year in the early 2000s to a 2021 peak of 363 transactions retiring just over \$90 billion, while cash holdings more than triple. Panel (b) turns to the cross section: a given firm is significantly more likely to tender in the years when its own cash position is high. Both patterns hold whether the sample ends in 2019 or 2024, so neither is an artifact of the COVID-19 period; Section 5.1 describes the data. These are correlations; in Section 5.2 we exploit the plausibly exogenous liquidity shock of the Tax Cuts and Jobs Act (TCJA) to show that more cash *causes* firms to repurchase their bonds.

The reason the benchmark rules out buybacks is instructive. The leverage rachet effect literature builds on Leland (1994) and there equity holders can always inject outside cash to cover a shortfall: dividends may be negative, so a dollar inside the firm is worth exactly a dollar in shareholders' pockets, and every default is a solvency default, chosen by shareholders who decline to keep paying. What the framework cannot produce is a liquidity-driven default, in which a firm with strictly positive going-concern value is nonetheless liquidated because it has run out of internal cash and outside cash is costly

(a) Aggregate Tender Offer Activity **(b)** Tender Offer Likelihood

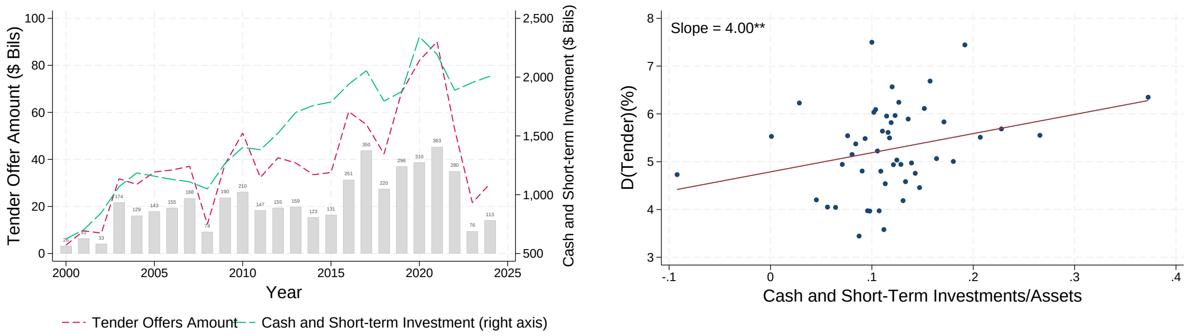

Figure I. Corporate bonds tender offers and corporate cash holdings. Panel (a) plots aggregate corporate bond tender offer activity of the nonfinancial U.S. firms in our sample over 2000–2024: bars show the number of tender offers each year (count above each bar), the dashed line shows the dollar amount of bonds retired through tender offers (left axis), and the long-dashed line shows aggregate cash and short-term investments (right axis), all in billions of 2010 dollars. Panel (b) is a binned scatter plot of the likelihood of a tender offer against the firm's cash position, measured as cash and short-term investments scaled by book assets, at the firm-year level over 2000–2019, controlling for firm and year fixed effects; the corresponding estimate is in Column (2) of Table B.1, Panel A. A one-standard-deviation increase in the cash-to-assets ratio raises the annual probability of a tender offer by 0.62 percentage points, about 12% of the unconditional frequency, and the estimate is significant at the 5% level.

or unavailable.[1] Absent that possibility, cash has no liquidity premium, and the firm has no reason to hold it.

One might therefore conjecture that turning on liquidity-driven default, and letting the firm actively manage its cash position, would let buybacks emerge along the equilibrium path. But it is not obvious ex ante why a liquidity constraint should help. A buyback is financed with the very internal cash that the constraint makes valuable: the dollar spent on retiring a bond is a dollar no longer available to avert a liquidity default. If anything, a firm that fears running out of cash should hoard it rather than hand it to creditors. Whether liquidity management works for or against buybacks is therefore a question about the relative value of a dollar hoarded and a dollar spent on debt, and answering it requires a model in which both the cash balance and the outstanding debt are state variables that the firm controls.

The rationale practitioners give for buybacks offers a clue, though not an answer. Treasurers and researchers alike cite "cheap market debt" as a leading motive for retiring bonds early. With bond prices set in equilibrium, however, it is hard to say what "cheap" is cheap relative to: the market price already reflects the firm's future policies, including the buyback itself. Indeed, DeMarzo and He (2021) show that the standard shareholder–creditor conflict of interest works in the opposite direction: however large the discount reflected in its credit spread, debt is too expensive to repurchase rather than too cheap, because the price the firm faces rises as it retires debt.

[1] Models of cash management with long-term debt that admit liquidity-driven default, notably Anderson and Carverhill (2012) and Gryglewicz (2011), fix the capital structure at the outset; Chen and He (2026) review this literature and note that endogenous debt adjustment in the presence of liquidity concerns remains largely unexplored.

We show, first in an illustrative two-period model (Section 2.2) and then in continuous time (Section 3), that a specific form of default risk—termination risk, the possibility that the firm's earning capacity is destroyed within the next instant—gives content to the idea of cheap market debt, and that it does so only when inside cash carries a liquidity premium. In the two-period model the advantage of buying back over hoarding is, per unit of face retired, the default probability times the liquidity premium on inside cash; in continuous time it is a distinct term in the debt adjustment rate: the expected loss of liquidity premium at the Poisson termination event. Both vanish without the liquidity friction, recovering the no-buyback benchmark.

Why must default risk enter, and why must it interact with a liquidity premium? Because a buyback is a trade between two parties who value the same dollar differently across states. Creditors value every state at par: the claim being retired pays one if the firm is repaid and nothing in default, and they price it at the probability of repayment. Retiring it with inside cash is therefore a fair bet in par-valued dollars: the equity holders give up cash in the default states, where it would have come back to them at par, and keep the debt payment in the states where the firm is repaid. If a dollar were worth par to the equity holders in every state, as in the setting of Leland (1994) where they can inject outside cash freely, this fair bet would create nothing. It creates value because inside cash is worth more than par in the states where it spares a fire sale, and the buyback moves resources into precisely those states. Both ingredients are needed: without default the price is par and there is no bet; without the liquidity premium the bet is fair for both sides. Section 2.2 makes this precise, with the wedge in closed form and its state-by-state anatomy in Table I.

We then move on to the continuous-time model, which is the paper's main contribution. The literature has lacked a framework that features both corporate liquidity and long-term debt and yet admits clean qualitative characterization; existing models with cash and long-term debt fix the capital structure or are solved numerically. We offer such a framework. Operating income follows an arithmetic Brownian motion until a Poisson termination shock ends it; the firm's debt is a consol that shareholders issue or repurchase, at a price set by competitive lenders who anticipate the firm's future policy; and the firm cannot raise outside equity, so it defaults the moment its cash is exhausted. We also make holding cash costly, so that the equity holders trade off hoarding it against paying it out as dividends.

Following DeMarzo and He (2021), we focus on smooth equilibria in which the firm adjusts its debt continuously, and we show that the model admits a closed-form solution. Two endogenous state variables, the cash balance W and the consol debt F, would ordinarily make the problem intractable. In a smooth equilibrium, however, the equity value can be solved *as if the firm never traded its debt*, so it becomes a function of W alone, with F entering as a parameter, and is available in closed form. The equilibrium debt price is then pinned down by the shareholders' marginal rate of substitution between debt and cash. We show that the optimal debt adjustment rate decomposes into three terms: a tax shield that favors issuance, a precautionary motive that favors issuance when cash is scarce, and a cash-carry wedge that favors buybacks when cash is abundant, the wedge being the sum of the return shortfall on idle cash and the termination-loss term described above. The state space partitions accordingly. The firm issues debt when cash is low, repurchases it above a threshold $\overline{W}(F_t)$, and pays dividends above the payout boundary $W^\star(F_t)$; buybacks occupy the region in between, where cash is costly to carry but still too valuable to distribute.

We show that the mechanism survives costly outside equity when the issuance rate is capped, and we characterize when it breaks: if equity can be raised at a proportional cost without limit, a smooth debt policy can never coexist with equity issuance in equilibrium, because reflecting cash at zero through recapitalization is inconsistent with the lenders' valuation of debt at that boundary. We leave to future research the dynamics of debt and cash when equity is costly but not prohibitively so, and the interaction of cash with other frictions such as transaction costs, reputation, or agency.

We then take the model to the data. Our sample covers all corporate bond tender offers by U.S. firms over 2000–2024, drawn from Bloomberg Corporate Actions and matched to Mergent FISD, with every record verified against SEC filings and press releases. To identify the effect of liquidity on buybacks we exploit the Tax Cuts and Jobs Act of 2017, which released an estimated \$1.7 trillion of foreign-held cash by ending the tax on repatriation. Sorting firms by pre-reform foreign income in a difference-in-differences design, we find that firms receiving a moderate liquidity shock respond on the debt margin: their probability of launching a tender offer rises by 4.8 percentage points a year, about four-fifths of the unconditional frequency, and they retire an additional 0.7 percent of their bonds outstanding. Firms receiving the largest shock respond instead on the equity margin, raising equity buybacks by 0.6 percent of market value while leaving bond buybacks essentially unchanged. The split is what the model's state space predicts: a windfall that leaves a firm in the buyback region is spent on debt, while one that carries it past the payout boundary is passed on to shareholders.

Related literature. The paper sits at the intersection of three literatures. The first studies leverage dynamics without commitment. Fischer et al. (1989) observed that shareholders who cannot commit will not reduce debt early; Admati et al. (2018) established the leverage ratchet effect in generality, and DeMarzo and He (2021) characterized the resulting dynamics in closed form, with the striking implication that the firm issues debt continuously and never repurchases it. Several recent papers ask how repurchases can arise once one departs from that benchmark. Malenko and Tsoy (2026) sustain repurchases through reputation, as an optimal time-consistent policy enforced by reversion to the no-commitment equilibrium, and obtain a stable regime in which the firm defends an interest-coverage target by issuing after good shocks and repurchasing after modest bad ones. Dick-Nielsen et al. (2025) study lumpy issuance under proportional costs with and without commitment, and Benzoni et al. (2022) characterize debt dynamics under fixed issuance costs. Our paper is complementary: we keep the frictionless no-commitment structure of DeMarzo and He (2021) intact and add only a cash balance and the possibility of running out of it, so that repurchases arise from liquidity management rather than from reputation, commitment devices, or transaction costs.[2]

The second literature models corporate cash under external financing frictions, treating cash management as an inventory problem in which the firm accumulates a buffer up to a payout boundary.[3] Fewer papers combine cash with long-term debt. Anderson and Carverhill (2012) solve numerically for a firm with a consol and a cash reserve, so that liquidity- and solvency-driven default coexist; Gryglewicz (2011) obtains closed forms by letting a sufficiently large buffer rule out liquidity default altogether; and Acharya et al. (2012) show that cash and credit spreads are positively related in the cross-section precisely because riskier firms hoard. In all of these the debt is set once. Our contribution is to let the firm adjust it continuously, and to show that the interaction of the cash buffer with the debt price is what switches the ratchet off.

The third literature is empirical work on bond repurchases and the corporate bond market.[4] Xu et al. (2022) show that firms buy back bonds when secondary-market liquidity deteriorates, transferring value from creditors; the liquidity we study is the firm's own cash rather than the market's, and the two interact through the bond price, as in He and Xiong (2012), He and Milbradt (2014), and Chen et al. (2018), where secondary-market illiquidity feeds back into default. Finally, our identification builds on the literature that treats the TCJA repatriation provision as a liquidity shock, notably Faulkender et al. (2019), Albertus et al. (2025), and Darmouni and Mota (2024); we add the debt margin of the response, which that literature has not examined.

The rest of the paper proceeds as follows. Section 2 describes the institutions of bond

[2] A related line of work restores commitment partially rather than abandoning it: DeMarzo (2019) shows that collateral lets the firm secure some creditors to the exclusion of others; Hu et al. (2024) and Dangl and Zechner (2021) obtain commitment through short-term debt and through the rollover frequency of exponentially retiring debt, respectively; and DeMarzo et al. (2023) evaluate debt-ceiling and issuance-cap rules in a sovereign setting. Related work on the maturity dimension includes Diamond and He (2014) and He and Milbradt (2016); Xu (2018) documents early refinancing through call provisions, at prices set in the indenture rather than in the market; and Li (2025) studies the dynamic mix of bank and market debt.

[3] See Bolton et al. (2011), Décamps et al. (2011), Hugonnier et al. (2015), and Riddick and Whited (2009); Opler et al. (1999) and Bates et al. (2009) document the size of the cash buffer and its secular growth.

[4] Mann and Powers (2007), Kruse et al. (2014), and Levy and Shalev (2017) describe the institutions and motives of debt tender offers and open-market repurchases, and Mao and Tserlukevich (2015) analyze creditor holdout in repurchase offers.

buybacks and lays out the two-period model. Section 3 develops the continuous-time model, and Section 4 extends it to costly outside equity. Section 5 describes the data and presents the TCJA evidence. Proofs and additional results are in the appendix.

2 Institutional Background and A Simple Model

This section provides background on corporate bond tender offers, and then lays out a simple two-period model that isolates the mechanism we study.

2.1 Bond Buybacks: Institutional Background

Corporate bonds are a major source of U.S. firm financing (roughly \$11.5 trillion outstanding and \$2.0 trillion of gross issuance in 2024; SIFMA 2025), and they leave a firm's balance sheet before maturity in three ways: at prices fixed in the indenture when the firm exercises a call provision, meets a sinking-fund requirement, or honors a holder's put; through an exchange for new securities; or at prices the market determines, when the firm repurchases bonds at its own discretion. We focus on the market-price channel, the quantitatively more important one: only a minority of bonds are callable at a fixed price (31% of U.S. corporate bonds at year-end 2003 according to Mann and Powers 2007, close to the 33% in our sample in 2003, a share that falls to 25% by 2020), and market-price repurchases have grown markedly over the past two decades.

Market-price repurchases take one of two forms: a *tender offer* or an *open-market repurchase* (Kruse et al., 2014; Levy and Shalev, 2017). In a tender offer, the issuer publicly announces the offer, distributes an offer to purchase to all holders of the issue, and pays the stated consideration to every holder who validly tenders within the offer window (subject to any cap on the amount sought and proration). Because it solicits all holders on uniform terms, the offer falls under Regulation 14E, issued under Section 14(e) of the Securities Exchange Act.[5] Rule 14e-1 requires that the offer remain open for at least 20 business days, and for at least ten business days after any change in the price or the amount sought.[6] The consideration is set in one of two ways. A fixed-price offer states a cash price per \$1,000 of principal at launch. A fixed-spread offer states a spread over the yield of a reference Treasury security and computes the price from that yield on a stated pricing date shortly before expiration, so the dollar price moves with Treasury yields during the offer while the yield spread over the Treasury is fixed; this mirrors how

[5] Regulation 14E applies to tender offers for any security. Convertible debt is additionally treated as an equity security for tender-offer purposes, so tender offers for convertible bonds must also comply with Rule 13e-4, including a Schedule TO filing and SEC review.

[6] Since the SEC's January 2015 no-action relief, an offer for any and all of a non-convertible debt issue may close in as few as five business days, provided it is not combined with a consent solicitation; this extended earlier relief that allowed any-and-all offers for investment-grade debt to close in seven to ten calendar days.

investment-grade bonds trade, and in our verified sample 83% of investment-grade offers use a fixed spread whereas 82% of high-yield offers use a fixed price. Either way, the consideration typically carries a premium over the market price (average 5.55%, median 3.24%, in Mann and Powers 2007), and offers that run the full 20 business days are often paired with consent solicitations that amend or strip restrictive covenants in exchange for a consent fee. Holders participate heavily: among offers in our verified sample with a stated purchase amount, issuers end up repurchasing 84% of the amount they seek on average (median 96%), and 41% of offers are filled in full. An open-market repurchase, by contrast, is not pre-announced: the issuer buys bonds through dealer-intermediated transactions at prevailing market prices, typically in a series of small trades in which the seller does not know that the issuer is the counterparty, a structure chosen to avoid moving the price against the issuer (Levy and Shalev, 2017).

Within the market-price channel, we focus on tender offers for two related reasons. First, they are observable: disclosure of a tender offer is *mandatory*, so the issuer must announce its terms—price, premium, size, and any consent fee—to all holders. Disclosure of open-market repurchases is *voluntary*: no rule requires it, issuers conceal their identity when buying, and any disclosure comes only ex post, if at all. Moreover, because the secondary bond market is illiquid, with an average bond trading only about eight days per year versus 243 for a typical stock (Levy and Shalev, 2017), open-market repurchases are hard to measure at the transaction level.

Second, recorded open-market repurchases are also far smaller. Our verified sample contains 4,347 tender offers retiring $1,078 billion of bonds, against 3,802 open-market repurchases retiring only $136 billion—a ratio of roughly eight to one in dollars despite comparable transaction counts. The gap is just as stark deal by deal: the average tender offer retires $231 million of an issue, the average open-market repurchase $28 million. From 2010 onward, tender offers retire $48.9 billion per year against $4.6 billion.[7] Kruse et al. (2014) likewise restrict attention to tender offers because they are more distinct and larger in magnitude than open-market repurchases.[8] This choice also suits our empirical setting in Section 5.2, which centers on large cash shocks: in an illiquid secondary market, a firm seeking to repurchase a substantial amount of debt in the open market must execute a series of purchases—a program that itself risks being deemed a *creeping* tender offer

[7]Because disclosure is voluntary, these figures may understate the channel; our choice rests less on open-market repurchases being small than on their size being unknowable from public records.

[8]This prominence of tender offers contrasts with equity, where repurchases proceed through the same two channels (Comment and Jarrell, 1991; Stephens and Weisbach, 1998) but roughly 90% of the dollar volume of actual share repurchases over 2004–2010 was executed in the open market (Busch and Obernberger, 2017); announcement-based evidence from earlier periods shows similar proportions (Ikenberry et al., 1995; Jagannathan et al., 2000). The difference reflects the economics of each transaction: equity repurchases are a recurring payout executed gradually in a liquid and anonymous market (Jagannathan et al., 2000), whereas a bond repurchase is a lumpy, issue-level transaction for which the tender offer is the natural mechanism, as creditors hold out when the issuer seeks a large amount (Mao and Tserlukevich, 2015) and shareholders reduce debt only episodically (Admati et al., 2018).

subject to the tender-offer rules under Regulation 14E, a constraint that also limits how large a program can remain undisclosed. The TCJA window bears this out: tender offer volume there runs well above its level outside, while open-market volume runs below, so the channel we study is the one that responds to the shock; Section 5.2 reports the figures.

Our interest is in the connection between market-price repurchases and corporate liquidity.[9] Practitioners describe bond buybacks as a use of excess cash, and in particular as a way to eliminate the negative carry of low-yielding cash held against higher-coupon liabilities, and firms describe their own repurchases in the same terms.[10] More broadly, a firm that accumulates cash beyond its needs eventually returns it to investors or pays down its debt (Opler et al., 1999). These accounts motivate our analysis of market-price bond repurchases as a margin of firms' cash-management decisions.

2.2 An Illustrative Two-Period Model

A static model with one source of risk, one debt claim, and one decision delivers the two results that organize Section 3: inside cash carries a liquidity premium, because it spares the firm a costly liquidation, and yet using that cash to retire debt at the market price dominates hoarding it. We keep the notation close to the continuous-time model throughout.

Setting. There are two dates, $t = 0$ and $t = 1$, and no discounting. The firm has debt with face value $F > 0$ due at date 1. The firm owns an existing asset that produces cash $\tilde{W} \geq 0$ at date 1, a random variable with c.d.f. Ψ and density ψ, and that is worth a further $A > 0$ to whoever holds it if the firm survives. The firm repays F from $\tilde{W}$; if $\tilde{W} < F$, it can cover the shortfall by liquidating the existing asset at a fire-sale discount, raising one dollar of cash for every $\theta > 1$ dollars of asset value sold. No covenant prevents the firm from paying dividends before servicing its debt, so the equity holders always retain the option to distribute $\tilde{W}$ to themselves and default, in which case creditors recover nothing, as in the continuous-time model. We assume $A/\theta < F < A$: the going-concern value exceeds the face value of debt, so that a firm with enough cash prefers to continue, but not by so much that liquidation alone could always avert default.

The dividend option favors hoarding against buybacks: after a bad realization the equity holders can pay themselves at date 1 and then default, so inside cash is never

[9] Throughout, "liquidity" means a firm's *corporate* liquidity—its cash holdings—rather than the *secondary-market* liquidity of its bonds (Bao et al., 2011; He and Xiong, 2012; He and Milbradt, 2014), although the two are linked: Xu et al. (2022) show that firms repurchase debt when bond-market liquidity deteriorates.

[10] Health Management Associates, in its 2008 Form 10-K, reported that it "used cash on hand to repurchase \$100.5 million of principal face amount . . . Notes in the open market at approximately 49.1% of their principal face value" (quoted in Levy and Shalev 2017).

trapped in the firm. It thereby makes inside cash strictly more valuable than outside cash,[11] and absent this option a buyback could be motivated simply by the wish to transfer value from creditors (Mao and Tserlukevich, 2015). Our aim is to show that buybacks at the equilibrium price arise even under an assumption that works against them.

The objects map directly onto the continuous-time model. Date-1 cash $\tilde{W}$ plays the role of the cash balance W_t that the firm accumulates from operating income, and F the role of the face amount F_t. The value of the existing asset, A, is the going-concern value that the termination shock destroys in continuous time. The parameter θ indexes the liquidity friction: at $\theta = 1$ assets can be sold at full value and there is no liquidity problem, which is the benchmark of DeMarzo and He (2021); as $\theta \to \infty$ the firm forfeits the entire going-concern value whenever it cannot repay from cash, the counterpart of liquidity-driven bankruptcy at $W_t = 0$. Finally, because $\tilde{W}$ has full support, in low realizations the firm defaults outright, the equity holders keep only their cash, and A is lost regardless of any marginal decision. This sudden loss of going-concern value is the static analogue of the Poisson termination shock, after which, by Proposition 1, the firm likewise distributes its cash and liquidates.

Equity values and debt prices. We start backwards from date 1 and fix the realized cash $\tilde{W} = W$. If $W \geq F$, the firm repays from cash and keeps the surplus together with the existing asset, which is worth $W - F + A > W$ to the equity holders, so they continue. If $W < F$, continuing means liquidating $\theta(F - W)$ of the asset to cover the shortfall, which leaves the equity holders $\theta(W - F) + A$, whereas defaulting means paying out W and walking away. Continuing is preferred if and only if $\theta(W - F) + A \geq W$, that is, $W \geq W_d$, where

$$W_d \equiv \frac{\theta F - A}{\theta - 1} \in (0, F) \tag{1}$$

is the default threshold; the two inequalities follow from $A/\theta < F < A$. Since $W_d > F - A/\theta$, the cash level below which liquidation could not cover the shortfall, the equity holders default before liquidation becomes infeasible: the fire-sale cost of continuing

[11] The analysis of buybacks with outside cash below implicitly assumes that a dollar of outside cash is gone by date 1. If instead the firm could choose when to inject it, and so avoid the liquidity cost, inside and outside cash would be equivalent.

already exceeds what they could keep. The equity payoff is therefore

$$V^1(W;F) = \begin{cases} W, & W < W_d, \\ \theta(W-F)+A, & W_d \le W < F, \\ W-F+A, & W \ge F, \end{cases} \tag{2}$$

which is continuous and piecewise linear in W, with slope one in the default region, where the equity holders lose the asset but keep their cash, $\theta > 1$ in the partial-liquidation region, where each dollar of cash spares the firm θ dollars of asset value, and one above it. Denote the probabilities of default, partial liquidation, and survival by (note that $d + \ell + s = 1$)

$$d \equiv \Psi(W_d), \qquad \ell \equiv \Psi(F) - \Psi(W_d), \qquad s \equiv 1 - \Psi(F). \tag{3}$$

Now move back to date 0. With no discounting, each claim is worth its expected date-1 payoff. Creditors are repaid in full if and only if $W \ge W_d$, so the debt price per unit of face is

$$p(F) = 1 - \Psi(W_d) = \ell + s < 1, \tag{4}$$

and equity is worth $V^0(F) = \mathbb{E}[V^1(\tilde{W};F)]$. Because V^1 is continuous at both thresholds, their dependence on F contributes nothing at the margin, and since the default payoff W does not depend on F at all,

$$-V^0_F = \theta\ell + s. \tag{5}$$

Retiring a unit of face is worth θ to the equity holders in the partial-liquidation states, one in the no-liquidation states, and nothing in default, where they walk away from the debt regardless of its size.

Compare this with the continuous-time model, where the equity value is a function $V(F,W)$ of two state variables that evolve endogenously, and $V_W(F,W)$ captures the liquidity premium on cash.[12] Just as in (2), a dollar of cash is worth more than one only when the firm is solvent but would otherwise have to liquidate part of its asset, and exactly one when it can repay outright or when it defaults and the dollar is simply paid out at par.

[12]One difference in notation: at date 0 the two-period firm holds no cash, so $V^0(F)$ depends on F alone and the marginal value of cash appears only as the slope of V^1; in continuous time the firm always carries a cash balance, so W is a state variable and V_W is defined at every instant.

Buyback with outside cash. To warm up, suppose first that at date 0 the equity holders repurchase Δ units of face with cash from outside the firm. Lenders price the debt at its reduced face, $p(F-\Delta)$, so equity is worth $V^0(F-\Delta) - \Delta \cdot p(F-\Delta)$ and

$$\frac{\partial}{\partial \Delta}\Big[V^0(F-\Delta) - \Delta \cdot p(F-\Delta)\Big]_{\Delta=0} = -V^0_F - p(F) = (\theta - 1)\,\ell \geq 0. \tag{6}$$

The gain is strictly positive whenever $\theta > 1$ and vanishes at $\theta = 1$.

Absent a liquidity friction, we are back to the classic setting of Leland (1994), where there is no difference between inside and outside cash, and retiring debt at its market price is a zero-sum transfer between the firm's claimants, as in DeMarzo and He (2021). What makes a buyback worthwhile here is $\theta > 1$: buyback alleviates the fire-sale cost in the partial-liquidation states. The opposite corner, $\theta \to \infty$, is the one our continuous-time model adopts: the firm has no liquidation value, and default destroys the entire going-concern value the moment cash runs out. The gain survives that limit: W_d rises to F, the partial-liquidation region in (3) collapses, ℓ converges to $(A-F)\psi(F)/\theta$, and (6) converges to $(A-F)\psi(F)$, the density of cash at the default boundary times the value that default destroys.

Inside cash: hoard or buyback? Now suppose the cash is already inside the firm, an amount w on the date-0 balance sheet, so that the equity holders may instead wait and deploy it at date 1. This is the decision at the heart of the continuous-time model.

Consider first hoarding the cash until date 1. Inside cash adds to date-1 cash, and its use is already encoded in (2): with $\tilde{W} = W$ realized, equity is worth $V^1(W + w; F)$. For infinitesimal w, the marginal value of a dollar of inside cash at date 0 is therefore the expected slope of V^1,

$$\left.\frac{\partial V^{0,\text{hoard}}}{\partial w}\right|_{w=0} = d \cdot 1 + \ell \cdot \theta + s \cdot 1 = 1 + (\theta - 1)\,\ell \geq 1. \tag{7}$$

A dollar inside the firm is worth its face value plus a liquidity premium $(\theta-1)\ell$, the fire-sale cost it spares in the partial-liquidation states. This is the static counterpart of $V_W(F_t, W_t) > 1$ in Corollary 3.1. The first term in (7) makes the role of default explicit: in the default states the cash is paid out before defaulting, so the dollar is worth exactly one and its premium is lost, precisely as at the termination shock in continuous time, where by Proposition 1 the firm distributes its cash at par.

Consider instead using the cash to repurchase debt at date 0. A dollar buys $1/p$ units of face at the price $p = \ell + s$, each worth $-V^0_F = \theta\ell + s$ to the equity holders, so the marginal value of a dollar spent on buyback is $(\theta\ell + s)/(\ell + s)$. Comparing with (7) and

Table I. Inside Cash: Hoard or Buyback? Gain, state by state, from retiring one unit of face with $p = \ell + s$ dollars of inside cash rather than hoarding them. V_W is the slope of V^1 in (2) and $\mathbb{1}$ indicates repayment.

Date-1 state	Prob.	Cash value V_W	Face repaid $\mathbb{1}$	Par part $\mathbb{1} - p$	Premium part $(V_W - 1)(\mathbb{1} - p)$
Default	d	1	0	$-p$	0
Partial liquidation	ℓ	θ	1	$1-p$	$(\theta-1)(1-p)$
Survival	s	1	1	$1-p$	0
Expected value				0	$(\theta-1)\,\ell\,d$

using $1-(\ell+s) = d$, we know for $\theta > 1$,

$$\underbrace{\frac{\theta\ell + s}{\ell + s}}_{\text{buyback}} - \underbrace{\left[1 + (\theta - 1)\,\ell\right]}_{\text{hoard}} = \frac{d}{1-d} \cdot \ell(\theta - 1) > 0 \tag{8}$$

Using inside cash to repurchase debt strictly dominates hoarding it, and the wedge is proportional to the default probability d.

To see why, note that a buyback is a trade between two parties who value the same dollar differently across states. In each date-1 state, let V_W denote the marginal value of a dollar of cash to the equity holders, the slope of V^1 in (2), and let $\mathbb{1}$ indicate whether the debt is repaid. Hoarding p dollars is worth $V_W\,p$ to them, and retiring one unit of face with those dollars is worth $V_W\mathbb{1}$, so the gain from the buyback is $V_W(\mathbb{1} - p)$. Table I decomposes this gain into a par part, $\mathbb{1} - p$, and a premium part, $(V_W - 1)(\mathbb{1} - p)$.

The par part is a fair bet. Creditors price the unit of face at $p = \ell + s$, the probability that it is repaid, so the equity holders give up p dollars at par in the default states and receive $1 - p = d$ at par in every state in which the firm is repaid; these transfers cancel in expectation. The selling creditors are thus paid in the default states in exchange for giving up payment in the repaid states, on actuarially fair terms. This fair bet is all a buyback is in the setting of Leland (1994), where the equity holders can inject outside cash freely, so that a dollar is worth par to them in every state and retiring debt at its market price is zero-sum. Neither the default state nor the survival state creates value on its own: in both, a dollar is worth par to the equity holders, exactly as it is to the creditors, so the trade merely reshuffles par-valued dollars. Value is created only in the partial-liquidation states, where the unit of face the equity holders keep is worth θ rather than one: the buyback places the fair bet in the states where cash carries a premium, moving resources from states where cash is worth par to states where it is worth more. Both ingredients are needed. Without default, $d = 0$, the price is par, $\mathbb{1} - p$ vanishes in every state, and there is no trade; without the liquidity premium, $\theta = 1$, the premium part vanishes and what remains is a fair bet. Per unit of face retired, the wedge is $d \cdot \ell(\theta - 1)$, the default probability times the liquidity premium, an observation that carries over to the continuous-time model.

At $\theta = 1$ both sides of (8) equal one and the firm is indifferent, as in DeMarzo and He (2021). The $\theta \to \infty$ limit carries over to inside cash as well: the wedge stays finite there, converging to $d\,(A - F)\,\psi(F)/s$, so repurchasing continues to dominate hoarding even when liquidation raises nothing.

The static model thus isolates why a cash-holding firm under a liquidity friction retires debt at the market price rather than hoarding, and exhibits the two ingredients that reappear in continuous time: a liquidity premium on internal cash and a source of sudden default that erases it. What it cannot deliver is the threshold structure of Section 3.4, in which the firm issues debt when cash is scarce and repurchases only when cash is abundant. That requires the dynamic elements the static model omits: the option to build a cash buffer gradually, the tax shield that rewards issuance, and the carry cost of holding cash against a positive discount rate.

3 The Model

This section develops the continuous-time counterpart of the two-period model of Section 2.2. Section 3.1 sets up the environment and Section 3.2 defines the equilibrium; Section 3.3 solves for equity value and the dividend boundary, Section 3.4 derives the debt adjustment rate and the buyback region, and Section 3.5 reports comparative statics.

3.1 The Continuous-Time Setting

Time is continuous, with $t \in [0, \infty)$. A firm is owned and managed by its equity holders, and is financed with a homogeneous class of debt borrowed from competitive lenders. All agents are risk neutral and discount future payoffs at the constant rate $\rho > 0$.

Operating income and termination risk. Let Z_t be a standard Brownian motion defined on a filtered complete probability space $(\Omega, \mathcal{F} = \{\mathcal{F}_t\}, \mathbb{P})$. Let N_t be an independent Poisson process with constant intensity $\lambda \geq 0$. Denote by $T \equiv \inf\{t \geq 0 : N_t = 1\}$ the arrival time of an earnings-termination shock that permanently destroys the firm's ability to generate operating income. The firm's cumulative operating income Y_t evolves according to

$$dY_t = (\mu dt + \sigma dZ_t)\,\mathbf{1}_{\{t<T\}}, \tag{9}$$

where μ and σ are positive constants. Before time T, the cumulative operating income process is an arithmetic Brownian motion. At time T, the firm permanently loses its capacity to generate operating income, and $dY_t = 0$ for all $t \geq T$. The termination shock is a one-time absorbing event with hazard rate λ, which all agents take into account when valuing the firm's claims.

Debt. The firm's debt is modeled as a consol with constant coupon rate $c > 0$. Let F_t denote the total face amount of debt outstanding at time t, so the required coupon payment over dt is $cF_t dt$. Following DeMarzo and He (2021), the firm adjusts its debt level through an endogenous issuance and buyback policy $d\Gamma_t$, where Γ_t represents the cumulative net debt issuance. Thus, the firm's debt outstanding F_t follows

$$dF_t = d\Gamma_t. \tag{10}$$

An increment $d\Gamma_t > 0$ denotes debt issuance, whereas $d\Gamma_t < 0$ denotes debt buyback. The firm cannot commit to a future debt policy, and competitive lenders price the firm's debt in anticipation of its equilibrium debt policy path.

Cash, taxes, and bankruptcy. We assume in the baseline that the firm cannot raise external equity, so that operating cash flow and debt issuance are its only sources of funds. This corresponds to a special case of the costly equity issuance problem in Section 4 where the issuance cost becomes prohibitively high. We adopt this restriction to remove equity issuance as a liquidity-management tool and focus instead on how the cash-holding trade-off shapes the firm's debt buyback decisions. Section 4 relaxes the restriction and shows that the debt buyback mechanism remains intact.

The firm pays corporate taxes at rate $\pi \in (0, 1)$. Coupon payments are tax-deductible and losses are fully offset. Cash held inside the firm earns no return.[13] Let W_t denote the firm's cash position at time t, then its dynamics are given by

$$dW_t = (1 - \pi) \left(\underbrace{dY_t}_{\substack{\text{operating}\\ \text{income}}} - \underbrace{cF_t dt}_{\substack{\text{coupon}\\ \text{payment}}} \right) + \underbrace{p_t d\Gamma_t}_{\substack{\text{issuance}\\ \text{proceeds}}} - \underbrace{dD_t}_{\substack{\text{dividend}\\ \text{payment}}}, \tag{11}$$

where p_t is the equilibrium debt price and D_t denotes the cumulative dividends paid out by the firm to its equity holders. The firm is forced into bankruptcy the moment it exhausts its cash, and we refer to such bankruptcy as liquidity-driven bankruptcy. Let $\tau \equiv \inf\{t \geq 0 : W_t = 0\}$ denote the time at which liquidity-driven bankruptcy occurs. Upon bankruptcy, the firm has zero recovery value and the equity holders forfeit their claims to future dividends. The prospect of this loss gives the equity holders a precautionary motive to conserve cash within the firm even though holding it may be

[13]Nothing hinges on the zero return on internal cash specifically. What matters is that the return on internal cash is strictly below ρ so that carrying cash is costly to the equity holders. In **??**, we solve a model in which internal cash earns return $r < \rho$ and show that the key economics remain unchanged.

costly.

Remark 1 (Interpreting debt adjustment)**.** *In our model, the debt adjustment policy $d\Gamma_t$ admits a specific institutional interpretation. At each time t, the firm announces the amount of debt it will issue or buyback and commits to it over the short interval from time t to $t + dt$. While the firm cannot commit to future debt policy beyond the instant, the current adjustment is public and lenders price the debt accordingly. When $d\Gamma_t > 0$, the firm issues debt: it announces an offering of a given size, and the price at which the debt is issued is set by lenders' valuation. This is how corporate bonds are issued in practice. When $d\Gamma_t < 0$, the firm repurchases debt: it announces an offer to repurchase its debt to all debt holders, whose valuation determines the price schedule the firm faces. This maps closely to a tender offer in practice. An open-market repurchase, by contrast, has a different structure—it is not pre-announced and the firm conceals its identity from the seller (see Section 2). We therefore interpret the model's debt adjustment as an issuance when $d\Gamma_t > 0$ and as a tender offer when $d\Gamma_t < 0$, the latter being the object of interest in our empirical analysis in Section 5.2.*

Remark 2 (Setting price versus setting quantity)**.** *In the model, the firm chooses the quantity of debt to issue or buyback while moving along the equilibrium price schedule. That matches practice for issuance. In a tender offer, however, the firm posts a price and the debt holders choose the quantity to surrender. Appendix C.6 shows that the distinction is immaterial here. The equilibrium map from quantity to price is strictly monotone and hence invertible, so optimizing over price yields the same first- and second-order conditions as optimizing over quantity. We use the quantity formulation throughout.*

3.2 Equilibrium

The equilibrium concept we focus on is that of Markov perfect equilibrium. The payoff-relevant state consists of the firm's face amount of debt outstanding F_t, its cash position W_t, and whether the termination shock has occurred. Let $V(F_t, W_t)$ and $\hat{V}(F_t, W_t)$ denote the equity holders' value before and after the termination shock, respectively. Then

$$V(F_t, W_t) = \max_{\{\Gamma_s, D_s\}} \mathbb{E}_t \left[\int_t^{\tau \wedge T} e^{-\rho(s-t)} dD_s + e^{-\rho(T-t)} \mathbf{1}_{\{T<\tau\}} \hat{V}(F_T, W_T) \right]. \tag{12}$$

The equity holders derive value from the dividends they receive. Acting on behalf of the firm, they choose the optimal debt adjustment and dividend payout policies $\{\Gamma_s, D_s\}_{s \geq t}$ to maximize their own value, subject to the cash dynamics in (11) and the equilibrium debt price schedule. Let $p(F_t, W_t)$ and $\hat{p}(F_t, W_t)$ denote the corresponding debt prices.

Before the termination shock,

$$p(F_t, W_t) = \mathbb{E}_t \left[\int_t^{\tau \wedge T} e^{-\rho(s-t)} c ds + e^{-\rho(T-t)} \mathbf{1}_{\{T<\tau\}} \hat{p}(F_T, W_T) \right], \tag{13}$$

which is the value lenders place on the coupon stream up to the liquidity-driven bankruptcy or the termination shock, whichever comes first, plus the post-termination value of the claim. The expectation is taken under the firm's equilibrium policy.

Equilibrium definition. An equilibrium consists of value and price functions $(V, \hat{V}, p, \hat{p})$ and Markov policies $\{\Gamma_t, D_t\}$ such that, in both the pre-termination and the post-termination regimes: (i) the policies maximize the equity holders' value given the corresponding debt price schedule, recognizing that debt adjustment moves the price along that schedule, and (ii) the debt price equals lenders' expected discounted payoff under the induced policies. The equilibrium is therefore a fixed point: the price at which the firm can trade its debt today reflects lenders' forecast of how the firm will behave in the future, and that behavior is itself optimal given the price.

Post-termination values. Because the termination shock is absorbing, the post-termination regime can be solved first and taken as a terminal condition for the pre-termination problem in (12)-(13), thereby allowing the remainder of the analysis to focus exclusively on the pre-termination regime.

Proposition 1 (Liquidation at termination shock)**.** *Upon the termination shock at time T, the firm distributes its entire cash balance and liquidates. For every realized state (F_T, W_T),*

$$\hat{V}(F_T, W_T) = W_T, \qquad \hat{p}(F_T, W_T) = 0. \tag{14}$$

Upon the termination shock at time T, the equity holders are left owning a firm that no longer generates any operating income. Their choice is whether to keep the firm alive for a little longer or wind it up immediately and take the cash. Keeping it alive is costly, as internal cash earns no return while coupon obligations continue to drain the cash balance. Nor can the equity holders borrow their way out of this, because lenders anticipate the firm's debt issuance and liquidation decisions and price the new debt accordingly. In equilibrium, debt issuance cannot generate gain from trade post-termination and the equity holders do not benefit from debt issuance. Consequently, with nothing to wait for and no way to create value by waiting, the equity holders distribute the cash and liquidate as soon as the termination shock occurs.

It is worth noting that the termination risk is not equivalent to a higher discount rate. A higher discount rate reduces the present value of all future payoffs. The termination shock instead destroys the firm's going-concern value—and with it the debt claim—while leaving its cash intact. Before termination, a marginal dollar of internal cash can be worth more than a dollar in the equity holders' pocket because it insures against liquidity-driven bankruptcy. Upon termination, this liquidity premium evaporates and the dollar is paid out at par. Section 3.4 shows that the expected loss of this liquidity premium creates an additional cost of carrying cash inside the firm.

Smooth debt adjustment. We henceforth restrict our attention to the pre-termination regime, $t < T$. Under certain conditions, the firm's optimal debt issuance or buyback policy is smooth over time, in the sense that discrete, lumpy adjustments are suboptimal. The lemma below provides a sufficient condition for smooth debt adjustment.

Lemma 2 (Optimality of smooth debt adjustment)**.** *Suppose that the equity holders' value function $V(F_t, W_t)$ satisfies*

$$V_{FF}(F_t, W_t) - 2V_{FW}(F_t, W_t)\frac{V_F(F_t, W_t)}{V_W(F_t, W_t)} + V_{WW}(F_t, W_t)\left(\frac{V_F(F_t, W_t)}{V_W(F_t, W_t)}\right)^2 > 0, \qquad (15)$$

for all $W_t > 0$. Then the firm's debt issuance and buyback policy is continuous in t, and along the equilibrium path,

$$p(F_t, W_t) = -\frac{V_F(F_t, W_t)}{V_W(F_t, W_t)}. \qquad (16)$$

The condition (15) is stated for all $W_t > 0$. Note that it is a verification condition on the endogenous value function rather than a restriction on the model's primitives. We proceed by using (16) to construct a candidate equilibrium and then verify (15) on the resulting solution in Section 3.3. (16) has a clean economic interpretation. Along the equilibrium path, the debt price equals the equity holders' marginal rate of substitution between debt and cash. Issuing additional debt increases the firm's leverage, but also strengthens its cash buffer. In equilibrium, the debt price balances these two marginal effects, leaving equity holders indifferent at the margin. With (16), we can also rewrite the condition (15) more intuitively as

$$\underbrace{V_W(F_t, W_t)}_{\text{marginal value of liquidity}} \cdot \underbrace{[p_F(F_t, W_t) + p_W(F_t, W_t)p(F_t, W_t)]}_{\text{price impact of marginal debt issuance}} < 0. \qquad (17)$$

The bracketed term is the price impact of marginal debt issuance. It includes both the direct effect of a higher face amount and the indirect effect of the cash raised by the issuance. Multiplying by V_W converts this price change into equity-value units.

Under the condition of Lemma 2, the firm adjusts its outstanding debt continuously, with changes of order dt. More specifically, the firm adjusts its debt level at the endogenous rate G_t, such that

$$d\Gamma_t = G_t dt. \tag{18}$$

The debt adjustment rate G_t may be positive or negative, with $G_t > 0$ denoting issuance and $G_t < 0$ denoting buyback.

3.3 Equity Value and the Dividend Policy

Liquidity-driven bankruptcy occurs when the firm's cash balance is depleted. The optimal dividend policy reflects a trade-off between distributing cash to the equity holders and preserving liquidity inside the firm. Paying dividends provides an immediate benefit to the equity holders, but it also reduces the firm's cash position and moves the firm closer to liquidity-driven bankruptcy. Because bankruptcy destroys equity value, the firm hoards cash when it is scarce and distributes it when it is plentiful. The firm's solvency region thus splits into a *dividend suspension region* $0 < W_t < W^\star(F_t)$ and a *dividend payout region* $W_t \geq W^\star(F_t)$, separated by an endogenous payout boundary $W^\star(F_t)$.

Dividends and the payout boundary. For a given debt level F_t, the firm pays out any excess cash whenever its cash position exceeds the endogenous dividend payout boundary. In this case, it is optimal for the firm to distribute the excess cash as a lump-sum dividend, bringing the firm's cash balance back to the dividend payout boundary $W^\star(F_t)$. Hence, the equity holders' value function in the payout region is given by

$$V(F_t, W_t) = V(F_t, W^\star(F_t)) + [W_t - W^\star(F_t)], \qquad \text{for } W_t \geq W^\star(F_t). \tag{19}$$

The first term on the right-hand side is the equity holders' value at the dividend payout boundary, while the second term is the excess cash distributed to the equity holders. Because equity holders are homogeneous, this distribution is equivalent to an equity buyback, and we read it as such when we return to the empirics in Section 5.2. Taking the limit as $W_t \to W^\star(F_t)$, we have the following smooth-pasting condition

$$V_W(F_t, W^\star(F_t)) = 1. \tag{20}$$

Intuitively, V_W represents the marginal value to the equity holders from an additional dollar held within the firm. It can be interpreted as the shadow price of liquidity, reflecting how much equity holders' value would increase if the firm had one more dollar of cash reserve. The condition (20) says that at the dividend payout boundary, the firm is just indifferent between distributing and retaining a dollar. Moreover, to help pin down the dividend payout boundary $W^\star(F_t)$, we also rely on the super-contact condition

$$V_{WW}(F_t, W^\star(F_t)) = 0. \tag{21}$$

Equity holders' value. The equity holders' value in the dividend payout region is given by (19). In the dividend suspension region, the equity holders receive no flow payment from the firm. With smooth debt adjustment, the dynamics for the firm's cash position (11) can then be written, on $\{t < T\}$, as

$$dW_t = \big[(1-\pi)(\mu - cF_t) + p_t G_t\big]dt + (1-\pi)\sigma dZ_t. \tag{22}$$

Pre-termination, the equity holders' value function $V(F_t, W_t)$ can be written recursively through the Hamilton-Jacobi-Bellman (HJB) equation

$$\begin{aligned}\rho V(F_t, W_t) = \max_{G_t} \Big\{ & G_t V_F(F_t, W_t) + \big[(1-\pi)(\mu - cF_t) + p_t G_t\big] V_W(F_t, W_t) \\ & + \frac{1}{2}(1-\pi)^2\sigma^2 V_{WW}(F_t, W_t) + \lambda\big[\hat{V}(F_t, W_t) - V(F_t, W_t)\big]\Big\},\end{aligned} \tag{23}$$

for $0 < W_t < W^\star(F_t)$. The left-hand side represents the required return by the equity holders. The first three terms on the right-hand side give the maximized expected change in value conditional on no termination shock, whereas the last term captures the expected change in value due to the termination shock. Debt issuance or buyback affects the equity holders' value both directly by changing the face amount of debt outstanding and indirectly by affecting the firm's cash balance through the proceeds from issuance or the cash used for buybacks.

The jump term in (23) does not involve the debt adjustment rate G_t. The first-order condition with respect to G_t is $V_F(F_t, W_t) + p_t V_W(F_t, W_t) = 0$, which is precisely (16). Substituting the first-order condition into the equity holders' HJB equation (23) cancels both G_t terms, and substituting $\hat{V}(F_t, W_t) = W_t$ from Proposition 1 leaves

$$(\rho + \lambda)V(F_t, W_t) = (1-\pi)(\mu - cF_t)V_W(F_t, W_t) + \frac{1}{2}(1-\pi)^2\sigma^2 V_{WW}(F_t, W_t) + \lambda W_t. \tag{24}$$

In equilibrium, the equity holders are indifferent between adjusting the firm's debt and leaving it alone, so their value function is the same as if debt level were fixed. Hence, although the state space is two-dimensional, (24) is an ordinary differential equation (ODE) in W_t while F_t can be treated as a parameter.[14] This gives us tractability in

[14]This is where the arithmetic specification in (9) is very helpful. The dynamic capital structure literature often models the operating income rate to follow a geometric Brownian motion, which introduces the income rate as an additional state variable but scale invariance reduces the state space to a two-dimensional one. What the arithmetic specification buys in this model is not fewer states but a

solving the model analytically.

For a given level of debt outstanding F_t, the general solution of (24) contains two constants and the unknown boundary $W^\star(F_t)$, which are pinned down by (20) and (21) at the dividend payout boundary $W_t = W^\star(F_t)$, together with the value-matching condition $V(F_t, 0) = 0$ at the liquidity-driven bankruptcy boundary.

Proposition 3 (Equity holders' value)**.** *Let*

$$\gamma(F_t) \equiv -\frac{(\mu - cF_t) + \sqrt{(\mu - cF_t)^2 + 2(\rho+\lambda)\sigma^2}}{(1-\pi)\sigma^2} < 0, \tag{25}$$

$$\eta(F_t) \equiv -\frac{(\mu - cF_t) - \sqrt{(\mu - cF_t)^2 + 2(\rho+\lambda)\sigma^2}}{(1-\pi)\sigma^2} > 0, \tag{26}$$

and define

$$\mathcal{K}(F_t) \equiv \frac{\rho\,\eta(F_t)}{(\rho+\lambda)\gamma(F_t)\big[\eta(F_t) - \gamma(F_t)\big]} < 0, \tag{27}$$

$$\mathcal{H}(F_t) \equiv -\frac{\rho\,\gamma(F_t)}{(\rho+\lambda)\eta(F_t)\big[\eta(F_t) - \gamma(F_t)\big]} > 0. \tag{28}$$

For a given debt level F_t, a dividend payout boundary $W^\star(F_t) > 0$ exists if and only if $F_t < \mu/c$, in which case it is the unique positive solution of

$$\eta(F_t)^2 e^{-\gamma(F_t)W^\star(F_t)} - \gamma(F_t)^2 e^{-\eta(F_t)W^\star(F_t)} = \frac{4\lambda\,(\mu - cF_t)\,\sqrt{(\mu - cF_t)^2 + 2(\rho+\lambda)\sigma^2}}{\rho\,(1-\pi)^2\sigma^4}. \tag{29}$$

The equity holders' value is

$$\begin{aligned} V(F_t, W_t) =& \frac{\lambda}{\rho+\lambda}\left[W_t + \frac{1-\pi}{\rho+\lambda}(\mu - cF_t)\right] \\ &+ \mathcal{K}(F_t)e^{\gamma(F_t)[W_t - W^\star(F_t)]} + \mathcal{H}(F_t)e^{\eta(F_t)[W_t - W^\star(F_t)]}, \end{aligned} \tag{30}$$

for $0 < W_t < W^\star(F_t)$, and

$$V(F_t, W_t) = V(F_t, W^\star(F_t)) + [W_t - W^\star(F_t)], \tag{31}$$

for $W_t \geq W^\star(F_t)$ with $V(F_t, W^\star(F_t))$ given by (19) evaluated at $W_t = W^\star(F_t)$.

Several features of Proposition 3 deserve comment. First, the sign of the dividend payout boundary $W^\star(F_t)$ is positive only over a restricted range of debt levels. As shown in the appendix, a positive payout boundary exists if and only if $F_t < \mu/c$. When $F_t \geq \mu/c$, the firm's debt-servicing burden becomes too onerous to be sustained by the expected

decoupled pair when applying the no-trade result of DeMarzo and He (2021): the HJB reduces to a family of ODEs in W_t indexed by F_t, which is what delivers the closed form.

cash flows from its assets-in-place. The equity holders therefore have no incentive to retain liquidity inside the firm: any positive cash balance is paid out immediately, and the firm is liquidated. Consequently, the firm's debt capacity is capped at μ/c.

Second, when $F_t < \mu/c$, (29) has a unique positive solution. Thus, for every sustainable debt level, there is a unique cash balance at which equity holders are indifferent at the margin between retaining and distributing cash. In the limit case without termination risk, that is $\lambda = 0$, the right-hand side of (29) vanishes, and the payout boundary has a closed form $W^\star(F_t) = \frac{2}{\gamma(F_t)-\eta(F_t)} \ln\big(\eta(F_t)/(-\gamma(F_t))\big)$. This expression provides a useful benchmark in which the payout decision reflects only the standard opportunity cost of retaining cash and its insurance value against liquidity-driven bankruptcy.

Third, consider the limit with $\rho \to 0$ while maintaining $\lambda = 0$. The right-hand side of (29) diverges at rate $1/\rho$, whereas the left-hand side remains finite for every finite value of $W^\star(F_t)$. It follows that $\lim_{\rho\to 0} W^\star(F_t) = \infty$ for $F_t < \mu/c$. In words, there is no finite payout boundary, and the firm retains all available cash until either liquidity-driven bankruptcy or the termination shock occurs. Intuitively, zero-yield internal cash is no longer costly relative to an immediate distribution when equity holders do not discount future payoffs. Retaining cash nevertheless continues to provide insurance against liquidity-driven bankruptcy, so distributing it before termination is weakly dominated.

Corollary 3.1 (Marginal value of liquidity). *For $F_t < \mu/c$ and $0 < W_t < W^\star(F_t)$,*

$$V_{WW}(F_t, W_t) = \frac{\rho}{\rho+\lambda} \frac{\gamma(F_t)\eta(F_t)}{\eta(F_t)-\gamma(F_t)} \left[e^{\gamma(F_t)[W_t - W^\star(F_t)]} - e^{\eta(F_t)[W_t - W^\star(F_t)]}\right] < 0. \tag{32}$$

Consequently $V_W(F_t, W_t) > 1$ on $(0, W^\star(F_t))$ with $V_W(F_t, W^\star(F_t)) = 1$. Moreover, $V_F(F_t, W_t) < 0$ for all $W_t \in (0, W^\star(F_t)]$, so the equilibrium debt price $p(F_t, W_t) = -V_F/V_W$ is strictly positive.

Corollary 3.1 formalizes the insurance value of internal liquidity. Within the dividend suspension region, an additional dollar of cash is worth more than its payout value because it protects equity holders against liquidity-driven bankruptcy. The inequality $V_{WW}(F_t, W_t) < 0$ further implies that this insurance value is greater when cash is more scarce.

In Figure II, we present a numerical solution by plotting the equity holders' value function against the relevant state variables. We use the following baseline parameter values: $\mu = 0.044$, $\sigma = 0.60$, $c = 0.0022$, $\rho = 0.03$, and $\pi = 0.05$, and $\lambda = 0.2$. Appendix C.7 verifies the condition (15) in Lemma 2 analytically in the dividend payout region and numerically in the dividend suspension region under the parameter values used in the analysis. Panel A plots the equity holders' value as a function of the firm's debt outstanding F_t, while fixing the cash position at $W_t = 0.5$. Panel B instead plots the equity holders' value as a function of the firm's cash position W_t, while fixing the face

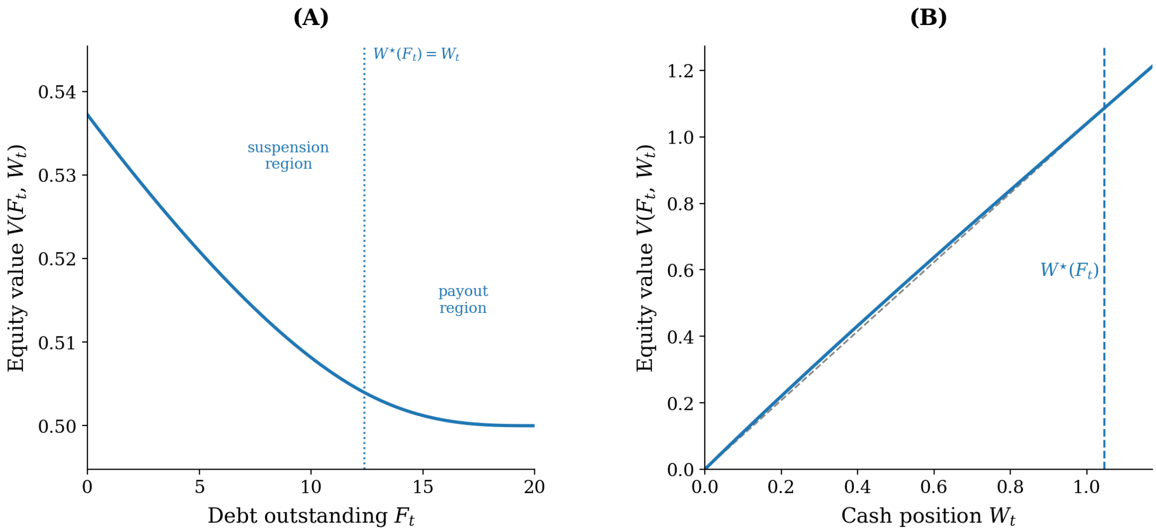

Figure II. Equity holders' value function. The figure plots the equity holders' value function $V(F_t, W_t)$ under the parameters $\mu = 0.044$, $\sigma = 0.60$, $c = 0.0022$, $\rho = 0.03$, and $\pi = 0.05$, and $\lambda = 0.2$. Panel A plots V as a function of the outstanding debt F_t, holding the cash position fixed at $W_t = 0.5$. Panel B plots V as a function of the cash position W_t, holding the face amount of debt fixed at $F_t = 0.5$. The vertical dashed line marks the dividend payout boundary $W^\star(F_t)$.

amount of debt at $F_t = 0.5$.

Panel A shows that the equity holders' value decreases with the firm's debt outstanding F_t. This is because more debt raises the firm's debt-servicing burden, which shifts continuation value to lenders and makes liquidity-driven bankruptcy more likely, thus hurting the equity holders. Panel B shows that the equity holders' value increases with its cash position W_t, consistent with Corollary 3.1. Intuitively, a larger cash buffer lowers bankruptcy risk by allowing the firm to better withstand bad cash-flow states. Moreover, the equity holders' value is concave in W_t in the dividend suspension region, implying that the cash buffer is more valuable when the firm is closer to running out of cash.

We then plot the dividend payout boundary $W^\star(F_t)$ in Figure III. Under the baseline parameters, the dividend payout boundary is decreasing in the amount of debt outstanding F_t. The main force here is a debt overhang effect. Higher debt levels increase debt service, causing more of the continuation value to accrue to lenders. As a result, the equity holders internalize less of the benefit from retaining cash inside the firm to avoid default, and therefore begin paying out cash sooner.

3.4 Debt Issuance and Buyback

The equilibrium debt price is derived using the pricing condition (16) and the equity holders' value function in Proposition 3. In Figure IV, we plot the equilibrium debt price against the relevant state variables using our baseline parameters. Panel A fixes the cash

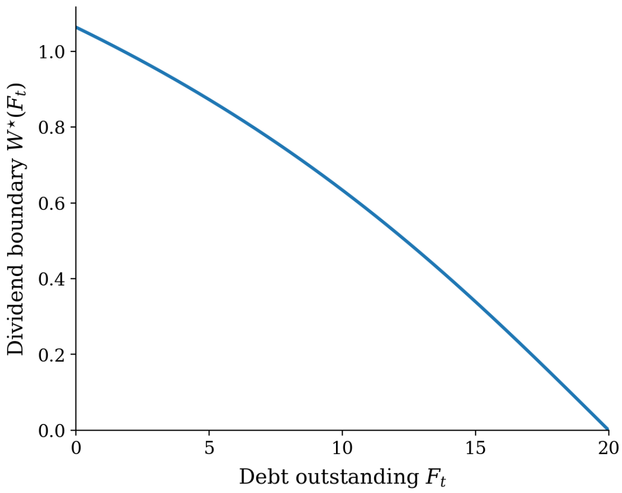

Figure III. Dividend payout boundary. The figure plots the dividend payout boundary $W^\star(F_t)$ as a function of the outstanding debt F_t, under the parameters $\mu = 0.044$, $\sigma = 0.60$, $c = 0.0022$, $\rho = 0.03$, and $\pi = 0.05$, and $\lambda = 0.2$.

position at $W_t = 0.5$ and shows that the debt price decreases in the amount of debt outstanding F_t. Panel B fixes the debt amount at $F_t = 0.5$ and shows that the debt price increases in the firm's cash position W_t throughout the dividend suspension region. Both patterns reflect the same logic: more debt and less cash both bring liquidity-driven bankruptcy closer, thereby shortening the coupon stream lenders expect to receive. In the payout region, however, the debt price remains constant in W_t. Any cash above $W^\star(F_t)$ is paid out to the equity holders, so from a lender's perspective the firm sits at $W^\star(F_t)$ and the debt price does not vary with the cash position.

We are interested in deriving the firm's optimal debt adjustment rate G_t in the dividend suspension region $0 < W_t < W^\star(F_t)$. In the dividend payout region, any cash above the payout boundary $W^\star(F_t)$ is immediately distributed, so the firm sits at the boundary and the relevant margin is the dividend rather than debt adjustment. Within the dividend suspension region, the equity holders' optimality leaves them indifferent over G_t, so their problem alone does not determine it. What pins down G_t is an equilibrium condition: the debt price implied by the equity holders' first-order condition (16) must also be a correct lender valuation, which satisfies the debt-price HJB equation. Using $\hat{p}(F_t, W_t) = 0$ from Proposition 1,

$$\begin{aligned}(\rho+\lambda)p(F_t,W_t) =& c + G_t p_F(F_t,W_t) + \big[(1-\pi)(\mu - cF_t) + p(F_t,W_t)G_t\big]p_W(F_t,W_t) \\ &+ \frac{1}{2}(1-\pi)^2\sigma^2 p_{WW}(F_t,W_t).\end{aligned} \tag{33}$$

Combining (33) with (24) yields the firm's optimal debt issuance and buyback policy, as captured by its debt adjustment rate G_t.

Proposition 4 (Optimal debt policy)**.** *For a given debt level F_t, the firm's optimal debt*

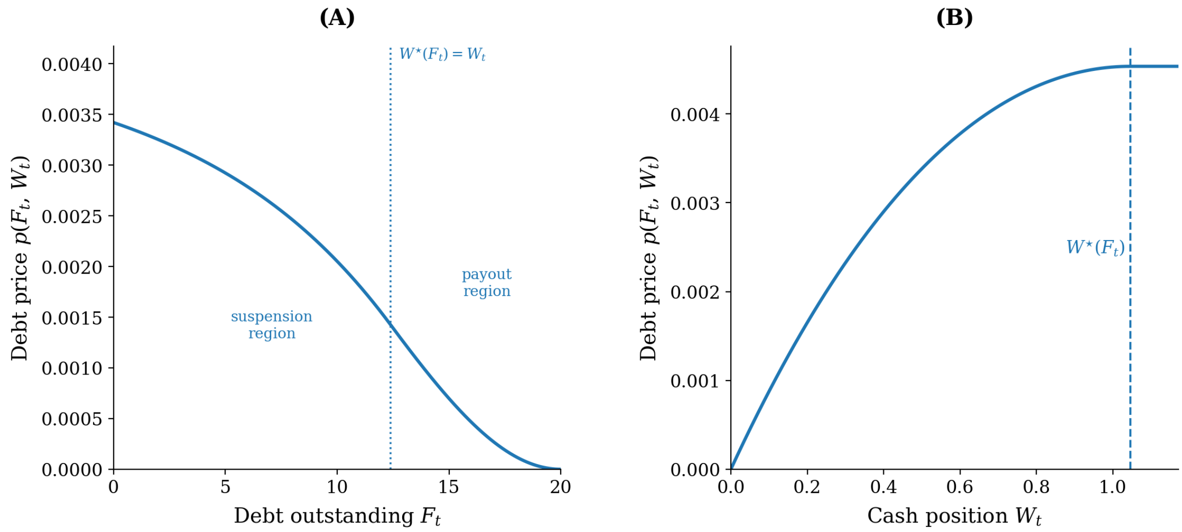

Figure IV. Equilibrium debt price. The figure plots the equilibrium debt price against the state variables, under the parameters $\mu = 0.044$, $\sigma = 0.60$, $c = 0.0022$, $\rho = 0.03$, and $\pi = 0.05$, and $\lambda = 0.2$. Panel A plots the debt price as a function of the outstanding debt F_t, holding the cash position fixed at $W_t = 0.5$. Panel B plots the debt price as a function of the cash position W_t, holding the face amount of debt fixed at $F_t = 0.5$. The vertical dashed line marks the dividend payout boundary $W^\star(F_t)$.

adjustment rate in the dividend suspension region $0 < W_t < W^\star(F_t)$ *is*

$$G_t = \frac{\pi c - \left[\rho + \lambda \dfrac{V_W(F_t, W_t) - 1}{V_W(F_t, W_t)}\right] p(F_t, W_t) + (1-\pi)^2\sigma^2 p_W(F_t, W_t) \dfrac{-V_{WW}(F_t, W_t)}{V_W(F_t, W_t)}}{-\left[p_F(F_t, W_t) + p(F_t, W_t) p_W(F_t, W_t)\right]}. \tag{34}$$

By (17) of Lemma 2 and the fact that $V_W(F_t, W_t) > 0$, the denominator of (34) is strictly positive. It measures the adverse price impact of issuing an additional unit of debt: issuance increases the debt amount F_t directly and also raises the cash balance W_t by the proceeds $p(F_t, W_t)$. Greater price impact causes the firm to adjust its debt more slowly. The direction of debt adjustment is set by the numerator, which can be interpreted as the net flow value of a marginal unit of debt to the equity holders. The numerator can be split into three terms.

Tax benefit. The first term, πc, is the flow tax benefit of carrying an additional unit of debt. Because coupon payments are tax deductible, one more unit of debt with coupon rate c reduces the firm's tax burden by πc per unit of time. This term is always positive, creating a baseline incentive for the equity holders to issue debt in order to capture the debt tax shield.

Cash-carry wedge. The second term,

$$-\left[\underbrace{\rho}_{\text{return shortfall}} + \underbrace{\lambda\frac{V_W(F_t,W_t)-1}{V_W(F_t,W_t)}}_{\text{expected termination loss}}\right]p(F_t,W_t), \tag{35}$$

captures a cash-carry wedge. It has two components. The first component, $-\rho p(F_t, W_t)$, reflects the return shortfall of internal cash. It arises because issuing an additional unit of debt raises the firm's cash position by $p(F_t, W_t)$. However, cash earns zero return inside the firm, while the equity holders require a rate of return $\rho > 0$. The return shortfall may be due to agency frictions or opportunity costs of alternative investments.

The second component, $-\lambda\frac{V_W(F_t,W_t)-1}{V_W(F_t,W_t)}p(F_t,W_t)$, captures the expected loss of liquidity premium upon termination. Recall from Corollary 3.1 that inside the dividend suspension region prior to the termination shock, a marginal dollar of internal cash is worth $V_W(F_t, W_t) > 1$ to the equity holders because it provides insurance against liquidity-driven bankruptcy. At termination, however, Proposition 1 shows that the same dollar is worth exactly one: the firm liquidates and the cash is distributed to the equity holders. Each dollar of cash held inside the firm thus carries an expected loss of $\lambda[V_W(F_t, W_t) - 1]$ in equity-value units per unit of time, or $\lambda[V_W(F_t, W_t) - 1]/V_W(F_t, W_t)$ per unit of time once converted into cash units.

Both components are strictly negative when $\rho > 0$ and $\lambda > 0$. Together, they form the cash-carry wedge, the force that generates an incentive to repurchase debt since the firm sitting on idle cash can reduce the carry cost by retiring liabilities. Although both components are costs of holding internal cash, they arise through distinct channels. The return shortfall component reflects the foregone return on cash, whereas the termination loss component reflects the risk that the termination shock destroys cash's liquidity premium. Taking $\rho \to 0$ eliminates the former, while setting $\lambda = 0$ removes the latter.

Precautionary liquidity. The last term in the numerator reflects the precautionary-liquidity motive for debt issuance, and can be further decomposed as

$$\underbrace{(1-\pi)^2\sigma^2}_{\text{cash flow risk}} \times \underbrace{p_W(F_t,W_t)}_{\text{debt-price sensitivity to cash}} \times \underbrace{\frac{-V_{WW}(F_t,W_t)}{V_W(F_t,W_t)}}_{\text{demand for liquidity insurance}}. \tag{36}$$

The decomposition highlights three forces. The last object, $-V_{WW}(F_t, W_t)/V_W(F_t, W_t)$, is a hedging demand term: formally the Arrow–Pratt curvature of equity value in cash, it measures the equity holders' aversion to low-cash states. It is strictly positive in the dividend suspension region where $V_W(F_t, W_t) > 0$ and $V_{WW}(F_t, W_t) < 0$, and vanishes in the payout region where the equity holders' value is affine in W_t. When this term is large, cash shortfalls are especially costly to equity holders, giving them a stronger precautionary motive to issue debt and build liquidity buffer.

This precautionary motive is scaled by the cash flow risk, $(1-\pi)^2\sigma^2$. Higher cash flow volatility makes liquidity buffer more valuable because the firm is more likely to enter states in which cash is scarce and external financing is costly. With deterministic cash flows the motive disappears entirely. Furthermore, the motive is also scaled by $p_W(F_t, W_t)$, the sensitivity of debt issuance price to the firm's cash position. For $p_W > 0$, debt issuance generates less proceeds precisely when the firm has less cash, that is, financing terms deteriorate in the states where liquidity is most valuable. A large p_W thus makes reliance on future issuance risky, strengthening the precautionary motive to issue debt early and accumulate liquidity while terms are still favorable.

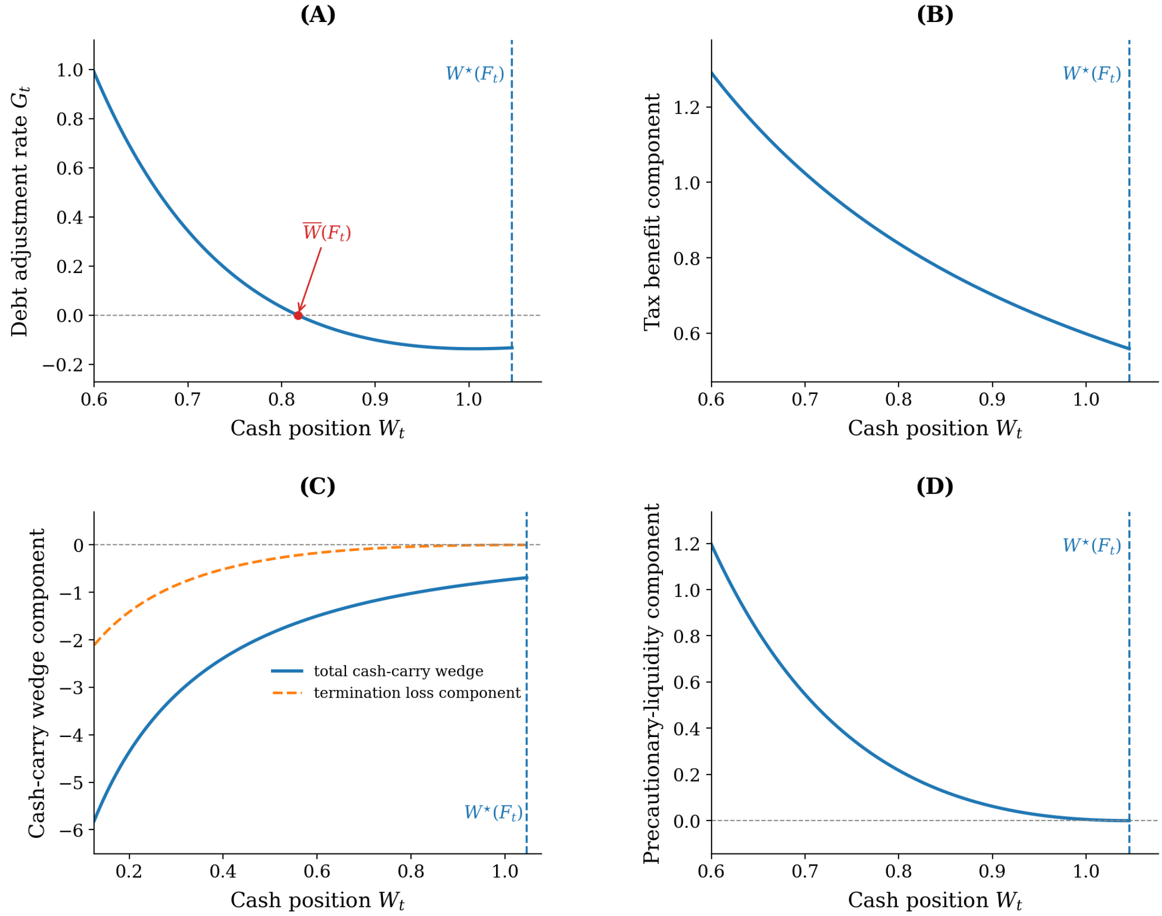

Figure V. Optimal debt adjustment rate. The figure plots the firm's optimal debt adjustment rate G_t, under the parameters $\mu = 0.044$, $\sigma = 0.60$, $c = 0.0022$, $\rho = 0.03$, and $\pi = 0.05$, and $\lambda = 0.2$. Panel A plots G_t as a function of the cash position W_t, holding the outstanding debt fixed at $F_t = 0.5$. Panels B–D plot the three components of G_t corresponding to the three terms in the numerator of equation (34): the tax shield term, the cash-carry-cost term (including the return shortfall component and the termination cost component), and the precautionary-liquidity term. The vertical dashed line marks the dividend payout boundary $W^\star(F_t)$.

Panel A of Figure V plots the firm's optimal debt adjustment rate G_t as a function of its cash position W_t, holding outstanding debt fixed at $F_t = 0.5$. Panels B-D then

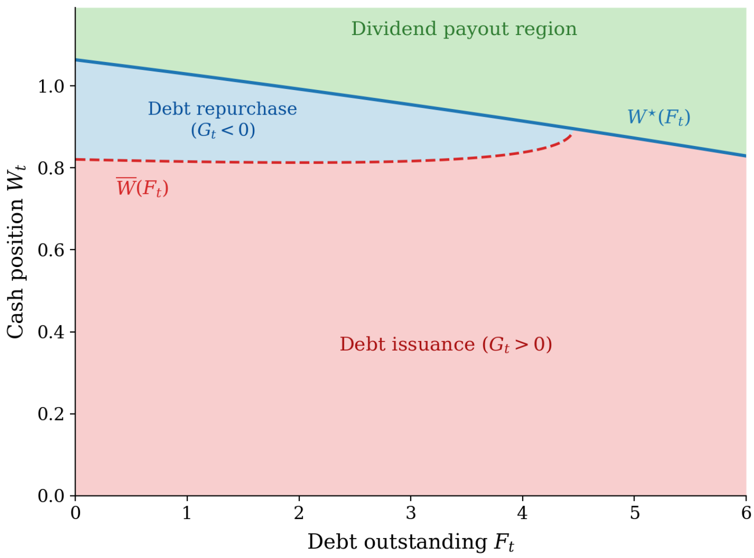

Figure VI. Illustration of state space. The figure illustrates the state space, with the outstanding debt F_t on the horizontal axis and the cash position W_t on the vertical axis, under the parameters $\mu = 0.044$, $\sigma = 0.60$, $c = 0.0022$, $\rho = 0.03$, and $\pi = 0.05$, and $\lambda = 0.2$. Within the dividend suspension region, the locus $\underline{W}(F_t)$ along which $G_t = 0$ separates the debt-buyback region ($G_t < 0$, blue) from the debt-issuance region ($G_t > 0$, red). The dividend payout boundary $W^\star(F_t)$ lies to the right of this locus, and the region beyond it is the dividend payout region (green).

decompose the adjustment rate into three components based on the three terms in the numerator of (34). When cash position is low, the tax and precautionary-liquidity terms dominate and the firm issues debt. As cash accumulates, the cash-carry wedge grows in importance and G_t turns negative. The crossing point is a cash threshold, $\overline{W}(F_t)$, above which the firm repurchases its own debt, which is the theoretical counterpart of the empirical pattern documented in Panel (b) of Figure I.

Figure VI illustrates the state space, with the debt level F_t on the horizontal axis and the cash position W_t on the vertical axis. Within the dividend suspension region, the locus $\overline{W}(F_t)$—along which the debt adjustment rate is $G_t = 0$—divides the states where the firm repurchases its debt ($G_t < 0$, in blue) from those where the firm issues debt ($G_t > 0$, in red). The payout boundary $W^\star(F_t)$ lies to its right, and beyond the payout boundary is the dividend payout region (in green).

The firm repurchases debt in the blue area bounded below by $\overline{W}(F_t)$ and above by $W^\star(F_t)$. Why does the firm retire debt rather than simply distribute cash to its equity holders? The model shows that the two policies are not interchangeable. Below the dividend payout boundary, the marginal value of internal cash exceeds one, $V_W(F_t, W_t) > 1$, as the cash buffer still provides valuable insurance against liquidity-driven bankruptcy. A dividend would convert a dollar that is worth more than one to the equity holders when held within the firm into a dollar in hand—a strict loss–which is why dividend distribution becomes optimal only at $W^\star(F_t)$. A debt buyback instead reduces cash and debt jointly along the equilibrium pricing condition, so that the equity holders incur no first-order loss at the debt-adjustment margin. Hence, when cash is abundant enough that the cash-carry wedge starts to dominate but not so abundant that outright distribution is warranted (i.e., $\overline{W}(F_t) < W_t < W^\star(F_t)$), debt buybacks provide the preferred way to deploy excess liquidity.

3.5 Comparative Statics

We examine how the firm's optimal debt adjustment policy and dividend payout boundary respond to the discount rate ρ, the termination shock arrival rate λ, and the cash flow volatility σ. The first two parameters enter the cash-carry wedge directly: ρ determines the return shortfall on internal cash, while λ governs the expected loss of its liquidity premium upon termination. The third parameter, σ, scales the precautionary-liquidity motive.

Cash-carry wedge. The cash-carry wedge consists of the return shortfall on internal cash (governed by ρ) and the expected termination loss of its liquidity premium (governed by λ). Both components are negative and create an incentive to repurchase debt. They operate through distinct channels, however, and thus have different equilibrium effects on the debt adjustment policy.

Return shortfall (ρ). Panels A and B of Figure VII present comparative statics with respect to the discount rate ρ. Since internal cash earns zero return, a higher ρ widens its return shortfall. Panel A plots the firm's debt adjustment rate G_t as a function of its cash position W_t, holding the face amount of debt outstanding fixed. The blue solid line uses the baseline parameters with $F_t = 0.5$, while the blue dashed line uses the baseline parameters with $F_t = 1$. The red dashed line returns to $F_t = 0.5$, but increases the discount rate from 0.03 to 0.045. Panel B plots the dividend payout boundary $W^\star(F_t)$ as a function of debt outstanding F_t, for $\rho = 0.03$ and $\rho = 0.045$ respectively.

Panel A shows that a higher discount rate moves the firm's debt adjustment rate downward. Intuitively, a higher discount rate widens the return shortfall and increases the cost of retaining cash inside the firm. Thus, issuing debt raises the cash drag associated with the additional liquidity, making debt issuance less attractive to the equity holders. As a result, the firm issues less debt or repurchases debt more rapidly. Moreover, Panel B shows that a higher discount rate lowers the dividend payout boundary. When future payoffs are discounted more heavily, the liquidity-insurance benefit of retaining cash becomes less valuable relative to an immediate distribution. The firm thus starts to pay out dividends at a lower cash level.

Expected termination loss (λ). Panels C and D of Figure VII report comparative

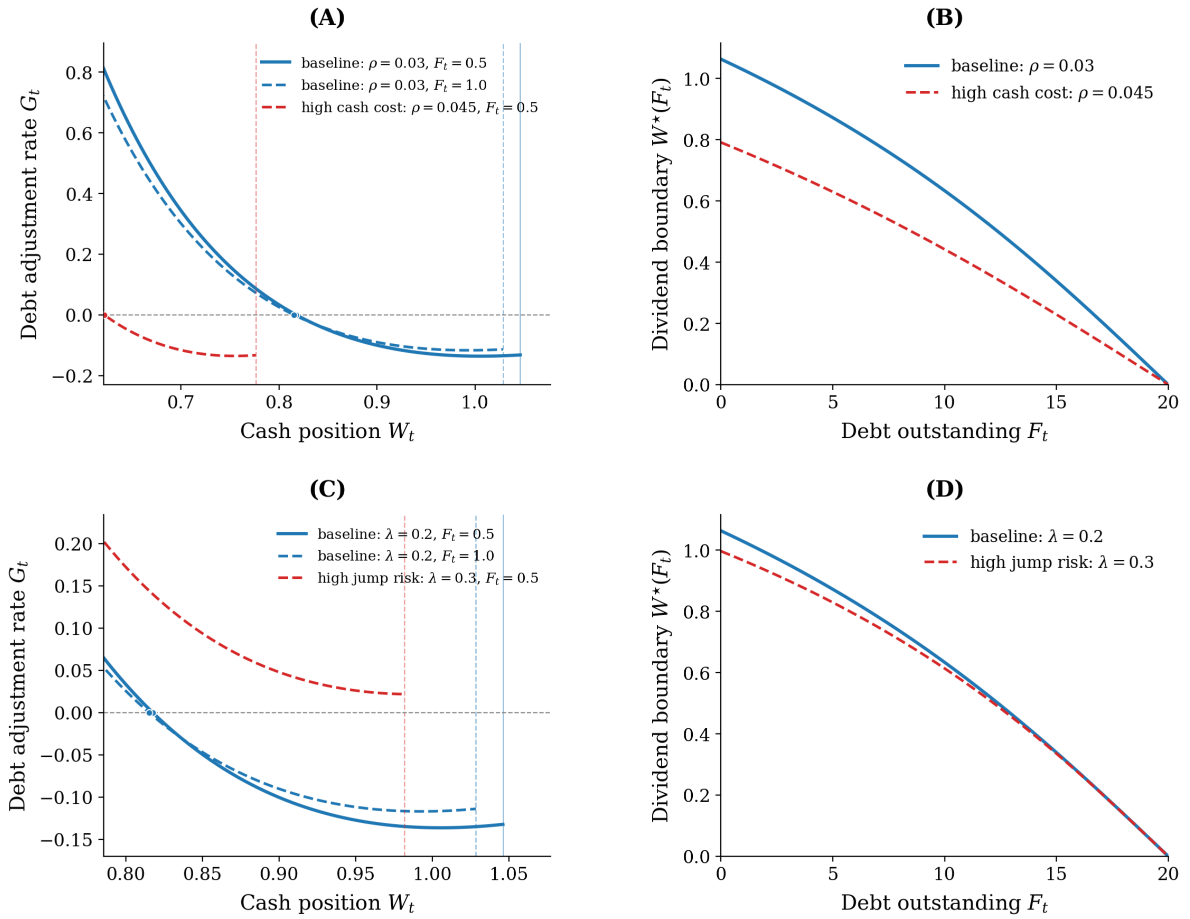

Figure VII. Comparative statics with respect to ρ and λ. The figure reports comparative statics with respect to the discount rate ρ (Panels A and B) and the Poisson intensity of the termination shock λ (Panels C and D). The baseline parameters are $\mu = 0.044$, $\sigma = 0.60$, $c = 0.0022$, $\rho = 0.03$, and $\pi = 0.05$, and $\lambda = 0.2$. Panels A and B increase ρ from 0.03 to 0.045. Panels C and D increase λ from 0.2 to 0.3. Panels A and C plot the debt adjustment rate G_t as a function of the cash position W_t, with the vertical lines marking the corresponding payout boundaries. Panels B and D plot the payout boundary $W^\star(F_t)$ against the outstanding debt F_t. The dashed blue curves in Panels A and C report the baseline policy at $F_t = 1.0$ for comparison. All unreported parameters are held at their baseline values. The label "jump risk" refers to termination risk.

statics with respect to the arrival rate of the termination shock λ. Panel C plots the firm's debt adjustment rate G_t against its cash position W_t, holding the outstanding debt F_t fixed. The blue solid line uses the baseline parameter values with $F_t = 0.5$, while the blue dashed line uses the same baseline parameter values with $F_t = 1$. The red dashed line returns to $F_t = 0.5$ but increases λ from 0.2 to 0.3. Panel D plots the dividend payout boundary $W^\star(F_t)$ against F_t, for $\lambda = 0.2$ and $\lambda = 0.3$ respectively.

Panel D shows that a higher termination risk lowers the dividend payout boundary. A shorter going-concern horizon reduces the expected liquidity-insurance benefit of a cash buffer, since the liquidity premium of internal cash is eliminated upon the termination shock. The firm thus distributes cash at a lower threshold. Panel C shows that the higher

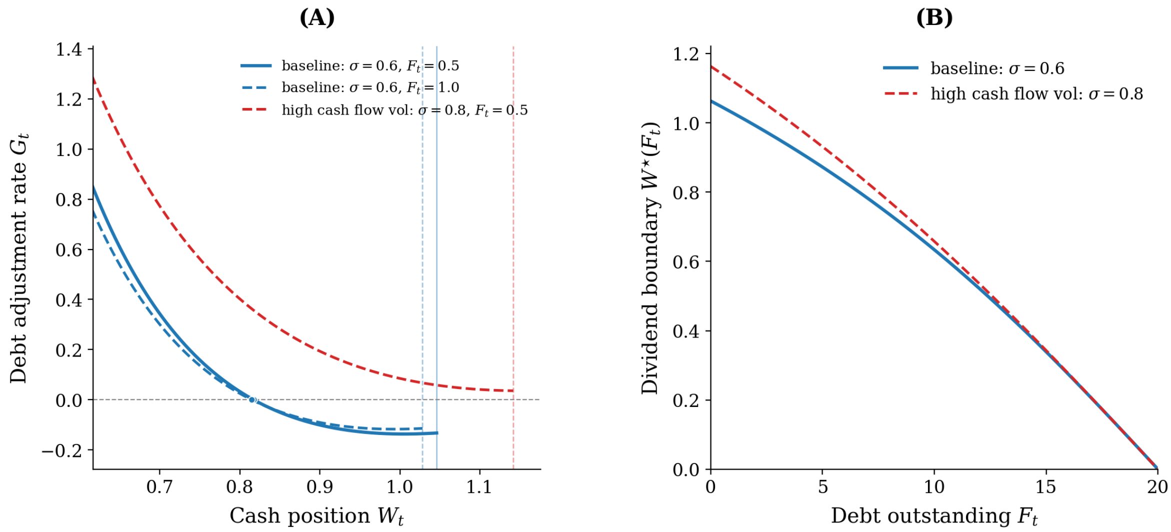

Figure VIII. Comparative statics with respect to σ. The figure reports comparative statics with respect to the cash-flow volatility σ. The baseline parameters are $\mu = 0.044$, $\sigma = 0.60$, $c = 0.0022$, $\rho = 0.03$, and $\pi = 0.05$, and $\lambda = 0.2$. Panel A plots the debt adjustment rate G_t as a function of the cash position W_t: the blue solid line uses the baseline parameters with $F_t = 0.5$, the blue dashed line uses the baseline parameters with $F_t = 1.0$, and the red dashed line uses $F_t = 0.5$ with σ increased to 0.8. The vertical lines mark the corresponding payout boundaries. Panel B plots the dividend payout boundary $W^{\star}(F_t)$ as a function of the outstanding debt F_t, for $\sigma = 0.6$ and $\sigma = 0.8$.

termination risk nevertheless raises the debt adjustment rate. A higher λ increases the rate at which internal cash is expected to lose its liquidity premium and hence discourages debt issuance. However, it also reduces the liquidity-insurance premium $V_W(F_t, W_t) - 1$. In the numerical example, the latter effect dominates, producing a higher rate of debt adjustment.

Precautionary liquidity. Figure VIII shows comparative statics with respect to the cash flow volatility σ. In Panel A, we plot the firm's debt adjustment rate G_t as a function of its cash position W_t, holding the debt amount F_t fixed. The blue solid line uses the baseline parameter values with $F_t = 0.5$, while the blue dashed line uses the same baseline parameter values with $F_t = 1$. The red dashed line returns to $F_t = 0.5$ but increases the cash flow volatility to $\sigma = 0.8$ from $\sigma = 0.6$. In Panel B, we plot the firm's dividend payout boundary $W^{\star}(F_t)$ as a function of F_t, comparing $\sigma = 0.6$ and $\sigma = 0.8$.

Higher cash flow volatility has a non-monotonic effect on the firm's debt adjustment policy. When the cash position is low, a higher σ lowers the debt adjustment rate G_t: the firm sits close to the liquidity-driven bankruptcy boundary, where greater volatility sharply raises default risk. Issuing debt brings in cash today, but it also adds to the future debt-servicing burden and so prices less favorably, and these costs dominate the precautionary benefit of raising additional cash. By contrast, when the cash balance is relatively high, a higher σ raises G_t: the precautionary value of additional liquidity becomes the dominant consideration, and the firm issues more debt.

On the other hand, the dividend payout boundary $W^\star(F_t)$ shifts upward under higher cash flow volatility. Intuitively, more volatile cash flows increase the precautionary value of liquidity: cash retained inside the firm becomes more valuable because it helps absorb adverse shocks and reduces the likelihood of liquidity-driven bankruptcy. The equity holders therefore require a larger cash buffer before paying dividends, so the firm begins distributing cash only at higher cash levels.

4 Costly Equity Issuance

Our model thus far rules out external equity financing. We now relax this restriction and allow the firm to replenish its cash balance by issuing equity at a proportional cost $\kappa > 0$. We proceed in two steps. First, we assume that the firm faces limited equity financing capacity over short horizons, and its equity issuance rate is bounded above by $\bar{I} < \infty$. Second, we consider the limit as $\bar{I} \to \infty$, in which the optimal equity issuance policy becomes a singular control. The extension serves several purposes: to show that the debt buyback mechanism in Proposition 4 does not depend on shutting down external equity financing, to characterize when the firm resorts to equity issuance, and to identify why smooth debt adjustment can fail when the equity issuance rate is unbounded.

4.1 Equity Issuance at a Bounded Rate

Let I_t denote the cumulative proceeds from external equity issuance, and let $\bar{I} \in (0, \infty)$ be an exogenous upper bound on the rate at which the firm can raise equity financing. An admissible equity issuance policy is a progressively measurable process $i_t \in [0, \bar{I}]$, generating

$$dI_t = i_t dt, \qquad \text{for } t < T. \tag{37}$$

This rules out instantaneous injections and caps the rate at which cash can be replenished, which is what preserves the possibility of liquidity-driven bankruptcy.

Issuing dI_t injects that much cash but costs incumbent equity holders $(1+\kappa)dI_t$: dI_t from dilution and κdI_t in issuance cost. As before, cash above a payout boundary $W^\star(F_t)$

is distributed immediately; below it, the equity holders' value function solves

$$(\rho+\lambda)V(F_t,W_t) = \max_{G_t, i_t\in[0,\bar{I}]} \Big\{ G_t V_F(F_t,W_t) + \left[(1-\pi)(\mu - cF_t) + p_t G_t + i_t\right] V_W(F_t,W_t) \\ + \frac{1}{2}(1-\pi)^2\sigma^2 V_{WW}(F_t,W_t) - (1+\kappa)i_t \Big\} + \lambda W_t, \tag{38}$$

where we have used $\hat{V}(F_t,W_t) = W_t$ from Proposition 1. The debt margin is unchanged: the first-order condition in G_t is still (16), and substituting it back annihilates the G_t terms, leaving F_t as a parameter. What is new is i_t, which enters the maximand linearly, so the optimal issuance policy is bang-bang: the firm issues at $i_t = \bar{I}$ whenever $V_W(F_t,W_t) > 1+\kappa$, and not at all otherwise. As we show in Appendix C.9, the switching point is a threshold $\underline{W}(F_t)$ defined by

$$V_W(F_t, \underline{W}(F_t)) = 1+\kappa. \tag{39}$$

It is useful to note that the resulting value function $V(F_t,W_t)$ (and therefore the debt price $p(F_t,W_t)$), $\underline{W}(F_t)$, and payout boundary $W^\star(F_t)$ are all independent of the equity issuance rate $\bar{I}$.

Below the threshold, the marginal value of internal cash exceeds the cost of an outside dollar, so the firm issues at the maximum rate to rebuild its buffer and delay liquidity-driven bankruptcy; above it, the firm stops.

Figure IX illustrates the state space. As in the baseline model, the dividend payout region sits above the payout boundary $W^\star(F_t)$, and within the dividend suspension region the locus $\overline{W}(F_t)$ still divides the states where the firm repurchases debt from those where it issues debt. What is new is the orange area along the bottom of the state space where the firm raises external equity—here cash is scarce enough that the firm issues debt and equity simultaneously. The area shrinks as the debt level rises, and $\underline{W}(F_t)$ reaches zero at $F_t = 8.67$. Beyond that point, the firm never raises external equity no matter how close it is to running out of cash, since an equity injection into a highly levered firm benefits primarily lenders rather than shareholders. Proposition 5 formalizes this.

Proposition 5 (External equity financing)**.** *For a given debt level F_t, the equity issuance threshold $\underline{W}(F_t) > 0$ so that the firm raises external equity whenever $0 < W_t < \underline{W}(F_t)$, if and only if $\kappa < \bar{\kappa}(F_t)$ where*

$$\bar{\kappa}(F_t) = \frac{\rho}{\rho+\lambda}\left[\frac{\eta(F_t)e^{-\gamma(F_t)W_0^\star(F_t)} - \gamma(F_t)e^{-\eta(F_t)W_0^\star(F_t)}}{\eta(F_t)-\gamma(F_t)} - 1\right], \tag{40}$$

with $\gamma(F_t), \eta(F_t)$ given by (25)–(26) and $W_0^\star(F_t)$ the baseline payout boundary of Proposition 3. Moreover, $\bar{\kappa}(F_t)$ is independent of I, is strictly decreasing on $[0,\mu/c)$, with

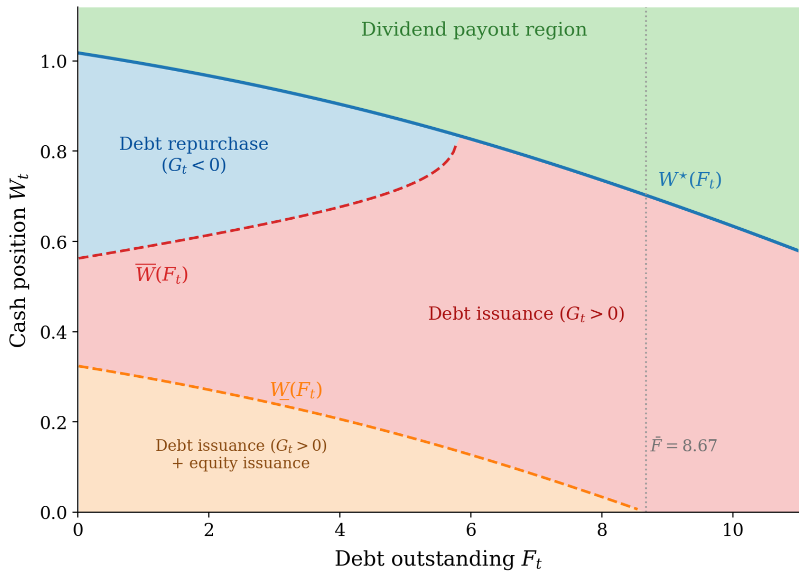

Figure IX. Illustration of state space with costly equity issuance. The figure illustrates the state space in the extension with costly external equity issuance subject to an upper bound on the issuance rate. The outstanding debt F_t is on the horizontal axis and the cash position W_t is on the vertical axis. The parameters are $\mu = 0.044$, $\sigma = 0.60$, $c = 0.0022$, $\rho = 0.03$, $\pi = 0.05$, $\lambda = 0.2$, $\kappa = 0.05$, and $\bar{I} = 1.0$. The dividend payout region lies above the payout boundary $W^\star(F_t)$. Within the dividend suspension region, the locus $G_t = 0$ separates the debt-buyback region from the debt-issuance region. The orange region along the bottom of the state space is the equity-issuance region, where the firm issues debt and equity simultaneously; its upper boundary, the equity-issuance threshold, reaches zero at $F_t = 8.67$.

$\bar{\kappa}(F_t) \to 0$ as $F_t \to \mu/c$.

In (39), $\bar{\kappa}(F_t)$ as a function of F_t, is the cutoff cost, so that if $\kappa < \bar{\kappa}(F_t)$ the firm finds it optimal to raise external equity given the debt burden. It does not depend on I because at this point the equity holders never issue equity. And it declines with the debt level F_t: an equity injection reduces bankruptcy risk, but as leverage rises a larger share of that benefit accrues to lenders while incumbent equity holders bear the issuance cost.

Corollary 5.1. *The firm never raises external equity if the equity issuance cost $\kappa \geq \bar{\kappa}(0)$.*

The endogenous threshold $\bar{\kappa}(0)$ plays an important role in the unbounded equity issuance case below. If $\kappa \geq \bar{\kappa}(0)$ the option is never exercised and the model collapses to the baseline of Section 3.1, so we restrict attention to $\kappa < \bar{\kappa}(0)$ throughout. The buyback margin survives: Figure IX shows the firm still repurchasing its debt when cash is abundant, so costly equity alters behavior near the bankruptcy boundary without disturbing the mechanism of Proposition 4.

Comparing Figure IX with Figure VI shows that the buyback region in fact expands once costly equity is allowed. Figure X plots the payout boundary $W^\star(F_t)$ and the issuance–buyback threshold $\overline{W}(F_t)$ against F_t for $\kappa = 0.05$ and 0.07; the firm repurchases debt between the two curves.

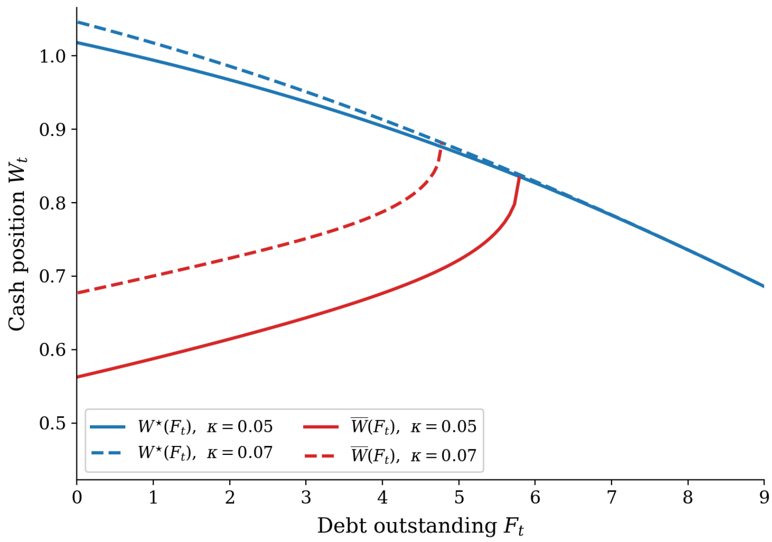

Figure X. Debt buyback region and equity issuance cost. The figure plots the dividend payout boundary $W^\star(F_t)$ and the debt issuance–buyback threshold $\underline{W}(F_t)$ as functions of the outstanding debt F_t, for the equity issuance costs $\kappa = 0.05$ and $\kappa = 0.07$. The remaining parameters are $\mu = 0.044$, $\sigma = 0.60$, $c = 0.0022$, $\rho = 0.03$, $\pi = 0.05$, $\lambda = 0.2$, and $\bar{I} = 1.0$. The firm repurchases its own debt in the region between the two curves.

A higher issuance cost raises both boundaries. $W^\star(F_t)$ rises slightly: dearer outside equity makes internal cash more valuable as protection against bankruptcy, so the firm is more reluctant to distribute it. $\overline{W}(F_t)$ rises by more, because the precautionary motive to issue debt and build a buffer strengthens, so the firm keeps issuing up to a higher cash level. On net the buyback region shrinks.

4.2 Unbounded Equity Issuance

The case with $\bar{I} \to \infty$ differs qualitatively from the finite-cap problem above: equity issuance is no longer an ordinary finite-rate control, and the cumulative proceeds I_t may be any adapted, right-continuous, nondecreasing process of finite variation. An increment dI_t adds to the firm's cash balance but costs the incumbent equity holders $(1+\kappa)dI_t$, so dividends and equity issuance are opposing singular controls. Ruling out profitable infinitesimal deviations in either direction implies that, if the firm issues equity along the equilibrium path, the equity value function satisfies the gradient constraint

$$1 \le V_W(F_t, W_t) \le 1 + \kappa. \tag{41}$$

Dividends are paid only where $V_W(F_t, W_t) = 1$, while equity issuance occurs only where $V_W(F_t, W_t) = 1 + \kappa$. Equity holders may of course instead exercise their default option and walk away with nothing.

This gradient constraint presumes smooth debt issuance, so that equity value remains the no-trade value. The central question is whether that policy can survive when equity issuance is unbounded, so that the firm recapitalizes at $W_t = 0$ rather than defaulting outright as in the baseline without equity issuance. The following proposition gives a negative answer, distinguishing two cases according to the issuance cost κ.

Proposition 6 (Unbounded equity issuance)**.** *Take the cutoff $\bar{\kappa}(0)$ given by (40). Then,*

(i) when $\kappa \geq \bar{\kappa}(0)$, it is never optimal for the firm to issue equity, and the equilibrium constructed in Section 3 remains valid; and

(ii) when $\kappa < \bar{\kappa}(0)$, the equilibrium with smooth debt policy fails to exist.

Part (i) follows directly from Proposition 5. Because $\bar{\kappa}(F_t)$ is strictly decreasing, $\kappa \geq \bar{\kappa}(0)$ implies $\kappa \geq \bar{\kappa}(F_t)$ at every debt level, so equity issuance is never optimal and the baseline smooth debt policy continues to apply. This case adds no economics beyond the baseline, since external equity is ruled out altogether.

Part (ii) is the substantive case, which we establish by contradiction. Suppose provisionally that debt is adjusted smoothly, so that the no-trade condition (16), $V_F(F_t, W_t) + p(F_t, W_t)V_W(F_t, W_t) = 0$, applies. Monotonicity of $\bar{\kappa}$ then yields a unique threshold F_κ satisfying $\bar{\kappa}(F_\kappa) = \kappa$ that separates two candidate regimes. For $F_t \geq F_\kappa$, equity issuance is never active and the candidate solution coincides with the baseline model.

For $F_t < F_\kappa$, the provisional smooth debt adjustment implies that cash is reflected at zero via equity issuance. Because equity can be raised instantaneously without a fixed cost while internal cash earns no return, the equity holders wait until cash reaches zero and then raise just enough to avert liquidity-driven bankruptcy. A marginal cash shortfall at the boundary is therefore absorbed by the equity holders and leaves the lenders' payoff unchanged, so a standard smooth-pasting argument makes the debt price locally insensitive to cash there—that is, $p_W(F_t, 0^+) = 0$.[15]

However, this contradicts the marginal no-trade condition implied by smooth debt adjustment. Throughout the recapitalization region, the marginal value of cash at the boundary is pinned at its replacement cost, $V_W(F_t, 0^+) = 1 + \kappa$, independently of the debt level, so that $V_{FW}(F_t, 0^+) = 0$. Differentiating the no-trade condition (16) with respect to W_t then gives

$$p_W = \frac{-V_{FW}V_W + V_{WW}V_F}{V_W^2} = \frac{-V_{WW}}{V_W} \cdot p > 0, \tag{42}$$

[15] At $W_t = 0$, consider a discrete-time approximation. Since the debt adjustment in F_t is always of order dt, we consider only the Brownian shocks to dW_t. The debt valuation then implies that

$$p(F_t, 0) = \frac{1}{2}\underbrace{p\big(F_t, 0 - \sigma\sqrt{dt}\big)}_{= p(F_t, 0),\ \text{reflection}} + \frac{1}{2}p\big(F_t, 0 + \sigma\sqrt{dt}\big) \quad \Longrightarrow \quad p\big(F_t, 0 + \sigma\sqrt{dt}\big) = p(F_t, 0),$$

which implies that $p_W(F_t, 0^+) = 0$.

where every object is evaluated at $(F_t, 0^+)$, the second equality uses $V_F = -pV_W$, and the inequality uses $V_W > 0$, $V_{WW} < 0$ and $p > 0$ from Corollary 3.1. This contradicts $p_W(F_t, 0^+) = 0$. Hence there is no equilibrium in which debt adjustment is smooth throughout the set of states with $F_t < F_\kappa$: smooth debt adjustment and equity issuance cannot coexist in equilibrium.

Intuitively, the debt valuation at the boundary is inconsistent with the equity holders' optimality condition for smooth debt adjustment. That condition, (16), reads $pV_W + V_F = 0$; consider how its left-hand side moves with cash at $W_t = 0^+$. As explained above the second term does not move ($V_W(F_t, 0) = 1 + \kappa$ regardless of F). The first term moves by $p_W V_W + pV_{WW}$. If the debt price satisfies the lenders' valuation equation, so that $p_W(F_t, 0^+) = 0$, this reduces to $pV_{WW} < 0$. Therefore the left-hand side of the first-order condition turns strictly negative just above the boundary, and the equity holders strictly prefer to repurchase debt, possibly in a lump sum.

5 Empirical Evidence

The model predicts that a firm retires debt precisely when its internal cash is abundant. We now take that prediction to the data. Section 5.1 describes the bond-level sample and the verification behind it, and Section 5.2 uses the TCJA repatriation shock to ask whether an exogenous increase in cash causes firms to repurchase their bonds.

5.1 Data Source and Sample

Our bond universe comprises all U.S. dollar-denominated corporate debentures, medium-term notes, and convertible issues of U.S.-domiciled firms in the Mergent Fixed Income Securities Database (FISD), excluding Yankee bonds, foreign-currency issues, and commercial paper.[16] FISD provides each issue's offering terms, covenants, Rule 144A status, and call schedule, and we measure each bond's time-varying amount outstanding from FISD's amount outstanding history.

Our tender offer sample includes all transactions under the "Tender Offer" action type in the fixed income segment of the Bloomberg Corporate Actions database over 2000–2024.[17] Because Bloomberg offers no bulk export, an automated pipeline navigates the Terminal page of each transaction, captures screenshots of the relevant fields, converts them to text by optical character recognition (OCR), and parses the text into a structured dataset. Each record gives the announcement and effective dates, the amounts

[16]Because we condition on the issuer's domicile rather than on where a bond is sold or traded, every bond in our sample is an obligation of a U.S. entity. Dollar bonds issued by foreign subsidiaries of U.S. parents are excluded.

[17]FISD also records some tender offers, but its coverage is incomplete and deteriorates after 2021; quarterly volumes from the two sources track each other closely through 2021.

offered, tendered, and repurchased, the price (fixed or fixed-spread), and any early tender premium and consent fee. We consolidate duplicates for paired tranches, exclude transactions in a bankruptcy window, and drop records with repurchased amounts above 115% of the offering amount (likely parsing errors). We also collect every record in the same segment's "Repurchase/Open Market" action type and screen these records similarly.[18]

Before any transaction enters the panel, we verify its channel, date, and amount against the issuer's SEC filings and press releases in a double-blind reading by two independent large language model agents (see Appendix A for details). Tender offer records, whose disclosure is mandatory, prove far more accurate: 99.0% of the 4,482 tender records are genuine cash tender offers, whereas only 51.2% of the 7,441 records in the "Repurchase/Open Market" group are genuine open-market purchases. The verified panel contains 4,347 tender offers ($1,078 billion) and 3,802 open-market repurchases ($136 billion), with verified amount corrections applied.

We link securities to their ultimate parent firms in Compustat using the issuer-level crosswalk of Fang (2023), exclude financial issuers, and aggregate to an annual firm-level panel. Each firm-year records tender offer and open-market repurchase activity, bonds outstanding at the beginning of the year, and Compustat characteristics, including cash and short-term investments scaled by book assets, our measure of the cash position, plus size and book leverage. The panel covers 31,705 firm-years and 45,109 bonds; 1,649 firm-years contain a tender offer.

5.2 The TCJA Repatriation Tax Shock

Exploiting the Tax Cuts and Jobs Act (TCJA) as an exogenous liquidity shock to firms' domestic operations, we now provide causal evidence that higher corporate cash holdings lead to bond buybacks.

Institutional background and empirical design. Prior to the TCJA, U.S. multinationals faced U.S. taxation on foreign earnings only upon repatriation, creating strong incentives to retain profits abroad and defer tax liabilities (see, e.g., Faulkender et al., 2019). The TCJA removed these incentives by eliminating the benefits of deferral: it imposed a one-time, mandatory transition tax on all previously unrepatriated earnings, regardless of whether firms repatriated the funds, and largely exempted future foreign earnings from U.S. repatriation taxes. The transition tax was applied at reduced rates of 15.5% on liquid foreign assets and 8% on illiquid assets, substantially below the pre-reform statutory rate. Once it was levied, repatriation was costless at the margin, freeing firms to redeploy foreign cash domestically. In aggregate, the reform unlocked as much as

[18] This group is a catch-all in which genuine open-market purchases sit alongside conversion settlements, negotiated purchases, exchange offers, puts, etc.

\$1.7 trillion in foreign-held cash for U.S. firms (Bureau of Economic Analysis, 2020), and the increase in liquidity primarily translated into higher payouts to shareholders, with more limited effects on domestic investment and employment (Albertus et al., 2025).

This setting maps naturally into our framework. The shock to accessible internal liquidity is plausibly exogenous, and its intensity varies across firms with the size of their foreign earnings. Through the lens of the model, how a U.S. firm deploys such a windfall depends on where it lands in the state space of Figure VI: a firm whose cash position ends up in the buyback region (above the threshold $\overline{W}(F_t)$ but below the payout boundary $W^{\star}(F_t)$) uses the marginal dollar to retire debt, whereas a firm pushed to or beyond the payout boundary passes the marginal dollar on to shareholders. The reform should therefore raise security buybacks among affected firms, with the split between the debt and equity margins determined by how far the windfall carries each firm toward the payout boundary.

Following Darmouni and Mota (2024), we evaluate how this increase in liquidity affected firms' buyback activity in a difference-in-differences design. We proxy for the intensity of the shock with foreign income, measured as pre-tax income from foreign operations (PIFO) in Compustat scaled by total sales, since firms with larger foreign earnings gained more accessible domestic capital from the reform. Our sample spans 2012–2019, with 2018–2019 constituting the post period.[19] Because our interest is in buybacks funded from internal liquidity, we exclude refinancing tender offers, defined as those for which the firm issued new bonds of similar size (within 5% of the amount tendered) in the 30 days before settlement.[20]

In our data, the TCJA window (2012–2019) is an unusually active one for tender offers, which retire about \$47.2 billion of bonds per year inside it against \$35.4 billion outside (in 2010 dollars). Open-market repurchases move the other way, retiring only \$2.8 billion per year inside the window against \$6.2 billion outside. This reinforces our focus on tender offers as the main channel for debt buybacks.

We sort firms into quartiles by their average foreign income-to-sales ratio over 2012–2016, so treatment assignment uses only pre-reform data. Quartiles 4 and 3 hold the highest and second-highest ratios; the bottom two serve as the comparison group.[21] We

[19]The sample includes 65 firms incorporated outside the United States, mostly in Ireland, Canada, Bermuda, and the United Kingdom. Excluding them does not change the results.

[20]Widening the size band to 10% of the amount tendered leaves the estimates in Table II essentially unchanged.

[21]The median of the firm-level average foreign income-to-sales ratio over 2012–2016 is 0.5% of sales, so the median firm earns essentially no foreign income. Below-median firms are thus a natural untreated benchmark: roughly 60% report foreign pre-tax losses, which imply no accumulated unrepatriated earnings and hence no exposure to the shock, and together they account for only 1.3% of aggregate pre-period foreign pre-tax income. We pool the bottom two quartiles rather than entering them separately because negative foreign income does not represent a lower dose of the shock.

Table II. TCJA Repatriation and Corporate Buyback Activities. This table reports the effects of the TCJA repatriation tax shock on firms' corporate bond and equity buybacks in the U.S. The unit of observation is at the firm-year level. The sample covers 2012–2019, with 2018–2019 as the post period. Firms are sorted into quartiles based on average foreign income relative to total sales over 2012–2016. Quartile 4 PIFO includes firms with the highest foreign income-to-sales ratios, and Quartile 3 PIFO includes the second-highest. The dependent variable is an indicator for corporate bond tender offers in Columns (1)–(2), the fraction of total bonds outstanding retired via tender offers in Columns (3)–(4), and the fraction of equity repurchased in Columns (5)–(6). Equity buybacks follow Fama and French (2001) and Almeida et al. (2016): they are measured as the increase in common treasury stock when available; otherwise, as stock buybacks minus stock issuances from the cash flow statement when treasury stock is zero in both the current and prior years. Negative values are set to zero. The fraction of equity repurchased is equity buybacks scaled by the market value of equity at the start of the year. All dependent variables are winsorized at the 1% and 99% levels. *, **, and *** indicate statistical significance at the 10%, 5%, and 1%, respectively.

	$\mathbb{1}$(Tender Offers)(%)		Frac(Tender Offers)(%)		Frac(Equity Repurchase)(%)	
	(1)	(2)	(3)	(4)	(5)	(6)
Quartile 3 PIFO × Post	4.714**	4.799**	0.699**	0.709**	0.071	0.047
	(1.990)	(1.990)	(0.324)	(0.324)	(0.303)	(0.301)
Quartile 4 PIFO × Post	3.158*	3.087*	0.279	0.272	0.543**	0.574**
	(1.841)	(1.841)	(0.286)	(0.286)	(0.275)	(0.277)
Leverage(t-1)		6.105**		0.721		-2.301***
		(2.908)		(0.527)		(0.477)
Adjusted R^2	0.103	0.103	0.054	0.054	0.288	0.295
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
NAICS3-year FE	Yes	Yes	Yes	Yes	Yes	Yes
Observations	5,598	5,571	5,595	5,568	5,468	5,453

estimate the following regression specification:

$$y_{it} = \beta \; Quartile\ 3\ PIFO_i \times Post_t + \delta \; Quartile\ 4\ PIFO_i \times Post_t + \gamma_i + \eta_{jt} + X_{it}'\theta + \epsilon_{it} \quad (43)$$

where y_{it} is an outcome variable for firm i in year t, *Post* is a dummy equal to one for years 2018 and 2019 and zero for years 2012–2017, γ_i and η_{jt} denote firm and three-digit NAICS industry-by-year fixed effects, and X_{it} collects controls. The firm fixed effects absorb time-invariant differences across firms; the industry-by-year effects absorb annual industry-level shocks, and also pick up other provisions of the TCJA, such as the corporate rate cut, bonus depreciation, and the limitation on interest deductibility, to the extent that their incidence varies at the industry level. Even-numbered columns additionally control for lagged book leverage. Standard errors are clustered at the firm level. The identifying assumption is that, absent the reform, the buyback activity of high- and low-foreign-income firms would have evolved in parallel within industry-years.

Empirical results. Table II reports estimates of equation (43), with the tender offer indicator in Columns (1)–(2), the fraction of total bonds outstanding retired via tender offers in Columns (3)–(4), and the fraction of equity repurchased in Columns (5)–(6). Following the reform, Quartile 3 firms become significantly more likely to conduct tender offers and retire a significantly larger share of their outstanding bonds through them. The pattern reverses on the equity margin: Quartile 4 firms significantly increase equity buybacks (Columns (5)–(6)), while Quartile 3 firms show no detectable change.

The estimated effects are economically meaningful. The Quartile 3 coefficient in Column (2) implies a 4.8 percentage point increase in the annual probability of conducting a tender offer after the reform, roughly four fifths of the 5.9% unconditional tender frequency in the estimation sample. The corresponding estimate in Column (4) implies that Quartile 3 firms retire an additional 0.71% of their outstanding bonds per year, on par to the unconditional mean of 0.80%. For Quartile 4, the tender offer estimates are smaller and less precise: the Column (2) coefficient implies a 3.1 percentage point increase in the likelihood of a tender offer, about half of the unconditional frequency and significant only at the 10% level, and the Column (4) estimate of 0.27% of bonds outstanding, about a third of the sample mean, is not statistically different from zero. On the equity margin, by contrast, the Quartile 4 estimate in Column (6) implies that equity buybacks rise by 0.57% of the market value of equity, about 30% of the 1.9% sample mean, and the estimate is statistically significant at the 5% level. The corresponding estimates for Quartile 3 are close to zero, 0.07% and 0.05% of the market value of equity in Columns (5) and (6), and far from statistically significant.

Taken together, the two groups adjust on different margins: firms receiving the largest liquidity shocks increase equity buybacks, whereas firms' receiving more moderate ones turn to bond buybacks. This split is consistent with the logic of the model. Quartile 4 firms are disproportionately large, cash-rich multinationals for which the deemed repatriation unlocked a large amount of liquidity for their domestic segments. For them, the large windfall pushes their U.S. cash holdings past the payout boundary. The excess cash received from the windfall is therefore paid out to shareholders. Quartile 3 firms, by contrast, receive a more moderate windfall that leaves them in the debt buyback region, where the model predicts that the marginal dollar is deployed to retire debt.

6 Conclusion

Firms routinely retire their bonds before maturity at market prices, even though the leverage ratchet logic of DeMarzo and He (2021) implies that a buyback at a fair price cannot benefit shareholders. This paper shows that corporate liquidity, interacted with termination risk, restores the gain. Creditors price every state at par, whereas the equity holders value inside cash above par in the survival states and at par in the termination states. A buyback therefore pays the creditors in the termination states in exchange for the survival states, moving resources to where the equity holders value them more. Neither ingredient works alone.

We embed this mechanism in a continuous-time model with cash and a continuously adjusted consol, which we solve in closed form. The firm issues debt when cash is scarce, buys it back when cash is abundant but below the payout boundary, and pays dividends above it. A hand-verified sample of bond tender offers over 2000–2024 confirms that cash-rich firms tender more, and the TCJA repatriation shock separates the two margins: moderately treated firms retire bonds, while the most treated pay out to shareholders.

Several directions remain open: the dynamics of cash and debt when outside equity is costly but not prohibitively so, the interaction of cash with transaction costs or agency conflicts, and the choice between tender offers and open-market repurchases, which our model treats as a single instrument and which voluntary disclosure makes hard to observe.

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Appendix A Verification of Bloomberg Records

This appendix describes how we verify the Bloomberg tender offer and open-market repurchase records against public disclosures (Section 5.1): every record is read in a double-blind design by two independent readers, and a record is accepted only when the two readers agree.

Before any transaction enters the panel, we verify it against public disclosures. For each record we assemble a packet of evidence from the SEC's EDGAR system: full-text searches for the bond's nine-digit CUSIP and for its coupon and maturity, and the issuer's filings in a window around the recorded date, including offer-to-purchase documents and press releases filed on Form 8-K and the debt footnotes of 10-Q and 10-K reports. Where filings are silent, we search newswires, company investor-relations pages, and the Internet Archive. A record whose initial packet does not settle the question receives fuller packets, up to the issuer's complete filings around the event.

Each packet is then read in a double-blind design by two independent readers, both large language model agents, who work blind to each other and to any prior verdict. Each reader reports whether the record is a genuine transaction in the recorded channel and, if not, what it actually is, along with the transaction date and the principal amount actually purchased, and must support each judgment with a verbatim quote. A record is settled only when both readers agree; silence never counts as confirmation. Disagreements are escalated to a fuller packet and a fresh pair of readers; the small residual that remains is ruled by us, not by the readers, under written standing rules that are applied consistently across all records. Where both readers reach a verdict, they agree on whether a record is genuine in 99.6% of tender reads and 97.2% of open-market reads; the remaining cases, in which at least one reader finds the packet insufficient, are the ones escalated to fuller evidence. Counting every read, including those in which a reader finds the packet insufficient, the two readers return the same verdict on genuineness in 93% of tender reads and 94% of open-market reads; where both readers report a principal amount, the two amounts agree within 5% in 98% of tender reads and 94% of open-market reads, and where both report a transaction date, the dates agree in 99% and 93% of reads, respectively.

As described in Section 2, a tender offer must be publicly announced and its results disclosed under the tender-offer rules, so the issuer itself states the terms of the offer and the amount accepted. Open-market buybacks are subject to no such disclosure requirement. We therefore expect, and find, the tender offer records to be far more accurate. Of the 4,482 tender offer records, 4,437 (99.0%) are genuine cash tender offers; the 45 rejected records are mostly exchange offers and open-market purchases misrecorded as tenders, and we also exclude 81 holder put exercises (1.8%). Among genuine tenders, the recorded amount is corrected in 3.0% of records and the announcement date in 0.7%. Of the 7,441 buyback records, 3,809 (51.2%) are genuine open-market purchases, accounting for 39% of the $357 billion recorded. The rejected records are conversion settlements of convertible bonds (995), privately negotiated purchases (609), exchange offers (330), holder put exercises (154), and calls (145), among other types of transactions. Among genuine open-market records, the recorded amount is off by more than 5% in 9.6% of cases, we apply the amount corrections in the panel. The verified panel contains 4,347 tender offers ($1,078 billion) and 3,802 open-market repurchases ($136 billion).

Appendix B Additional Empirical Results

Table B.1. Corporate Bond Tender Offers and Firms' Cash Position

This table examines the relationship between a firm's cash position and corporate bond tender offer activities. The unit of observation is at the firm-year level. The dependent variable $\mathbb{1}(\textit{Tender Offers})\%$ in Columns (1)-(2) is an indicator of tender offer activities. The dependent variable *Frac(Tender Offers)%* in Columns (3)-(4) is the dollar amount of bonds repurchased through tender offers during the year, scaled by the firm's total dollar amount of corporate bonds outstanding at the beginning of the year. The independent variable is a firm's cash and short-term investments relative to book assets. Standard errors are clustered at the firm level and reported in parentheses. *, **, and *** indicate statistical significance at the 10%, 5%, and 1%, respectively.

A. Cash Position, 2000-2019

	$\mathbb{1}$(Tender Offers)(%)		Frac(Tender Offers)(%)	
	(1)	(2)	(3)	(4)
Cash/Assets	6.820***	4.004**	7.908**	7.508**
	(2.068)	(2.021)	(3.094)	(3.138)
Adjusted R^2	0.036	0.046	0.006	0.048
Firm FE	Yes	Yes	No	No
Year FE	No	Yes	No	Yes
Observations	24,598	24,598	1,284	1,284

B. Cash Position, 2000-2024

	$\mathbb{1}$(Tender Offers)(%)		Frac(Tender Offers)(%)	
	(1)	(2)	(3)	(4)
Cash/Assets	4.560**	1.798	8.595***	7.671***
	(1.861)	(1.814)	(2.992)	(2.959)
Adjusted R^2	0.042	0.051	0.007	0.069
Firm FE	Yes	Yes	No	No
Year FE	No	Yes	No	Yes
Observations	29,947	29,947	1,582	1,582

Appendix C Proofs and Derivations

Notation and Preliminaries

Throughout the appendix we suppress time subscripts where no confusion arises and write (F, W) for a generic state. We use the after-tax primitives

$$\tilde{\mu} \equiv (1-\pi)\mu, \qquad \tilde{c} \equiv (1-\pi)c, \qquad \tilde{\sigma} \equiv (1-\pi)\sigma, \tag{A.1}$$

and define

$$b(F) \equiv \tilde{\mu} - \tilde{c}F, \qquad \Delta(F) \equiv \sqrt{b(F)^2 + 2(\rho+\lambda)\tilde{\sigma}^2}, \qquad \theta \equiv \frac{\lambda}{\rho+\lambda}, \quad 1-\theta = \frac{\rho}{\rho+\lambda}. \tag{A.2}$$

Note $b(F) > 0$ if and only if $F < \mu/c$, and $b'(F) = -\tilde{c}$, $\Delta'(F) = -\tilde{c}b/\Delta$.

For each F, let $\mathcal{L}_F$ denote the second-order operator associated with the pre-jump problem,

$$\mathcal{L}_F f(W) \equiv \tfrac{1}{2}\tilde{\sigma}^2 f''(W) + b(F) f'(W) - (\rho+\lambda) f(W). \tag{A.3}$$

Its characteristic roots are

$$\gamma(F) = -\frac{b(F)+\Delta(F)}{\tilde{\sigma}^2} < 0, \qquad \eta(F) = -\frac{b(F)-\Delta(F)}{\tilde{\sigma}^2} > 0, \tag{A.4}$$

with the elementary identities

$$\gamma + \eta = -\frac{2b}{\tilde{\sigma}^2}, \qquad \eta - \gamma = \frac{2\Delta}{\tilde{\sigma}^2}, \qquad \gamma\eta = -\frac{2(\rho+\lambda)}{\tilde{\sigma}^2}, \qquad \tilde{\sigma}^2\gamma + b = -\Delta, \qquad \tilde{\sigma}^2\eta + b = \Delta. \tag{A.5}$$

Differentiating (A.4) with respect to F gives

$$\gamma'(F) = -\frac{\tilde{c}}{\Delta(F)}\gamma(F), \qquad \eta'(F) = \frac{\tilde{c}}{\Delta(F)}\eta(F), \qquad \gamma''(F) = -\frac{2(\rho+\lambda)\tilde{c}^2}{\Delta(F)^3}, \quad \eta''(F) = \frac{2(\rho+\lambda)\tilde{c}^2}{\Delta(F)^3}. \tag{A.6}$$

The following two lemmas are used repeatedly. The first is a direct computation from (A.5); the second is a maximum-principle argument that replaces the explicit sign computations of the pure-diffusion benchmark.

Lemma 7. *For any constant ω, the operator $\mathcal{L}_F$ satisfies*

$$\mathcal{L}_F\left[e^{\gamma W}\right] = 0, \qquad \mathcal{L}_F\left[e^{\eta W}\right] = 0, \tag{A.7}$$

$$\mathcal{L}_F\left[We^{\gamma W}\right] = -\Delta\, e^{\gamma W}, \qquad \mathcal{L}_F\left[We^{\eta W}\right] = +\Delta\, e^{\eta W}, \tag{A.8}$$

$$\mathcal{L}_F\left[W^2e^{\gamma W}\right] = \tilde{\sigma}^2 e^{\gamma W} - 2\Delta\, We^{\gamma W}, \qquad \mathcal{L}_F\left[W^2e^{\eta W}\right] = \tilde{\sigma}^2 e^{\eta W} + 2\Delta\, We^{\eta W}, \tag{A.9}$$

$$\mathcal{L}_F[1] = -(\rho+\lambda), \qquad \mathcal{L}_F[W] = b - (\rho+\lambda)W. \tag{A.10}$$

The same identities hold with W replaced by $W-\omega$ in the exponents, since $e^{\gamma(W-\omega)} = e^{-\gamma\omega}e^{\gamma W}$ and $e^{-\gamma\omega}$ is a constant for fixed F. Consequently, particular solutions of $\mathcal{L}_F f = s$ for the resonant sources are

$$s = e^{\gamma W}: \ f = -\tfrac{1}{\Delta}We^{\gamma W}; \qquad s = We^{\gamma W}: \ f = -\tfrac{1}{2\Delta}W^2e^{\gamma W} - \tfrac{\tilde{\sigma}^2}{2\Delta^2}We^{\gamma W}, \tag{A.11}$$

and analogously with $\gamma \mapsto \eta$ and $\Delta \mapsto -\Delta$.

Proof. For (A.8), with $f = We^{\gamma W}$ we have $f' = (1+\gamma W)e^{\gamma W}$ and $f'' = (2\gamma + \gamma^2 W)e^{\gamma W}$, so $\mathcal{L}_F f = e^{\gamma W}\left[\tilde{\sigma}^2\gamma + b + W(\tfrac{1}{2}\tilde{\sigma}^2\gamma^2 + b\gamma - (\rho+\lambda))\right] = e^{\gamma W}(\tilde{\sigma}^2\gamma + b) = -\Delta e^{\gamma W}$, using that γ solves the characteristic equation and the last identity in (A.5). For (A.9), with $f = W^2 e^{\gamma W}$ we get $\mathcal{L}_F f = e^{\gamma W}\left[\tilde{\sigma}^2 + 2W(\tilde{\sigma}^2\gamma + b)\right]$. The remaining identities are immediate. The particular solutions in (A.11) follow by matching coefficients. □

Lemma 8. *Fix $F < \mu/c$ and let $g : [0, W^\star(F)] \to \mathbb{R}$ be C^2 with $\mathcal{L}_F g > 0$ on $(0, W^\star(F))$, $g(0) = 0$ and $g'(W^\star(F)) = 0$. Then $g < 0$ on $(0, W^\star(F)]$ and $g'(0) < 0$.*

Proof. Suppose $g(W_0) > 0$ for some $W_0 \in (0, W^\star]$, and let $\bar{W}$ be a maximizer of g on $[0, W^\star]$, so $g(\bar{W}) > 0$. If $\bar{W} \in (0, W^\star)$ then $g'(\bar{W}) = 0$ and $g''(\bar{W}) \leq 0$, whence $\mathcal{L}_F g(\bar{W}) = \frac{1}{2}\tilde{\sigma}^2 g''(\bar{W}) - (\rho + \lambda)g(\bar{W}) < 0$, a contradiction. If $\bar{W} = W^\star$ then $g'(W^\star) = 0$ and, by continuity of $\mathcal{L}_F g$ up to the boundary, $\frac{1}{2}\tilde{\sigma}^2 g''(W^\star) = \mathcal{L}_F g(W^\star) + (\rho + \lambda)g(W^\star) > 0$; hence g is strictly convex with a stationary point at $W^\star$, so $g(W) > g(W^\star)$ for W slightly below $W^\star$, again a contradiction. Finally $\bar{W} = 0$ is impossible because $g(0) = 0 < g(W_0)$. Therefore $g \leq 0$ on $[0, W^\star]$. Since $g(0) = 0$, this forces $g'(0) \leq 0$; and if $g'(0) = 0$ then $\frac{1}{2}\tilde{\sigma}^2 g''(0) = \mathcal{L}_F g(0) > 0$, so $g > 0$ just to the right of 0, a contradiction. Hence $g'(0) < 0$, and $g < 0$ on $(0, W^\star]$ because $g \equiv 0$ on any subinterval would contradict $\mathcal{L}_F g > 0$. □

Appendix C.1 Proof of Proposition 1

Condition on $\mathcal{F}_T$ and let (F_T, W_T) denote the state the firm carries into the post-jump regime, with $W_T \geq 0$. Let $\hat{\tau} \equiv \inf\{t \geq T : W_t = 0\}$. On $\{t \geq T\}$ the equity holders solve

$$\hat{V}(F_T, W_T) = \sup_{\{\Gamma_t, D_t\}_{t \geq T}} \mathbb{E}_T\left[\int_{[T,\hat{\tau}]} e^{-\rho(t-T)} dD_t \middle| \mathcal{F}_T\right], \tag{A.12}$$

subject to the cash dynamics in the post-jump regime

$$dW_t = -(1-\pi)cF_t dt + \hat{p}_t d\Gamma_t - dD_t. \tag{A.13}$$

What follows characterizes $\hat{V}$ and $\hat{p}$ as functions on the post-jump state space, and the arguments below are generic states (F, W). Debt can be adjusted freely in either direction. Fix a state (F, W) with $W > 0$ and consider a marginal adjustment of size $d\Gamma$. It moves the state by $(dF, dW) = (d\Gamma, \hat{p}(F, W)d\Gamma)$, so its marginal effect on the equity holders' value is $[\hat{V}_F(F, W) + \hat{p}(F, W)\hat{V}_W(F, W)]d\Gamma$, the price impact of the trade entering only at second order. Since both signs of $d\Gamma$ are feasible and the coefficient is linear and unbounded in $d\Gamma$, a finite value requires

$$\hat{V}_F(F, W) + \hat{p}(F, W)\hat{V}_W(F, W) = 0. \tag{A.14}$$

Hence, the dividend control satisfies

$$\max\left\{-\rho\hat{V}(F, W) - (1-\pi)cF\hat{V}_W(F, W), 1 - \hat{V}_W(F, W)\right\} = 0, \tag{A.15}$$

where the first branch corresponds to dividend suspension and the second corresponds to dividend payout.

Fix a state (F, W) with $W > 0$. Paying out the entire cash balance and liquidating at once is feasible from there and delivers W to the equity holders. Thus, $\hat{V}(F, W) \geq W$. Furthermore $\hat{V}$ is non-decreasing in W. To see this, take $W' > W$. From (F, W') the firm can distribute $W' - W$ at once, which leaves it at (F, W) yielding $W' - W + \hat{V}(F, W)$, so

$$\hat{V}(F, W') \geq (W' - W) + \hat{V}(F, W) > \hat{V}(F, W). \tag{A.16}$$

Hence, $-\rho\hat{V}(F,W)-(1-\pi)cF\hat{V}_W(F,W)<0$. Then (A.15) implies $\hat{V}_W(F,W)=1$. Hence, the firm pays out its entire cash balance at time T to the equity holders, and $\hat{V}(F,W)=W$. Thus, $\hat{p}(F_T,W_T)=0$.

Appendix C.2 Proof of Lemma 2

Prior to the termination shock, consider a firm with debt outstanding F and cash position W. Suppose that the firm issues Δ face amount of debt, then the debt level becomes $F+\Delta$, while the cash position becomes $W+\hat{p}(\Delta)\Delta$, where we let

$$\hat{p}(\Delta)\equiv p(F+\Delta,W+\hat{p}(\Delta)\Delta). \tag{A.17}$$

The firm's problem is

$$\max_{\Delta}\ V(F+\Delta,W+\hat{p}(\Delta)\Delta). \tag{A.18}$$

For the proposed smooth debt adjustment policy to be optimal, $\Delta=0$ must be optimal in (A.18). Taking the first-order condition with respect to Δ yields

$$\begin{aligned}&\frac{\partial}{\partial\Delta}V(F+\Delta,W+\hat{p}(\Delta)\Delta)\\&=V_F(F+\Delta,W+\hat{p}(\Delta)\Delta)+V_W(F+\Delta,W+\hat{p}(\Delta)\Delta)[\hat{p}(\Delta)+\hat{p}'(\Delta)\Delta],\end{aligned} \tag{A.19}$$

which must equal zero at $\Delta=0$, implying that

$$V_F(F,W)+V_W(F,W)p(F,W)=0. \tag{A.20}$$

That is,

$$p(F,W)=-\frac{V_F(F,W)}{V_W(F,W)}. \tag{A.21}$$

Note that this first-order condition holds along the equilibrium path. Then differentiating with respect to F and W respectively, we get

$$p_F=-\frac{V_{FF}V_W-V_FV_{FW}}{V_W^2}=-\frac{V_{FF}+pV_{FW}}{V_W}, \tag{A.22}$$

$$p_W=-\frac{V_{FW}V_W-V_FV_{WW}}{V_W^2}=-\frac{V_{FW}+pV_{WW}}{V_W}. \tag{A.23}$$

For simplicity, we have suppressed arguments F and W in (A.22) and (A.23), which hold for all (F,W) in the dividend suspension region. Then we have

$$\begin{aligned}V_W(p_F+p_Wp)&=V_W\left(-\frac{V_{FF}+pV_{FW}}{V_W}-\frac{V_{FW}+pV_{WW}}{V_W}p\right)\\&=-V_{FF}-2pV_{FW}-p^2V_{WW}\\&=-\left[V_{FF}-2V_{FW}\frac{V_F}{V_W}+V_{WW}\left(\frac{V_F}{V_W}\right)^2\right].\end{aligned} \tag{A.24}$$

From (A.17), for any δ we have

$$\hat{p}'(\delta)=p_F(F+\delta,W+\hat{p}(\delta)\delta)+p_W(F+\delta,W+\hat{p}(\delta)\delta)[\hat{p}(\delta)+\hat{p}'(\delta)\delta]. \tag{A.25}$$

Rearranging, we get

$$\hat{p}'(\delta) = \frac{p_F(F+\delta, W+\hat{p}(\delta)\delta) + p_W(F+\delta, W+\hat{p}(\delta)\delta)p(F+\delta, W+\hat{p}(\delta)\delta)}{1-\delta p_W(F+\delta, W+\hat{p}(\delta)\delta)}. \tag{A.26}$$

There exists a small $0<\delta<\bar{\delta}$ such that $\delta p_W(F+\delta, W+p(\delta)\delta)<1$, that is the denominator of (A.26) is positive. Moreover, suppose the condition (15) in Lemma 2 holds throughout the dividend suspension region, then

$$V_W(F+\delta, W+\hat{p}(\delta)\delta)\hat{p}'(\delta)<0. \tag{A.27}$$

To check for global optimality, suppose that the firm chooses any $\Delta>0$, then the firm's gain is given by

$$\begin{aligned} &V(F+\Delta, W+\hat{p}(\Delta)\Delta) - V(F,W) \\ &= \int_0^{\Delta} \left[V_F(F+\delta, W+\hat{p}(\delta)\delta) + V_W(F+\delta, W+\hat{p}(\delta)\delta)(\hat{p}(\delta)+\delta\hat{p}'(\delta))\right] d\delta \\ &= \int_0^{\Delta} \delta V_W(F+\delta, W+\hat{p}(\delta)\delta)\hat{p}'(\delta) d\delta < 0, \end{aligned} \tag{A.28}$$

where the first equality follows from the first-order condition (A.20). Analogously, we can also show that for any $\Delta<0$, $V(F+\Delta, W+\hat{p}(\Delta)\Delta)-V(F,W)<0$.

Appendix C.3 Proof of Proposition 3

Substituting $\hat{V}(F,W)=W$ from Proposition 1 into the HJB equation (23) gives

$$\begin{aligned}(\rho+\lambda)V(F,W) = \max_G \Big\{ & GV_F(F,W) + \big[b(F)+p(F,W)G\big]V_W(F,W) \\ & + \tfrac{1}{2}\tilde{\sigma}^2 V_{WW}(F,W)\Big\} + \lambda W.\end{aligned} \tag{A.29}$$

The jump term contributes λW, which does not involve G. The first-order condition in G is therefore $V_F + pV_W = 0$, i.e., (16), and substituting it back annihilates both G terms. Hence, treating F as a parameter, $V(F,\cdot)$ solves the ODE

$$\mathcal{L}_F V(F,W) + \lambda W = 0, \qquad \text{i.e.} \qquad \tfrac{1}{2}\tilde{\sigma}^2 V_{WW} + b(F)V_W - (\rho+\lambda)V + \lambda W = 0, \tag{A.30}$$

on $0<W<W^\star(F)$, subject to

$$V(F,0)=0, \qquad V_W\big(F, W^\star(F)\big)=1, \qquad V_{WW}\big(F, W^\star(F)\big)=0. \tag{A.31}$$

A. General Solution

Look for a particular solution of (A.30) of the affine form $V_p(W)=a_1W+a_0$. Substituting and matching coefficients,

$$-(\rho+\lambda)a_1+\lambda=0, \qquad ba_1-(\rho+\lambda)a_0=0 \quad\Longrightarrow\quad a_1=\frac{\lambda}{\rho+\lambda}=\theta, \qquad a_0=\frac{\lambda b}{(\rho+\lambda)^2}=\frac{\theta b}{\rho+\lambda}. \tag{A.32}$$

This particular solution has a direct interpretation. Under the passive policy of never distributing and never defaulting, W_t has drift b and $W_T = W + b(T-t)$ plus a martingale, so

$$\mathbb{E}_t\big[e^{-\rho(T-t)}W_T\big] = \frac{\lambda}{\rho+\lambda}W + b\int_0^\infty e^{-\rho s}s\,\lambda e^{-\lambda s}\mathrm{d}s = \frac{\lambda}{\rho+\lambda}W + \frac{\lambda b}{(\rho+\lambda)^2} = V_p(W), \tag{A.33}$$

the value of simply waiting for the jump and collecting the cash.

By Lemma 7, the homogeneous solutions are $e^{\gamma(F)W}$ and $e^{\eta(F)W}$, so the general solution of (A.30) is

$$V(F,W) = K(F)e^{\gamma(F)W} + H(F)e^{\eta(F)W} + \theta W + \frac{\theta b(F)}{\rho+\lambda}. \tag{A.34}$$

Differentiating and writing $\gamma(F)$ and $\eta(F)$ as γ and η respectively for brevity,

$$V_W(F,W) = \gamma K e^{\gamma W} + \eta H e^{\eta W} + \theta, \tag{A.35}$$

$$V_{WW}(F,W) = \gamma^2 K e^{\gamma W} + \eta^2 H e^{\eta W}. \tag{A.36}$$

B. Boundary Conditions at the Payout Boundary

The super-contact condition $V_{WW}(F,W^\star) = 0$ in (A.31) and (A.36) give

$$H(F)e^{\eta W^\star} = -\frac{\gamma^2}{\eta^2}K(F)e^{\gamma W^\star}. \tag{A.37}$$

Substituting into the smooth-pasting condition $V_W(F,W^\star) = 1$ and using (A.35),

$$\gamma K e^{\gamma W^\star}\left(1-\frac{\gamma}{\eta}\right) = 1-\theta \qquad \Longrightarrow \qquad K(F)e^{\gamma(F)W^\star(F)} = \frac{(1-\theta)\eta}{\gamma(\eta-\gamma)} = \mathcal{K}(F), \tag{A.38}$$

and then from (A.37),

$$H(F)e^{\eta(F)W^\star(F)} = -\frac{(1-\theta)\gamma}{\eta(\eta-\gamma)} = \mathcal{H}(F), \tag{A.39}$$

which are exactly (27) and (28). Since $\gamma < 0 < \eta$ we have $\mathcal{K} < 0 < \mathcal{H}$. Substituting (A.38)–(A.39) into (A.34) yields the representation (30),

$$V(F,W) = \mathcal{K}(F)e^{\gamma(F)[W-W^\star(F)]} + \mathcal{H}(F)e^{\eta(F)[W-W^\star(F)]} + \theta\left[W + \frac{b(F)}{\rho+\lambda}\right], \tag{A.40}$$

and, writing $\widehat{W} \equiv W - W^\star(F)$,

$$V_W(F,W) = (1-\theta)\,\frac{\eta e^{\gamma\widehat{W}} - \gamma e^{\eta\widehat{W}}}{\eta-\gamma} + \theta, \tag{A.41}$$

$$V_{WW}(F,W) = (1-\theta)\,\frac{\gamma\eta}{\eta-\gamma}\left[e^{\gamma\widehat{W}} - e^{\eta\widehat{W}}\right]. \tag{A.42}$$

Note that (A.41) and (A.42) deliver $V_W(F,W^\star) = (1-\theta)+\theta = 1$ and $V_{WW}(F,W^\star) = 0$, as required.

C. The Payout Boundary

The remaining condition is $V(F,0) = 0$. Evaluating (A.40) at $W = 0$,

$$\mathcal{K}e^{-\gamma W^\star} + \mathcal{H}e^{-\eta W^\star} + \frac{\theta b}{\rho+\lambda} = 0. \tag{A.43}$$

Dividing by $1-\theta = \rho/(\rho+\lambda)$ and using (27)-(28),

$$\frac{\eta}{\gamma(\eta-\gamma)}e^{-\gamma W^\star} - \frac{\gamma}{\eta(\eta-\gamma)}e^{-\eta W^\star} + \frac{\lambda b}{\rho(\rho+\lambda)} = 0. \tag{A.44}$$

Multiplying (A.44) by $\gamma\eta(\eta-\gamma) < 0$ and using $\gamma\eta(\eta-\gamma) = -4(\rho+\lambda)\Delta/\tilde{\sigma}^4$ from (A.5) gives the boundary equation (29),

$$\Xi\big(W^\star(F); F\big) = \frac{4\lambda\, b(F)\Delta(F)}{\rho\, \tilde{\sigma}^4}, \qquad \text{where} \qquad \Xi(W; F) \equiv \eta^2 e^{-\gamma W} - \gamma^2 e^{-\eta W}. \tag{A.45}$$

Existence and uniqueness. Differentiating Ξ in W,

$$\Xi_W(W; F) = -\gamma\eta^2 e^{-\gamma W} + \gamma^2\eta e^{-\eta W} = \gamma\eta\big[-\eta e^{-\gamma W} + \gamma e^{-\eta W}\big] > 0, \tag{A.46}$$

since $\gamma\eta < 0$ and the bracket is strictly negative. Hence $\Xi(\cdot; F)$ is strictly increasing, with $\Xi(W; F) \to +\infty$ as $W \to \infty$ because $-\gamma > 0$. At $W = 0$, by (A.5),

$$\Xi(0; F) = \eta^2 - \gamma^2 = (\eta-\gamma)(\eta+\gamma) = -\frac{4b(F)\Delta(F)}{\tilde{\sigma}^4}. \tag{A.47}$$

Therefore (A.45) has a (unique) strictly positive root if and only if $\Xi(0; F)$ lies strictly below the right-hand side, i.e.

$$-\frac{4b\Delta}{\tilde{\sigma}^4} < \frac{4\lambda b\Delta}{\rho\tilde{\sigma}^4} \iff -\rho\, b < \lambda\, b \iff (\rho+\lambda)\, b(F) > 0 \iff b(F) > 0 \iff F < \frac{\mu}{c}. \tag{A.48}$$

When $F \geq \mu/c$ the unique root is $W^\star(F) \leq 0$: any positive cash holding is immediately paid out to the equity holders and the firm is liquidated, so the firm's debt capacity is μ/c, exactly as in the benchmark without jump risk. Note that the threshold in (A.48) does not depend on λ: after dividing through by the common positive factor $4\Delta(F)/\tilde{\sigma}^4$, the inequality reduces to $(\rho+\lambda)b(F) > 0$, and since $\rho+\lambda > 0$ for every $\lambda \geq 0$ this is simply $b(F) > 0$. Jump risk shifts the level of the payout boundary but not the debt level at which it hits zero.

The benchmark $\lambda = 0$. When $\lambda = 0$ the right-hand side of (A.45) vanishes, so $\eta^2 e^{-\gamma W^\star} = \gamma^2 e^{-\eta W^\star}$, i.e. $e^{(\eta-\gamma)W^\star} = \gamma^2/\eta^2$, giving the closed form

$$W^\star(F)\Big|_{\lambda=0} = \frac{2}{\gamma(F)-\eta(F)} \ln\!\left(\frac{\eta(F)}{-\gamma(F)}\right). \tag{A.49}$$

In that case $\mathcal{H}e^{-\eta W^\star} = -\mathcal{K}e^{-\gamma W^\star}$ and (A.40) collapses to the two-term form $V = K(F)\big(e^{\gamma W} - e^{\eta W}\big)$ of the pure-diffusion model. For $\lambda > 0$ the affine particular solution breaks this symmetry, which is why no closed form for $W^\star$ survives.

D. Value at the Boundary

Evaluating the ODE (A.30) at $W = W^\star(F)$ and using $V_W = 1$, $V_{WW} = 0$ there yields

$$V\big(F, W^\star(F)\big) = \frac{b(F) + \lambda W^\star(F)}{\rho+\lambda}. \tag{A.50}$$

Differentiating (A.30) in W and evaluating at $W^\star$ gives, in addition,

$$(\rho+\lambda)\cdot 1 = b\cdot 0 + \tfrac{1}{2}\tilde{\sigma}^2 V_{WWW}(F, W^\star) + \lambda \qquad \Longrightarrow \qquad V_{WWW}\big(F, W^\star(F)\big) = \frac{2\rho}{\tilde{\sigma}^2} > 0. \tag{A.51}$$

Proof of Corollary 3.1. Expression (32) is (A.42). For $W < W^\star(F)$ we have $\widehat{W} < 0$, hence $\gamma\widehat{W} > 0 > \eta\widehat{W}$ and the bracket in (A.42) is strictly positive; since $\gamma\eta/(\eta-\gamma) < 0$ and $1-\theta > 0$, we conclude $V_{WW}(F, W) < 0$ on $(0, W^\star(F))$. Because $V_W(F, W^\star) = 1$ and V_W is strictly decreasing, $V_W(F, W) > 1$ for $0 < W < W^\star(F)$. Finally, $V_F < 0$ follows from Lemma 8 applied to $g \equiv V_F$: as shown below, g satisfies

$\mathcal{L}_F g = \tilde{c}\,V_W > 0$, $g(F,0)=0$ and $g_W(F,W^\star)=0$. Hence $V_F<0$ on $(0,W^\star]$ and $p=-V_F/V_W>0$. ■

Appendix C.4 Debt Issuance and Buyback

This appendix derives the equilibrium debt price in closed form and then proves Proposition 4. Throughout, $F<\mu/c$ is fixed and we write $\widehat{W}\equiv W-W^\star(F)$, $\gamma\equiv\gamma(F)$, $\eta\equiv\eta(F)$, $\Delta\equiv\Delta(F)$, $b\equiv b(F)$.

A. The equilibrium debt price

Rather than differentiating the closed-form solution (A.40) directly— which would require the derivative of the implicitly defined boundary $W^\star(F)$ —we obtain V_F as the solution of an ODE in the same family. Differentiating (A.30) with respect to F and using $b'(F)=-\tilde{c}$, the function $u(F,\cdot)\equiv V_F(F,\cdot)$ satisfies

$$\mathcal{L}_F u(F,W)=\tilde{c}\,V_W(F,W),\qquad 0<W<W^\star(F). \tag{A.52}$$

Its boundary conditions are obtained by differentiating the boundary conditions of V with respect to F. Since $V(F,0)=0$ for every F,

$$u(F,0)=0. \tag{A.53}$$

Differentiating the smooth-pasting condition $V_W(F,W^\star(F))=1$ gives

$$V_{FW}\big(F,W^\star\big)+V_{WW}\big(F,W^\star\big)\,W^{\star\prime}(F)=0,$$

and since $V_{WW}(F,W^\star)=0$,

$$u_W\big(F,W^\star(F)\big)=V_{FW}\big(F,W^\star(F)\big)=0. \tag{A.54}$$

Note that (A.53)–(A.54) do not involve $W^{\star\prime}(F)$, which is what makes this route tractable. Note also that (A.52)– (A.54) are exactly the hypotheses of Lemma 8, since $V_W>0$; this completes the proof of $V_F<0$ used in Corollary 3.1.

By (A.41), the source in (A.52) is

$$\tilde{c}V_W=\tilde{c}\left[\frac{(1-\theta)\eta}{\eta-\gamma}e^{\gamma\widehat{W}}-\frac{(1-\theta)\gamma}{\eta-\gamma}e^{\eta\widehat{W}}+\theta\right]. \tag{A.55}$$

Applying Lemma 7 term by term—the two exponentials are resonant and require the factor W, while the constant is handled by $\mathcal{L}_F[1]=-(\rho+\lambda)$—a particular solution is

$$-\frac{\tilde{c}}{\Delta}\,W\,\Phi(F,W)-\frac{\tilde{c}\,\theta}{\rho+\lambda},\qquad\text{where}\qquad \Phi(F,W)\equiv\frac{1-\theta}{\eta-\gamma}\left[\eta e^{\gamma\widehat{W}}+\gamma e^{\eta\widehat{W}}\right]. \tag{A.56}$$

Hence

$$V_F(F,W)=A(F)e^{\gamma\widehat{W}}+B(F)e^{\eta\widehat{W}}-\frac{\tilde{c}}{\Delta}\,W\,\Phi(F,W)-\frac{\tilde{c}\,\theta}{\rho+\lambda} \tag{A.57}$$

for constants $A(F),B(F)$ pinned down by (A.53)–(A.54). Differentiating (A.57),

$$V_{FW}(F,W)=A\gamma e^{\gamma\widehat{W}}+B\eta e^{\eta\widehat{W}}-\frac{\tilde{c}}{\Delta}\Big[\Phi(F,W)+W\,\Phi_W(F,W)\Big],\qquad \Phi_W=\frac{(1-\theta)\gamma\eta}{\eta-\gamma}\left[e^{\gamma\widehat{W}}+e^{\eta\widehat{W}}\right]. \tag{A.58}$$

Imposing (A.54) at $\widehat{W}=0$ and using $\Phi(F,W^\star)=(1-\theta)(\eta+\gamma)/(\eta-\gamma)=-(1-\theta)b/\Delta$ and $\Phi_W(F,W^\star)=2(1-\theta)\gamma\eta/(\eta-\gamma)=-2\rho/\Delta$ from (A.5),

$$\gamma A+\eta B=-\frac{\tilde{c}}{\Delta^2}\Big[(1-\theta)\,b+2\rho\,W^\star(F)\Big]\equiv R_1(F). \tag{A.59}$$

Imposing (A.53) at $W = 0$, where the term proportional to W drops out,

$$A\,e^{-\gamma W^\star} + B\,e^{-\eta W^\star} = \frac{\tilde{c}\,\theta}{\rho+\lambda} = \frac{\tilde{c}\,\lambda}{(\rho+\lambda)^2} \equiv R_2(F). \tag{A.60}$$

Writing $E_\gamma \equiv e^{-\gamma W^\star(F)}$, $E_\eta \equiv e^{-\eta W^\star(F)}$ and $D \equiv \gamma E_\eta - \eta E_\gamma < 0$, the linear system (A.59)–(A.60) solves as

$$A(F) = \frac{R_1 E_\eta - \eta R_2}{D}, \qquad B(F) = \frac{\gamma R_2 - R_1 E_\gamma}{D}. \tag{A.61}$$

The equilibrium debt price then follows from (16),

$$p(F,W) = -\frac{V_F(F,W)}{V_W(F,W)}, \tag{A.62}$$

with V_F given by (A.57)–(A.61) and V_W by (A.41). When $\lambda = 0$ we have $\theta = 0$ and $R_2 = 0$, and (A.57)–(A.61) reduce to the expression for V_F in the pure-diffusion benchmark.

The price at the payout boundary and the slope of $W^\star$. Evaluating (A.62) at $W = W^\star(F)$ where $V_W = 1$ gives $p^\star(F) = -V_F(F, W^\star(F))$. Evaluating the ODE (A.52) at $W = W^\star(F)$, where $u_W = 0$, $V_W = 1$ and $u = -p^\star$, yields

$$\tfrac{1}{2}\tilde{\sigma}^2\, V_{FWW}\big(F,W^\star\big) = \tilde{c} + (\rho+\lambda)V_F\big(F,W^\star\big), \quad \text{i.e.} \quad V_{FWW}\big(F,W^\star\big) = \frac{2\big[\tilde{c} - (\rho+\lambda)p^\star(F)\big]}{\tilde{\sigma}^2}. \tag{A.63}$$

Differentiating the super-contact condition $V_{WW}(F, W^\star(F)) = 0$ in F gives

$$V_{FWW}(F,W^\star) + V_{WWW}(F,W^\star)W^{\star\prime}(F) = 0, \tag{A.64}$$

so with (A.51),

$$W^{\star\prime}(F) = -\frac{V_{FWW}(F,W^\star)}{V_{WWW}(F,W^\star)} = -\frac{2[\tilde{c} - (\rho+\lambda)p^\star]/\tilde{\sigma}^2}{2\rho/\tilde{\sigma}^2} = \frac{(\rho+\lambda)p^\star(F) - \tilde{c}}{\rho}, \tag{A.65}$$

Combining with (A.50),

$$\phi(F) \equiv V\big(F, W^\star(F)\big) - W^\star(F) = \frac{b(F) - \rho W^\star(F)}{\rho+\lambda}, \qquad \phi'(F) = \frac{-\tilde{c} - \rho W^{\star\prime}(F)}{\rho+\lambda} = -p^\star(F), \tag{A.66}$$

and $\phi''(F) = -\rho W^{\star\prime\prime}(F)/(\rho+\lambda) = -p^{\star\prime}(F)$.

B. Second derivative in the debt level

For the denominator of (34) we also need V_{FF}. Differentiating (A.52) with respect to F and using $V_{FW} = u_W$, the function $v(F,\cdot) \equiv V_{FF}(F,\cdot)$ satisfies

$$\mathcal{L}_F v(F,W) = 2\tilde{c}\,V_{FW}(F,W), \qquad 0 < W < W^\star(F), \tag{A.67}$$

with boundary conditions obtained as before. Since $V_F(F,0) = 0$ for all F,

$$v(F,0) = 0, \tag{A.68}$$

and differentiating $V_{FW}(F, W^\star(F)) = 0$ from (A.54) gives $V_{FFW}(F,W^\star) + V_{FWW}(F,W^\star)W^{\star\prime}(F) = 0$, so using (A.63) and (A.65),

$$v_W\big(F, W^\star(F)\big) = -V_{FWW}(F,W^\star)\,W^{\star\prime}(F) = \frac{2\big[\tilde{c} - (\rho+\lambda)p^\star(F)\big]^2}{\rho\,\tilde{\sigma}^2} \geq 0. \tag{A.69}$$

By (A.58), the source of (A.67) can be written as

$$2\tilde{c}\,V_{FW} = c_1 e^{\gamma\widehat{W}} + c_2 e^{\eta\widehat{W}} + c_3\,W e^{\gamma\widehat{W}} + c_4\,W e^{\eta\widehat{W}}, \tag{A.70}$$

with

$$c_1 = 2\tilde{c}\left[A\gamma - \frac{\tilde{c}(1-\theta)\eta}{\Delta(\eta-\gamma)}\right], \quad c_2 = 2\tilde{c}\left[B\eta - \frac{\tilde{c}(1-\theta)\gamma}{\Delta(\eta-\gamma)}\right], \quad c_3 = c_4 = -\frac{2\tilde{c}^2(1-\theta)\gamma\eta}{\Delta(\eta-\gamma)}. \tag{A.71}$$

Applying Lemma 7, a particular solution of (A.67) is

$$v_p(F,W) = -\frac{c_3}{2\Delta}W^2 e^{\gamma\widehat{W}} + \frac{c_4}{2\Delta}W^2 e^{\eta\widehat{W}} - \left(\frac{c_1}{\Delta} + \frac{c_3\tilde{\sigma}^2}{2\Delta^2}\right) W e^{\gamma\widehat{W}} + \left(\frac{c_2}{\Delta} - \frac{c_4\tilde{\sigma}^2}{2\Delta^2}\right) W e^{\eta\widehat{W}}, \tag{A.72}$$

so that

$$V_{FF}(F,W) = \tilde{A}(F)e^{\gamma\widehat{W}} + \tilde{B}(F)e^{\eta\widehat{W}} + v_p(F,W), \tag{A.73}$$

where $(\tilde{A},\tilde{B})$ solve the linear system

$$\tilde{A}e^{-\gamma W^\star} + \tilde{B}e^{-\eta W^\star} = -v_p(F,0), \qquad \gamma\tilde{A} + \eta\tilde{B} = v_W\big(F,W^\star(F)\big) - \partial_W v_p\big(F,W^\star(F)\big), \tag{A.74}$$

with $v_W(F,W^\star)$ given by (A.69). Finally, we have

$$p_W = -\frac{V_{FW}(F,W) + p(F,W)V_{WW}(F,W)}{V_W(F,W)}, \qquad p_F = -\frac{V_{FF}(F,W) + p(F,W)V_{FW}(F,W)}{V_W(F,W)}, \tag{A.75}$$

which together with (A.41), (A.42), (A.58), (A.73) and (A.62) gives every object entering (34) in closed form, up to the scalar root $W^\star(F)$ of (A.45).

Two byproducts are worth recording. First, evaluating (A.75) at $W = W^\star(F)$, where $V_{FW} = V_{WW} = 0$ and $V_W = 1$, gives

$$p_W\big(F,W^\star(F)\big) = 0, \qquad p_F\big(F,W^\star(F)\big) = -V_{FF}\big(F,W^\star(F)\big) = p^{\star\prime}(F), \tag{A.76}$$

so the verification condition of Lemma 2 at the payout boundary is exactly $p^{\star\prime}(F) < 0$. Second, (A.69) shows $v_W(F,W^\star) \geq 0$: the second derivative V_{FF} is increasing in W at the boundary whenever $p^\star \neq \tilde{c}/(\rho+\lambda)$.

C. Proof of Proposition 4

Since $\hat{p} = 0$ by Proposition 1, the lender's valuation satisfies, in the dividend suspension region,

$$(\rho+\lambda)p(F,W) = c + G\big[p_F(F,W) + p(F,W)p_W(F,W)\big] + b\,p_W(F,W) + \tfrac{1}{2}\tilde{\sigma}^2 p_{WW}(F,W). \tag{A.77}$$

Indeed, $\rho p\,\mathrm{d}t = c\,\mathrm{d}t + \mathbb{E}[\mathrm{d}p]$ where $\mathbb{E}[\mathrm{d}p]$ includes the jump to $\hat{p} = 0$ at rate λ, contributing $-\lambda p\,\mathrm{d}t$.

Rewrite the equity ODE (A.30) as

$$(\rho+\lambda)V = b\,V_W + \tfrac{1}{2}\tilde{\sigma}^2 V_{WW} + \lambda W. \tag{A.78}$$

Differentiating (A.78) with respect to F and to W respectively gives

$$(\rho+\lambda)V_F = -\tilde{c}V_W + b\,V_{FW} + \tfrac{1}{2}\tilde{\sigma}^2 V_{FWW}, \tag{A.79}$$

$$(\rho+\lambda)V_W = b\,V_{WW} + \tfrac{1}{2}\tilde{\sigma}^2 V_{WWW} + \lambda. \tag{A.80}$$

From (16), $V_F = -pV_W$, so

$$V_{FW} = -p_W V_W - pV_{WW}, \tag{A.81}$$
$$V_{FWW} = -p_{WW} V_W - 2p_W V_{WW} - pV_{WWW}. \tag{A.82}$$

Substituting (A.81)–(A.82) into (A.79),

$$\begin{aligned} -(\rho+\lambda)p\, V_W = &-\tilde{c}V_W - b\big[p_W V_W + pV_{WW}\big] \\ &- \tfrac{1}{2}\tilde{\sigma}^2\big[p_{WW}V_W + 2p_W V_{WW} + pV_{WWW}\big]. \end{aligned} \tag{A.83}$$

Multiplying (A.80) by p and adding it to (A.83), the terms in pV_{WW} and pV_{WWW} cancel and we obtain

$$0 = -\tilde{c}V_W - b\,p_W V_W - \tfrac{1}{2}\tilde{\sigma}^2\big[p_{WW}V_W + 2p_W V_{WW}\big] + \lambda p. \tag{A.84}$$

Because $V_W > 0$ by Corollary 3.1, dividing by V_W gives

$$0 = -\tilde{c} - b\,p_W - \tfrac{1}{2}\tilde{\sigma}^2\left[p_{WW} + 2p_W\frac{V_{WW}}{V_W}\right] + \frac{\lambda p}{V_W}. \tag{A.85}$$

Adding (A.85) to (A.77) annihilates $b\,p_W$ and $\frac{1}{2}\tilde{\sigma}^2 p_{WW}$ and leaves, using $c - \tilde{c} = \pi c$,

$$(\rho+\lambda)p = \pi c + G\big[p_F + p\,p_W\big] - \tilde{\sigma}^2 p_W\frac{V_{WW}}{V_W} + \frac{\lambda p}{V_W}. \tag{A.86}$$

Collecting the terms in p,

$$(\rho+\lambda)p - \frac{\lambda p}{V_W} = \left[\rho + \lambda\,\frac{V_W - 1}{V_W}\right]p, \tag{A.87}$$

and rearranging (A.86) gives

$$G = \frac{\pi c - \left[\rho + \lambda\dfrac{V_W(F,W)-1}{V_W(F,W)}\right]p(F,W) + \tilde{\sigma}^2 p_W(F,W)\left(-\dfrac{V_{WW}(F,W)}{V_W(F,W)}\right)}{-\big[p_F(F,W) + p(F,W)p_W(F,W)\big]}, \tag{A.88}$$

which is (34) after substituting $\tilde{\sigma}^2 = (1-\pi)^2\sigma^2$. The denominator is strictly positive by (17) and $V_W > 0$.

Appendix C.5 Discrete-Time Optimization

This appendix rederives (34) from a discrete-time deviation argument, which also delivers the second-order condition and the size of the gain from a deviation. Fix a state (F, W) inside the dividend suspension region and recall $\mathcal{L}_F$ from (A.3). It is convenient to write the diffusion generator alone as

$$\mathcal{D}_F f(F,W) \equiv b(F)f_W(F,W) + \tfrac{1}{2}\tilde{\sigma}^2 f_{WW}(F,W). \tag{A.89}$$

Let $h > 0$ denote a short time interval. At time t the firm changes the face amount of its debt by Δ, with $\Delta > 0$ denoting issuance and $\Delta < 0$ denoting buyback, and commits not to readjust debt until time $t+h$. Let $\hat{p}_h(\Delta)$ denote the equilibrium debt price for this trade. Conditional on no jump in $[t, t+h)$, the end-of-period cash position is

$$W_h^{\Delta} = W_t + \hat{p}_h(\Delta)\Delta + (1-\pi)\big[\mu - c(F_t+\Delta)\big]h + \tilde{\sigma}\sqrt{h}\,\varepsilon, \qquad \varepsilon \sim N(0,1). \tag{A.90}$$

The jump arrives in $[t, t+h)$ with probability $\lambda h + o(h)$, in which case, by Proposition 1, equity holders receive the cash on hand, $W_t + \hat{p}_h(\Delta)\Delta + O(h)$, and lenders receive nothing beyond accrued coupons.

Competitive lenders therefore require

$$\hat{p}_h(\Delta) = \mathbb{E}_t\left[\int_0^h e^{-(\rho+\lambda)s} c\,\mathrm{d}s + e^{-(\rho+\lambda)h}\, p\big(F_t + \Delta, W_h^{\Delta}\big)\right], \tag{A.91}$$

and the equity holders' value from the deviation is

$$J_h(\Delta) = e^{-(\rho+\lambda)h}\,\mathbb{E}_t\left[V\big(F_t + \Delta, W_h^{\Delta}\big)\right] + \lambda h\Big[W_t + \hat{p}_h(\Delta)\Delta\Big] + o(h), \tag{A.92}$$

where the second term is the jump branch, $\int_0^h \lambda e^{-(\rho+\lambda)s}\hat{V}(\cdot)\mathrm{d}s = \lambda h\,\hat{V}(F_t + \Delta, W_t + \hat{p}_h(\Delta)\Delta) + o(h)$, evaluated using $\hat{V}(F, W) = W$.

A. First derivative

At $\Delta = 0$ we have $W_h^0 = W_t + b\,h + \tilde{\sigma}\sqrt{h}\,\varepsilon$, and differentiating (A.90),

$$\left.\frac{\partial W_h^{\Delta}}{\partial \Delta}\right|_{\Delta=0} = \hat{p}_h(0) - \tilde{c}\,h. \tag{A.93}$$

From (A.91),

$$\begin{aligned}\hat{p}_h(0) &= ch + \big[1 - (\rho+\lambda)h\big]\Big[p(F_t, W_t) + \mathcal{D}_F p(F_t, W_t)h\Big] + o(h)\\ &= \mathbb{E}_t\big[p(F_t, W_h^0)\big] + \big[c - (\rho+\lambda)p(F_t, W_t)\big]h + o(h),\end{aligned} \tag{A.94}$$

using $\mathbb{E}_t[p(F_t, W_h^0)] = p(F_t, W_t) + \mathcal{D}_F p(F_t, W_t)h + o(h)$. Differentiating the first term of (A.92) and using $V_F(F_t, W_h^0) = -p(F_t, W_h^0)V_W(F_t, W_h^0)$,

$$\begin{aligned}&\mathbb{E}_t\Big[V_W(F_t, W_h^0)\big(\hat{p}_h(0) - p(F_t, W_h^0) - \tilde{c}h\big)\Big]\\ &\quad= \mathbb{E}_t\big[V_W(F_t, W_h^0)\big]\Big(\hat{p}_h(0) - \mathbb{E}_t\big[p(F_t, W_h^0)\big] - \tilde{c}h\Big) - \mathrm{Cov}_t\Big(V_W(F_t, W_h^0),\, p(F_t, W_h^0)\Big)\\ &\quad= V_W(F_t, W_t)\Big[\pi c - (\rho+\lambda)p(F_t, W_t) - \tilde{\sigma}^2 p_W(F_t, W_t)\frac{V_{WW}(F_t, W_t)}{V_W(F_t, W_t)}\Big]h + o(h),\end{aligned} \tag{A.95}$$

using $c - \tilde{c} = \pi c$ and $\mathrm{Cov}_t(V_W, p) = \tilde{\sigma}^2 V_{WW} p_W\, h + o(h)$. The jump branch of (A.92) contributes $\lambda h\,\hat{p}_h(0) = \lambda\, p(F_t, W_t)h + o(h)$ to the derivative. Adding the two pieces and defining

$$N(F_t, W_t) \equiv \pi c - \left[\rho + \lambda\frac{V_W(F_t, W_t) - 1}{V_W(F_t, W_t)}\right] p(F_t, W_t) + \tilde{\sigma}^2 p_W(F_t, W_t)\left(-\frac{V_{WW}(F_t, W_t)}{V_W(F_t, W_t)}\right), \tag{A.96}$$

we obtain, using $-(\rho+\lambda)p + \lambda p/V_W = -\big[\rho + \lambda(V_W - 1)/V_W\big]p$,

$$J_h'(0) = V_W(F_t, W_t)\,N(F_t, W_t)\,h + o(h). \tag{A.97}$$

This reproduces the numerator of (34) and confirms the continuous-time derivation of Appendix C.4 by an independent route.

B. Second derivative

Differentiating (A.91) with respect to Δ and evaluating at zero,

$$\begin{aligned}\hat{p}_h'(0) &= e^{-(\rho+\lambda)h}\,\mathbb{E}_t\Big[p_F(F_t, W_h^0) + p_W(F_t, W_h^0)\big(\hat{p}_h(0) - \tilde{c}h\big)\Big]\\ &= p_F(F_t, W_t) + p_W(F_t, W_t)p(F_t, W_t) + o(1).\end{aligned} \tag{A.98}$$

Let

$$\mathcal{Q}(F_t,W_t) \equiv -\big[p_F(F_t,W_t) + p_W(F_t,W_t)p(F_t,W_t)\big] > 0, \tag{A.99}$$

which is positive by (17), so that $\hat{p}_h'(0) = -\mathcal{Q} + o(1)$. A second-order differentiation of the first term of (A.92) gives

$$\begin{aligned} J_h''(0) = V_{FF}(F_t,W_t) + 2p(F_t,W_t)V_{FW}(F_t,W_t) + p(F_t,W_t)^2 V_{WW}(F_t,W_t) \\ - 2\mathcal{Q}(F_t,W_t)V_W(F_t,W_t) + o(1) = -V_W(F_t,W_t)\mathcal{Q}(F_t,W_t) + o(1), \end{aligned} \tag{A.100}$$

The jump branch of (A.92) contributes only $O(h)$ to $J_h''(0)$ and therefore does not affect the leading term.

C. Optimization

Taking the first-order condition $J_h'(\Delta_h^*) = 0$ and expanding around $\Delta = 0$,

$$0 = J_h'(0) + J_h''(0)\Delta_h^* + o(h) + o(\Delta_h^*), \tag{A.101}$$

so by (A.97) and (A.100), $\mathcal{Q}\Delta_h^* = Nh + o(h) + o(\Delta_h^*)$. Hence $\Delta_h^* = O(h)$ and

$$\Delta_h^* = \frac{N(F_t,W_t)}{\mathcal{Q}(F_t,W_t)}\,h + o(h), \tag{A.102}$$

which is precisely $G_t h$ with G_t as in (34). When $\overline{W}(F_t) < W_t < W^\star(F_t)$ we have $N(F_t,W_t) < 0$ and therefore $\Delta_h^* < 0$: the firm repurchases. Finally, by Taylor expansion,

$$J_h(\Delta_h^*) - J_h(0) = \tfrac{1}{2}\,V_W(F_t,W_t)\,\frac{N(F_t,W_t)^2}{\mathcal{Q}(F_t,W_t)}\,h^2 + o(h^2) > 0, \tag{A.103}$$

so in the buyback region repurchasing Δ_h^* in face amount of debt results in a strictly positive gain of order h^2. Note that the jump enters (A.103) only through N, and specifically through the liquidation wedge: holding everything else fixed, a larger λ makes N more negative in the buyback region and hence the gain from a buyback larger.

Appendix C.6 Equivalence Between Setting Price and Setting Quantity

In the dividend suspension region $0 < W < W^\star(F)$, Corollary 3.1 establishes $V_W(F,W) > 0$. Suppose the first- and second-order conditions in Lemma 2 hold. Since (16) is equivalent to (17), i.e.

$$V_W(F,W)\big[p_F(F,W) + p_W(F,W)p(F,W)\big] < 0, \tag{A.104}$$

and $V_W > 0$, we have

$$p_F(F,W) + p_W(F,W)p(F,W) < 0. \tag{A.105}$$

The inequality (A.105) implies that the debt price is decreasing in debt issuance. To see this, suppose the firm has debt outstanding F and cash position W and issues Δ face amount of debt, so the debt level becomes $F+\Delta$ and the cash position becomes $W + p(\Delta)\Delta$, where, as in Appendix C.2,

$$p(\Delta) \equiv p\big(F+\Delta, W + p(\Delta)\Delta\big), \qquad p'(\Delta) = \left.\frac{p_F + p_W p}{1 - \Delta p_W}\right|_{(F+\Delta,\,W+p(\Delta)\Delta)}. \tag{A.106}$$

Thus $p'(0) < 0$ and the debt price $p(\Delta)$ is locally strictly decreasing. Since $p'(\cdot)$ is continuous, there exists $\epsilon > 0$ such that $p'(\delta) < 0$ for all $\delta \in (0, \epsilon)$; extending along the path, for any $\Delta < \bar{\Delta}$,

$$p(\bar{\Delta}) - p(\Delta) = \int_{\Delta}^{\bar{\Delta}} p'(\delta)\,\mathrm{d}\delta < 0. \tag{A.107}$$

Hence $p(\Delta)$ is strictly decreasing in Δ, so the mapping from Δ to $p(\Delta)$ is one-to-one.

In the main model, the equity holders maximize by choosing debt issuance Δ:

$$\max_{\Delta}\ g(\Delta) \equiv V\big(F + \Delta, W + p(\Delta)\Delta\big). \tag{A.108}$$

Now suppose the equity holders maximize by choosing the debt price instead. Since the mapping from Δ to $p(\Delta)$ is one-to-one, we can write their problem as

$$\max_{p}\ \mathfrak{h}(p) \equiv V\big(F + \Delta(p), W + p\,\Delta(p)\big) = g\big(\Delta(p)\big). \tag{A.109}$$

By the chain rule, $\mathfrak{h}'(p) = g'(\Delta(p))\Delta'(p)$, so the first-order condition is identical whether the equity holders maximize by choosing quantity or price. Similarly, $\mathfrak{h}''(p) = g''(\Delta(p))[\Delta'(p)]^2 + g'(\Delta(p))\Delta''(p)$, and at an interior optimum $g'(\Delta(p)) = 0$, so

$$\mathfrak{h}''(p) = g''(\Delta)\big|_{\Delta=\Delta(p)}\big[\Delta'(p)\big]^2, \tag{A.110}$$

and the second-order condition is also identical under the two formulations. Jump risk plays no role in this argument beyond its effect on the equilibrium objects V and p, since a trade is executed at a single instant.

Appendix C.7 Verification of Optimality Condition in Lemma 2

We verify the optimality condition in Lemma 2 in this appendix.

A. The dividend payout region

From Proposition 3, in the dividend payout region the equity holders' value function is affine in cash, $V(F, W) = V(F, W^\star(F)) + [W - W^\star(F)]$, so (16) reduces to $\phi''(F) > 0$ where $\phi(F) \equiv V(F, W^\star(F)) - W^\star(F)$. By (A.66),

$$\phi(F) = \frac{b(F) - \rho W^\star(F)}{\rho + \lambda}, \qquad \phi''(F) = -\frac{\rho}{\rho + \lambda} W^{\star\prime\prime}(F) = -p^{\star\prime}(F). \tag{A.111}$$

Hence, in the payout region the verification condition is equivalent to each of the following three statements:

$$\phi''(F) > 0 \iff W^{\star\prime\prime}(F) < 0 \iff p^{\star\prime}(F) < 0 \iff V_{FF}\big(F, W^\star(F)\big) > 0, \tag{A.112}$$

the last equivalence following from (A.76). That is: the condition holds if and only if the payout boundary is concave in the debt level, equivalently if and only if the equilibrium debt price at the boundary is decreasing in the amount of debt outstanding. This is a considerably more transparent statement than in the pure-diffusion benchmark, and it is the natural object to check numerically since $W^\star(\cdot)$ is computed as part of the solution.

The benchmark $\lambda = 0$ in closed form. When $\lambda = 0$, $W^\star$ is available in closed form from (A.49) and (A.112) can be verified analytically. Substituting (A.49) into (A.40) with $\theta = 0$ and using (A.5) gives

$V(F, W^\star(F)) = (\gamma+\eta)/(\gamma\eta) = b/\rho$, so

$$\phi(F) = \frac{b(F)}{\rho} - \frac{2\tilde{\sigma}^2}{\Delta(F)} \ln\left(\frac{\Delta(F)+b(F)}{\Delta(F)-b(F)}\right), \qquad \Delta(F) = \sqrt{b(F)^2 + 2\rho\tilde{\sigma}^2}. \tag{A.113}$$

Because $b(F) = \tilde{\mu} - \tilde{c}F$ is affine with $b'(F) = -\tilde{c}$, we have $\phi''(F) = \tilde{c}^2\hat{\phi}''(b)$ where

$$\hat{\phi}(b) = \frac{b}{\rho} - \frac{2\tilde{\sigma}^2}{\Delta(b)} \ln\frac{\Delta(b)+b}{\Delta(b)-b}, \qquad \Delta(b) = \sqrt{b^2 + 2\rho\tilde{\sigma}^2}. \tag{A.114}$$

Let $\tilde{s} \equiv \tilde{\sigma}\sqrt{2\rho}$ and $u \equiv b/\tilde{s} > 0$, so $\Delta(b) = \tilde{s}\sqrt{1+u^2}$ and

$$\ln\frac{\Delta(b)+b}{\Delta(b)-b} = \ln\left(\frac{\sqrt{1+u^2}+u}{\sqrt{1+u^2}-u}\right) = 2\ln\left(\sqrt{1+u^2}+u\right) = 2\,\mathrm{arcsinh}\,u. \tag{A.115}$$

Hence $\hat{\phi}(b) = \frac{b}{\rho} - \frac{2\tilde{\sigma}^2}{\tilde{s}}\,h(u)$ with $h(u) \equiv \mathrm{arcsinh}(u)/\sqrt{1+u^2}$, and $\hat{\phi}''(b) = -\frac{2\tilde{\sigma}^2}{\tilde{s}^3}h''(u)$, so the condition $\phi''(F) > 0$ is equivalent to $h''(u) < 0$. A direct computation gives

$$(1+u^2)^{5/2}h''(u) = P(u) \equiv (2u^2-1)\,\mathrm{arcsinh}\,u - 3u\sqrt{1+u^2}. \tag{A.116}$$

We have $P(0) = 0$ and

$$P'(u) = 4\left(u\,\mathrm{arcsinh}\,u - \sqrt{1+u^2}\right) \equiv 4Q(u), \tag{A.117}$$

with $Q(0) = -1$ and $Q'(u) = \mathrm{arcsinh}\,u > 0$ for $u > 0$. Thus Q increases monotonically from $Q(0) = -1$ and changes sign exactly once, at some $u_0 > 0$. Consequently P is strictly decreasing on $(0, u_0)$ and strictly increasing for $u > u_0$. Since $P(0) = 0$ and P first decreases, $P(u) < 0$ on $(0, u_1)$ where u_1 is the unique positive root of P, i.e., the unique positive solution of $(2u^2-1)\,\mathrm{arcsinh}\,u = 3u\sqrt{1+u^2}$, so that $u_1 \approx 2.70$. Hence $h''(u) < 0$ and $\phi''(F) > 0$ for all $u \in (0, u_1)$, or equivalently for all

$$F > \frac{\mu - \sigma u_1\sqrt{2\rho}}{c}. \tag{A.118}$$

Under the baseline parameters this bound is negative and the condition therefore holds on the entire relevant debt range $[0, \mu/c)$.

For $\lambda > 0$ no closed form for $W^\star$ is available, and we verify $W^{\star\prime\prime}(F) < 0$ numerically over the relevant debt range, using (A.65) to compute $W^{\star\prime}$ analytically from $p^\star$.

B. The dividend suspension region

In the dividend suspension region the condition (16) is checked numerically on the solved model, by evaluating the denominator of (34), that is $\mathcal{Q}(F, W) = -[p_F + p\,p_W]$ from (A.99), on a grid over $(F, W) \in [0, \mu/c) \times (0, W^\star(F))$ and confirming it is strictly positive throughout. All objects entering $\mathcal{Q}$ are available in closed form: V_W from (A.41), V_{WW} from (A.42), V_{FW} from (A.58), V_{FF} from (A.73), and p, p_W, p_F from (A.62) and (A.75), each evaluated at the scalar root $W^\star(F)$ of (A.45).

We verify the condition numerically for both the dividend payout region and the dividend suspension region. Our parameter values are: $\mu = 0.044$, $\sigma = 0.60$, $c = 0.0022$, $\rho = 0.03$, and $\pi = 0.05$, and $\lambda = 0.2$. The results are presented in Figure C.1.

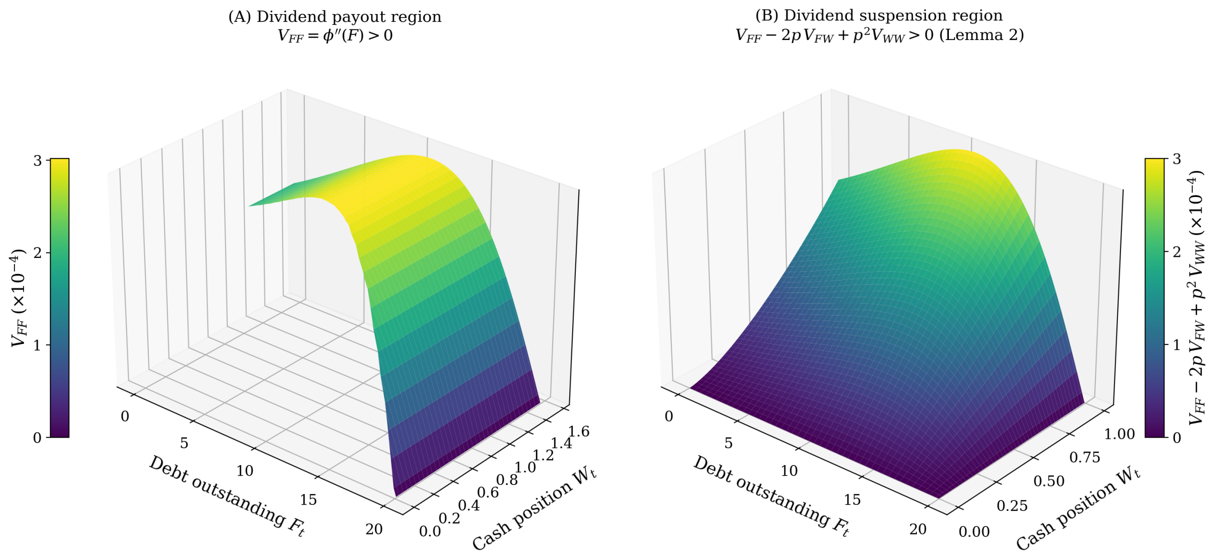

Figure C.1. Numerical verification of Lemma 2 condition. The figure numerically verifies the optimality condition in Lemma 2 under the parameters $\mu = 0.044$, $\sigma = 0.60$, $c = 0.0022$, $\rho = 0.03$, and $\pi = 0.05$, and $\lambda = 0.2$ for both the dividend payout region (panel A) and the dividend suspension region (panel B).

Appendix C.8 Optimality of Payout Policy in Dividend Payout Region

We prove that when the firm is above the payout boundary, that is $W > W^\star(F)$, it is optimal to distribute excess cash and leave debt unchanged. In the payout region, the candidate value function for the equity holders is

$$V(F,W) = V\big(F, W^\star(F)\big) + W - W^\star(F) = \phi(F) + W, \qquad W \geq W^\star(F), \tag{A.119}$$

where $\phi(F) \equiv V(F, W^\star(F)) - W^\star(F)$ is given in closed form by (A.66). As established in Appendix C.7, $\phi''(F) > 0$ over the relevant debt range.

Under the candidate policy, whenever $W > W^\star(F)$ the firm distributes excess cash immediately and returns to the payout boundary $W^\star(F)$. Thus the candidate debt price satisfies

$$p(F,W) = p\big(F, W^\star(F)\big) \equiv p^\star(F), \qquad W \geq W^\star(F), \tag{A.120}$$

and by (A.66),

$$p^\star(F) = -\phi'(F), \qquad \text{hence} \qquad p^{\star\prime}(F) = -\phi''(F) < 0. \tag{A.121}$$

Thus, under the candidate equilibrium, debt becomes cheaper when the firm has more debt outstanding. Note that this is the corrected sign relative to a pure-diffusion statement: $\phi'' > 0$ is exactly what makes $p^{\star\prime} < 0$.

A. Deviations that remain in the payout region

Suppose the firm is at (F, W) with $W \geq W^\star(F)$ and follows the candidate policy. Now suppose it deviates by trading δ in face amount of debt. The post-trade state becomes $(F + \delta, W + \hat{p}(\delta)\delta)$ where

$\hat{p}(\delta) = p(F+\delta, W+\hat{p}(\delta)\delta)$. If the post-trade state remains in the payout region, then under the candidate pricing rule $\hat{p}(\delta) = p^\star(F+\delta)$. Denote by $J(\delta)$ the equity holders' continuation value after the deviation, then using (A.119) and (A.121),

$$J(\delta) = W + \phi(F+\delta) + p^\star(F+\delta)\delta = W + \phi(F+\delta) - \delta\,\phi'(F+\delta). \tag{A.122}$$

Hence

$$J(\delta) - J(0) = \phi(F+\delta) - \delta\phi'(F+\delta) - \phi(F) = -\int_0^\delta s\,\phi''(F+s)\,\mathrm{d}s < 0 \tag{A.123}$$

for every $\delta \neq 0$, since $\phi'' > 0$.

B. Deviations that enter the suspension region

Now consider a deviation that moves the firm into the dividend suspension region, that is $W+\hat{p}(\delta)\delta < W^\star(F+\delta)$. For such states, the candidate value function is the suspension-region solution (A.40). Define $J(\delta) = V(F+\delta, W+\hat{p}(\delta)\delta)$ with $\hat{p}(\delta) = p(F+\delta, W+\hat{p}(\delta)\delta)$. Differentiating,

$$J'(\delta) = V_F\big(F+\delta, W+\hat{p}(\delta)\delta\big) + V_W\big(F+\delta, W+\hat{p}(\delta)\delta\big)\big[\hat{p}(\delta) + \hat{p}\,'(\delta)\delta\big]. \tag{A.124}$$

In the suspension region, the candidate solution satisfies the first-order condition $V_F + V_W\hat{p}(\delta) = 0$, so

$$J'(\delta) = \delta\, V_W\big(F+\delta, W+\hat{p}(\delta)\delta\big)\,\hat{p}\,'(\delta). \tag{A.125}$$

Since $V_W > 0$ by Corollary 3.1 and $\hat{p}\,'(\delta) < 0$, we have $J'(\delta) > 0$ for $\delta < 0$ and $J'(\delta) < 0$ for $\delta > 0$. Hence $J(\delta) < J(0)$ for all nonzero feasible deviations, and the candidate payout policy is optimal.

Appendix C.9 Costly Equity Issuance with Bounded Issuance Rate

In this appendix, we relax the assumption of no external equity issuance and instead assume the firm can raise external equity at a proportional cost, subject to an upper bound on the rate at which funds can be raised. Let I_t denote the cumulative external equity issuance, assumed absolutely continuous with $\mathrm{d}I_t = i_t\mathrm{d}t$ and $i_t \in [0, \bar{I}]$, where $\bar{I} \in (0,\infty)$ is an exogenous cap on the equity issuance rate. Since the firm liquidates at the jump by Proposition 1, equity is raised only on $\{t < T\}$.

The firm's cash dynamics become, on $\{t < T\}$,

$$\mathrm{d}W_t = \big[(1-\pi)(\mu - cF_t) + p_tG_t + i_t\big]\mathrm{d}t + (1-\pi)\sigma\,\mathrm{d}Z_t - \mathrm{d}D_t, \tag{A.126}$$

and the equity holders' problem is

$$V(F_t, W_t) = \max_{\{G_s, D_s, i_s\}} \mathbb{E}_t\left[\int_t^{\tau\wedge T} e^{-\rho(s-t)}\big(\mathrm{d}D_s - (1+\kappa)\mathrm{d}I_s\big) + e^{-\rho(T-t)}\mathbf{1}_{\{T<\tau\}}W_T\right]. \tag{A.127}$$

Because i_t is bounded, the firm's cash position can reach zero in finite time with positive probability. As in the baseline model, bankruptcy occurs when the firm depletes cash at $W_t = 0$. Since bankruptcy is costly, the equity holders conserve liquidity by suspending dividends and only distribute cash when it is sufficiently abundant.

When dividends are suspended, the equity holders' value function solves

$$(\rho+\lambda)V(F,W) = \max_{G,\, i\in[0,\bar{I}]} \Big\{ G V_F(F,W) + \big[b(F) + p\,G + i\big] V_W(F,W) + \tfrac{1}{2}\tilde{\sigma}^2 V_{WW}(F,W) - (1+\kappa) i \Big\} + \lambda W, \tag{A.128}$$

where we have used $\hat{V}(F,W) = W$. The control i enters through the linear form $[V_W(F,W) - (1+\kappa)]i$, so the issuance policy is bang-bang,

$$i(F,W) = \begin{cases} \bar{I} & \text{if } V_W(F,W) > 1+\kappa, \\ 0 & \text{if } V_W(F,W) < 1+\kappa. \end{cases} \tag{A.129}$$

We conjecture and later verify that $V_W(F,\cdot)$ is strictly decreasing. The firm issues equity when it is close to liquidity-driven default, and the equity issuance region is an interval $[0, \underline{W}(F))$ adjacent to the default boundary, where $\underline{W}(F)$ is defined by

$$V_W\big(F, \underline{W}(F)\big) = 1+\kappa. \tag{A.130}$$

Hence the dividend suspension region divides into an equity issuance sub-region (sub-region ℓ) and an equity non-issuance sub-region (sub-region h).

A. Sub-region h in closed form

In the dividend suspension region (both sub-regions), the first-order condition with respect to G in (A.128) is again $p = -V_F/V_W$, and substituting it back annihilates the G terms, so F can be treated as a parameter. We solve from the payout boundary inwards. In sub-region h the equity holders receive no flow payment and $i = 0$, so $V^h(F,\cdot)$ solves the same equation as the baseline,

$$\tfrac{1}{2}\tilde{\sigma}^2 V^h_{WW} + b(F) V^h_W - (\rho+\lambda) V^h + \lambda W = 0, \tag{A.131}$$

whose general solution, by Appendix C.3, is

$$V^h(F,W) = K(F) e^{\gamma(F)W} + H(F) e^{\eta(F)W} + \theta W + \frac{\theta b(F)}{\rho+\lambda}, \tag{A.132}$$

with γ, η as given in (A.4). Imposing the value-matching and super-contact conditions at the dividend payout boundary, $V^h_W(F, W^\star(F)) = 1$ and $V^h_{WW}(F, W^\star(F)) = 0$, yields $K(F) e^{\gamma W^\star} = \mathcal{K}(F)$ and $H(F) e^{\eta W^\star} = \mathcal{H}(F)$, so that with $\widehat{W} \equiv W - W^\star(F)$,

$$V^h(F,W) = \mathcal{K}(F) e^{\gamma \widehat{W}} + \mathcal{H}(F) e^{\eta \widehat{W}} + \theta \left[W + \frac{b(F)}{\rho+\lambda} \right], \tag{A.133}$$

$$V^h_W(F,W) = (1-\theta) \frac{\eta e^{\gamma \widehat{W}} - \gamma e^{\eta \widehat{W}}}{\eta - \gamma} + \theta, \tag{A.134}$$

$$V^h_{WW}(F,W) = (1-\theta) \frac{\gamma \eta}{\eta - \gamma} \left[e^{\gamma \widehat{W}} - e^{\eta \widehat{W}} \right]. \tag{A.135}$$

Since $\gamma < 0 < \eta$, for $W < W^\star(F)$ we have $e^{\gamma \widehat{W}} > e^{\eta \widehat{W}}$ and consequently $V^h_{WW}(F,W) < 0$. Thus, given F, $V^h_W(F,\cdot)$ is strictly decreasing on $(0, W^\star(F))$, which verifies the conjecture made above. Note that unlike in the baseline model, $W^\star(F)$ is not determined by (A.45) here; it is pinned down jointly with $\underline{W}(F)$ below.

The equity issuance threshold $\underline{W}(F)$ can be expressed as a function of the payout boundary through

(A.130) and (A.134):

$$(1-\theta)\,\frac{\eta(F)e^{\gamma(F)[\underline{W}(F)-W^\star(F)]}-\gamma(F)e^{\eta(F)[\underline{W}(F)-W^\star(F)]}}{\eta(F)-\gamma(F)}=1+\kappa-\theta. \tag{A.136}$$

Because the left-hand side is strictly decreasing in $\underline{W}-W^\star$ and equals $1-\theta$ at $\underline{W}=W^\star$, equation (A.136) determines the *gap* $\underline{W}(F)-W^\star(F)<0$ uniquely for any $\kappa>0$.

B. Proof of Proposition 5

The construction above presumes $\underline{W}(F)>0$. Since $V_W(F,\cdot)$ is strictly decreasing on $(0,W^\star(F))$, the firm never finds it optimal to issue equity if and only if $V_W(F,0)\le 1+\kappa$, in which case the solution coincides with the baseline model of Proposition 3. Let V denote that baseline value function. Evaluating (A.41) at $W=0$,

$$V_W(F,0)=(1-\theta)\,\frac{\eta(F)e^{-\gamma(F)W^\star(F)}-\gamma(F)e^{-\eta(F)W^\star(F)}}{\eta(F)-\gamma(F)}+\theta, \tag{A.137}$$

where $W^\star(F)$ is the baseline payout boundary solving (A.45). Thus sub-region ℓ is empty if and only if $\kappa\ge\bar\kappa(F)$ with

$$\bar\kappa(F)\equiv V_W(F,0)-1=\frac{\rho}{\rho+\lambda}\left[\frac{\eta(F)e^{-\gamma(F)W^\star(F)}-\gamma(F)e^{-\eta(F)W^\star(F)}}{\eta(F)-\gamma(F)}-1\right], \tag{A.138}$$

where we used $\theta-1=-(1-\theta)=-\rho/(\rho+\lambda)$. This is (40). Given F, for $\kappa<\bar\kappa(F)$ the equity issuance sub-region is nonempty and $\underline{W}(F)>0$.

Sign and monotonicity of $\bar\kappa$. Write $\Lambda(x)\equiv\big[\eta e^{-\gamma x}-\gamma e^{-\eta x}\big]/(\eta-\gamma)$, so that $\bar\kappa(F)=(1-\theta)[\Lambda(W^\star(F))-1]$. Then $\Lambda(0)=1$ and

$$\Lambda'(x)=\frac{-\gamma\eta\big[e^{-\gamma x}-e^{-\eta x}\big]}{\eta-\gamma}>0\qquad\text{for } x>0, \tag{A.139}$$

since $-\gamma\eta>0$ and $-\gamma>0>-\eta$. Hence $\bar\kappa(F)>0$ whenever $W^\star(F)>0$, i.e. whenever $F<\mu/c$; $\bar\kappa$ is strictly increasing in the payout boundary; and $\bar\kappa(F)\to 0$ as $F\to\mu/c$, since $W^\star(F)\to 0$ by (A.48). Equity issuance is therefore not a device that equity holders use to rescue the firm at the limit of its debt capacity.

For monotonicity in F, note that $\bar\kappa(F)=V_W(F,0)-1$ and the evaluation point $W=0$ does not move with F, so

$$\bar\kappa'(F)=V_{FW}(F,0)=u_W(F,0), \tag{A.140}$$

where $u=V_F$ solves (A.52)–(A.54). Since $\mathcal{L}_F u=\tilde c V_W>0$, $u(F,0)=0$ and $u_W(F,W^\star(F))=0$, Lemma 8 delivers $u_W(F,0)<0$. Hence $\bar\kappa'(F)<0$ on $[0,\mu/c)$, so $\bar\kappa$ is strictly decreasing, and if $\kappa\ge\bar\kappa(0)$ the firm never raises external equity at any debt level. This completes the proof of Proposition 5.

Two comparative statics in λ follow immediately from (A.138). The explicit factor $\rho/(\rho+\lambda)$ is decreasing in λ, and $\bar\kappa$ is strictly increasing in $W^\star(F)$ by (A.139), which is itself decreasing in λ. Both channels lower the ceiling on the equity issuance cost for which outside equity is used at all.

C. Sub-region ℓ and the location of the boundaries

For a given F, if $\kappa<\bar\kappa(F)$ then sub-region ℓ is nonempty. In this sub-region dividends are suspended while $i=\bar I$, so the HJB equation (A.128) becomes, after substituting the first-order condition in G,

$$(\rho+\lambda)V^\ell(F,W)=\big[b(F)+\bar I\big]V^\ell_W(F,W)+\tfrac12\tilde\sigma^2V^\ell_{WW}(F,W)-(1+\kappa)\bar I+\lambda W. \tag{A.141}$$

An affine particular solution $\theta W + C_\ell(F)$ requires $-(\rho+\lambda)\theta + \lambda = 0$, which holds, and

$$C_\ell(F) = \frac{\theta\big[b(F)+\bar{I}\big] - (1+\kappa)\bar{I}}{\rho+\lambda}. \tag{A.142}$$

Treating F as a parameter, the general solution is therefore

$$V^\ell(F,W) = M(F)e^{\nu(F)W} + N(F)e^{\varphi(F)W} + \theta W + C_\ell(F), \tag{A.143}$$

where the characteristic roots now use the drift $b(F)+\bar{I}$ and the effective discount rate $\rho+\lambda$:

$$\nu(F) = -\frac{\big[b(F)+\bar{I}\big] + \sqrt{\big[b(F)+\bar{I}\big]^2 + 2(\rho+\lambda)\tilde{\sigma}^2}}{\tilde{\sigma}^2} < 0, \tag{A.144}$$

$$\varphi(F) = -\frac{\big[b(F)+\bar{I}\big] - \sqrt{\big[b(F)+\bar{I}\big]^2 + 2(\rho+\lambda)\tilde{\sigma}^2}}{\tilde{\sigma}^2} > 0. \tag{A.145}$$

The value-matching and smooth-pasting conditions at $\underline{W}(F)$ require

$$V^\ell\big(F,\underline{W}(F)\big) = V^h\big(F,\underline{W}(F)\big) \equiv \bar{V}^h(F), \qquad V^\ell_W\big(F,\underline{W}(F)\big) = V^h_W\big(F,\underline{W}(F)\big) = 1+\kappa, \tag{A.146}$$

where, from (A.133),

$$\bar{V}^h(F) = \mathcal{K}(F)e^{\gamma(F)[\underline{W}(F)-W^\star(F)]} + \mathcal{H}(F)e^{\eta(F)[\underline{W}(F)-W^\star(F)]} + \theta\left[\underline{W}(F) + \frac{b(F)}{\rho+\lambda}\right], \tag{A.147}$$

and the gap $\underline{W}(F) - W^\star(F)$ is pinned down by (A.136). Writing $m \equiv M(F)e^{\nu\underline{W}}$, $n \equiv N(F)e^{\varphi\underline{W}}$ and

$$\Theta(F) \equiv \bar{V}^h(F) - \theta\,\underline{W}(F) - C_\ell(F), \tag{A.148}$$

the two conditions (A.146) read $m+n=\Theta$ and $\nu m + \varphi n = 1+\kappa-\theta$, so

$$m = \frac{\varphi(F)\Theta(F) - (1+\kappa-\theta)}{\varphi(F)-\nu(F)}, \qquad n = \frac{(1+\kappa-\theta) - \nu(F)\Theta(F)}{\varphi(F)-\nu(F)}. \tag{A.149}$$

Hence, for $0 \le W < \underline{W}(F)$,

$$V^\ell(F,W) = m\,e^{\nu(F)[W-\underline{W}(F)]} + n\,e^{\varphi(F)[W-\underline{W}(F)]} + \theta W + C_\ell(F). \tag{A.150}$$

The equity issuance boundary $\underline{W}(F)$ is finally determined by the zero recovery condition at default, $V^\ell(F,0) = 0$, that is

$$m\,e^{-\nu(F)\underline{W}(F)} + n\,e^{-\varphi(F)\underline{W}(F)} + C_\ell(F) = 0. \tag{A.151}$$

Since the gap $\underline{W}(F) - W^\star(F)$ is pinned down by (A.136)—so that $\bar{V}^h$ and Θ in (A.147)–(A.148) depend on $\underline{W}(F)$ only through $\underline{W}$ itself and the known gap—equation (A.151) is a scalar equation in $\underline{W}(F)$, and $W^\star(F)$ then follows from the gap. Setting $\lambda = 0$ (so $\theta = 0$) recovers the corresponding system in the pure-diffusion benchmark.

D. Verification of concavity

For $b(F) + \bar{I} \ge 0$, we show that $V_{WW}(F,W) < 0$ for all $0 \le W < W^\star(F)$. This has already been established for sub-region h: since $\gamma < 0 < \eta$, for $W < W^\star(F)$ we have $e^{\gamma\widehat{W}} > e^{\eta\widehat{W}}$ and consequently $V^h_{WW}(F,W) < 0$ by (A.135), so $V^h_W(F,\cdot)$ is strictly decreasing on $(0, W^\star(F))$. Moreover, the smooth-pasting condition at the equity issuance boundary and C^2 matching imply $V^\ell_{WW}(F,\underline{W}(F)) =$

$V^h_{WW}(F, \underline{W}(F)) < 0$.

We now show $V^\ell_{WW}(F,W) < 0$ on $[0, \underline{W}(F))$. Differentiating (A.141) with respect to W yields

$$\tfrac{1}{2}\tilde{\sigma}^2 V^\ell_{WWW} + \big[b(F) + \bar{I}\big] V^\ell_{WW} - (\rho+\lambda) V^\ell_W + \lambda = 0, \tag{A.152}$$

that is,

$$V^\ell_{WWW}(F,W) = \frac{2}{\tilde{\sigma}^2}\Big\{(\rho+\lambda)V^\ell_W(F,W) - \lambda - \big[b(F)+\bar{I}\big]V^\ell_{WW}(F,W)\Big\}. \tag{A.153}$$

Let $W_0 \equiv \inf\big\{W < \underline{W}(F) : V^\ell_{WW} < 0 \text{ on } (W, \underline{W}(F)]\big\}$, which is well defined by the previous paragraph and continuity. On $(W_0, \underline{W}(F)]$ we have $V^\ell_{WW} < 0$, so V^ℓ_W is strictly decreasing there and therefore $V^\ell_W(F,W) > V^\ell_W(F, \underline{W}(F)) = 1+\kappa > 1$. Consequently $(\rho+\lambda)V^\ell_W - \lambda > (\rho+\lambda) - \lambda = \rho > 0$, and since $b(F)+\bar{I} \geq 0$ and $V^\ell_{WW} < 0$, the second bracketed term in (A.153) is also nonnegative. Hence $V^\ell_{WWW} > 0$ on $(W_0, \underline{W}(F)]$, that is V^ℓ_{WW} is strictly increasing there. Therefore for every $W \in (W_0, \underline{W}(F))$,

$$V^\ell_{WW}(F,W) < V^\ell_{WW}\big(F, \underline{W}(F)\big) < 0, \tag{A.154}$$

and by continuity $V^\ell_{WW}(F, W_0) \leq V^\ell_{WW}(F, \underline{W}(F)) < 0$. If $W_0 > 0$ this contradicts the definition of W_0 as an infimum, since strict negativity at W_0 extends to a left neighborhood. Therefore $W_0 = 0$ and $V^\ell_{WW} < 0$ on all of $[0, \underline{W}(F))$.

Note that the presence of jump risk strengthens rather than weakens this argument: the term $-\lambda$ in (A.153) is more than offset by $(\rho+\lambda)V^\ell_W > (\rho+\lambda)$, precisely because $V^\ell_W > 1+\kappa > 1$ in the equity issuance sub-region.

E. Optimal debt adjustment rate

In both sub-region ℓ and sub-region h, the firm's equilibrium debt price satisfies

$$p(F,W) = -V_F(F,W)/V_W(F,W). \tag{A.155}$$

The equity issuance term $[V_W - (1+\kappa)]i$ is independent of F up to V_W, and the additional drift i enters both the equity ODE and the debt-price HJB symmetrically, and we obtain

$$G = \frac{\pi c - \left[\rho + \lambda \dfrac{V_W(F,W)-1}{V_W(F,W)}\right] p(F,W) + \tilde{\sigma}^2 p_W(F,W)\left(-\dfrac{V_{WW}(F,W)}{V_W(F,W)}\right)}{-\big[p_F(F,W) + p(F,W)p_W(F,W)\big]}. \tag{A.156}$$

The issuance cap $\bar{I}$ and the equity issuance cost κ do not appear explicitly, but they affect G through their effect on the equity holders' value function $V(F,W)$ and the equilibrium debt price $p(F,W)$. Note also that the liquidation wedge is strictly larger in sub-region ℓ than in sub-region h, since $V_W > 1+\kappa$ there, so jump risk raises the cost of debt-financed cash by most in exactly those states where the firm is simultaneously raising outside equity.

Appendix C.10 Costly Equity Issuance with $\bar{I} \to \infty$

This appendix develops the contradiction argument used in Section 4 with $\bar{I} \to \infty$. We first assume that debt adjustment is smooth. That assumption yields a one-dimensional reflected-cash candidate. We then show that the candidate is valid only in the region where equity issuance is inactive. In the low-debt recapitalization region, the candidate violates a necessary lender boundary condition, so the smooth-debt conjecture is false.

A. Singular-control problem under the smooth-debt conjecture

Let I be an adapted, right-continuous, nondecreasing process of finite variation. An increment dI_t

adds one dollar to cash and costs incumbent shareholders $(1+\kappa)dI_t$. Define

$$b(F) = (1-\pi)(\mu - cF), \qquad \widetilde{\sigma} = (1-\pi)\sigma, \qquad q = \rho + \lambda.$$

Under the maintained conjecture $d\Gamma_t = G_t dt$, the pre-termination cash dynamics are

$$dW_t = [b(F_t) + p_t G_t]dt + \widetilde{\sigma}\, dZ_t + dI_t - dD_t.$$

The dynamic programming equation is

$$\max\left\{\sup_G \left[GV_F + (b+pG)V_W + \frac{1}{2}\widetilde{\sigma}^2 V_{WW} + \lambda W - qV\right],\ 1 - V_W,\ V_W - (1+\kappa)\right\} = 0. \tag{A.157}$$

The corresponding complementary-slackness conditions are

$$1 \le V_W \le 1+\kappa, \qquad \int_0^t \mathbf{1}_{\{V_W > 1\}} dD_s = 0, \qquad \int_0^t \mathbf{1}_{\{V_W < 1+\kappa\}} dI_s = 0. \tag{A.158}$$

If the supremum over the unbounded two-sided smooth debt rate is finite, its coefficient must vanish:

$$V_F + pV_W = 0. \tag{A.159}$$

Consequently the candidate continuation equation becomes

$$\frac{1}{2}\widetilde{\sigma}^2 V_{WW} + b(F)V_W - qV + \lambda W = 0. \tag{A.160}$$

Equation (A.160) is derived under the smooth-debt conjecture. It must not be imposed after that conjecture has been rejected.

B. The reflected-cash candidate

Fix $F < \mu/c$ and let $\theta = \lambda/q$. The characteristic roots of (A.160) are

$$\gamma(F) = \frac{-b(F) - \sqrt{b(F)^2 + 2q\widetilde{\sigma}^2}}{\widetilde{\sigma}^2} < 0, \qquad \eta(F) = \frac{-b(F) + \sqrt{b(F)^2 + 2q\widetilde{\sigma}^2}}{\widetilde{\sigma}^2} > 0. \tag{A.161}$$

Define

$$K(F) = \frac{(1-\theta)\eta(F)}{\gamma(F)[\eta(F) - \gamma(F)]}, \qquad H(F) = -\frac{(1-\theta)\gamma(F)}{\eta(F)[\eta(F)-\gamma(F)]}. \tag{A.162}$$

When recapitalization is optimal, the candidate value for $0 < W < W^*_\infty(F)$ is

$$V^{\text{cash}}_\infty(F,W) = K(F)e^{\gamma(F)[W - W^*_\infty(F)]} + H(F)e^{\eta(F)[W - W^*_\infty(F)]} + \theta W + \frac{b(F)}{q}. \tag{A.163}$$

For $W \ge W^*_\infty(F)$, extend the value linearly:

$$V^{\text{cash}}_\infty(F,W) = V^{\text{cash}}_\infty(F, W^*_\infty(F)) + W - W^*_\infty(F). \tag{A.164}$$

The constants in (A.162) impose

$$V_W(F, W^*_\infty(F)) = 1, \qquad V_{WW}(F, W^*_\infty(F)) = 0.$$

The lower reflection condition $V_W(F,0^+) = 1+\kappa$ gives

$$\frac{\rho}{q}\frac{\eta(F)e^{-\gamma(F)W^*_\infty(F)} - \gamma(F)e^{-\eta(F)W^*_\infty(F)}}{\eta(F)-\gamma(F)} + \frac{\lambda}{q} = 1+\kappa. \tag{A.165}$$

The left-hand side is strictly increasing in $W^*_\infty(F)$ from one, so (A.165) has a unique positive solution for every $\kappa > 0$.

The boundary $W = 0$ can also be obtained as the limit of the finite-cap problem. If $\underline{W}^{\bar{I}}(F)$ denotes the threshold below which the firm issues at rate $\bar{I}$, then

$$\underline{W}^{\bar{I}}(F) \longrightarrow 0, \qquad W^*_{\bar{I}}(F) \longrightarrow W^*_\infty(F) \quad \text{as } \bar{I} \to \infty. \tag{A.166}$$

Thus the issuance region becomes a vanishing boundary layer and, when recapitalization has priority over default, I is the minimal regulator that prevents cash from becoming negative.

C. The debt threshold and the baseline region

Let V^0 and W^*_0 denote the baseline value and payout boundary without equity issuance. Define

$$\bar{\kappa}(F) = V^0_W(F,0^+) - 1 = \frac{\rho}{q}\frac{\eta(F)e^{-\gamma(F)W^*_0(F)} - \gamma(F)e^{-\eta(F)W^*_0(F)}}{\eta(F)-\gamma(F)} - 1. \tag{A.167}$$

The function $\bar{\kappa}(F)$ is strictly decreasing on $[0,\mu/c)$ and converges to zero as $F \uparrow \mu/c$. Therefore, if $0 < \kappa < \bar{\kappa}(0)$, there is a unique $F_\kappa \in (0,\mu/c)$ such that

$$\bar{\kappa}(F_\kappa) = \kappa. \tag{A.168}$$

For $F \geq F_\kappa$, the cost of equity issuance is at least its marginal benefit at zero cash. Equity issuance is inactive, $W = 0$ remains a default boundary, and the candidate coincides with the baseline model. Accordingly, the smooth-debt formula in Section 3.4 is valid in this region subject to the verification condition in Lemma 2.

For $F < F_\kappa$, shareholders prefer recapitalization to default. The candidate cash process is reflected on $[0, W^*_\infty(F)]$ and satisfies

$$\int_0^t \mathbf{1}_{\{W_s>0\}} dI_s = 0, \qquad \int_0^t \mathbf{1}_{\{W_s<W^*_\infty(F_s)\}} dD_s = 0. \tag{A.169}$$

This is the region in which the smooth-debt conjecture will fail.

D. Contradiction in the recapitalization region

In the continuation region, lender value under a smooth debt rate G satisfies

$$qp = c + G[p_F + pp_W] + b(F)p_W + \frac{1}{2}\tilde{\sigma}^2 p_{WW}. \tag{A.170}$$

At the lower reflection boundary, Itô's formula contributes $p_W(F,0^+)dI_t$. The equity injection makes no direct payment to lenders. Absence of an uncompensated predictable finite-variation term therefore requires

$$p_W(F,0^+) = 0. \tag{A.171}$$

Suppose, toward a contradiction, that debt adjustment is smooth for $F < F_\kappa$. Then (A.159) gives $p = -V_F/V_W$. Since $V_W(F,0^+) = 1+\kappa$ throughout the recapitalization interval, $V_{FW}(F,0^+) = 0$.

Differentiating the price identity with respect to W gives

$$p_W(F,0^+) = \frac{V_F(F,0^+)V_{WW}(F,0^+)}{(1+\kappa)^2}. \tag{A.172}$$

The price is positive, so (A.159) and $V_W = 1+\kappa > 0$ imply $V_F(F,0^+) < 0$. Moreover, direct differentiation of (A.163) yields

$$V_{WW}(F,0^+) = (1-\theta)\frac{\gamma(F)\eta(F)}{\eta(F)-\gamma(F)}\left[e^{-\gamma(F)W_\infty^*(F)} - e^{-\eta(F)W_\infty^*(F)}\right] < 0. \tag{A.173}$$

Hence (A.172) implies $p_W(F,0^+) > 0$, contradicting (A.171). This proves that the smooth-debt conjecture is incompatible with the singular recapitalization policy for every $F < F_\kappa$.