Intermediation via Credit Chains*

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Abstract

The modern financial system features complicated intermediation chains, with each layer performing some degree of credit/maturity transformation. We develop a dynamic model where an ultimate borrower obtains funds from overlapping-generation households via layers of funds, forming a credit chain. Each intermediary fund in the chain faces rollover risks. The model delivers new insights regarding the benefits of intermediation via layers: by shortening the maturity of liquidated assets, the chain structure insulates interim negative fundamental shocks and protects the underlying cash-flows from being discounted heavily during bad times. We show the equilibrium chain length minimizes run risks and is constrained efficient.

Keywords: Financial intermediation, Debt runs, Shadow banking, Dynamic economy, Money.

JEL codes: D85, G21, G23, G33, E44, E51

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I Introduction

Since the mid-1980, the nature of financial intermediation has been changed in a dramatic way by the emergence of securitization, giving rise to a more market-based financial system. Shadow banking can be viewed as the product of this market-based financial system, which contributed to the U.S. real estate market boom prior to the 2007–09 financial crisis. Although the underlying economic mechanism of shadow banking has been well studied by many leading scholars (Adrian and Shin, 2009, 2013; Duffie, 2019), our paper focuses on one missing piece in this literature. Adrian et al. (2012) explain it vividly:

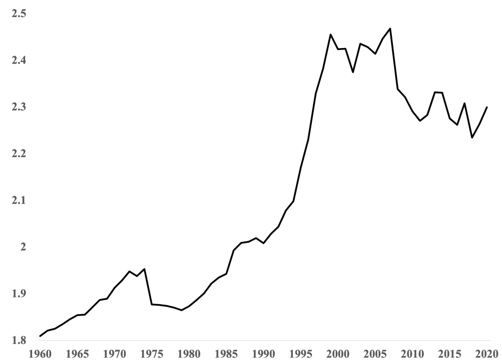
> Like the traditional banking system, the shadow banking system conducts credit intermediation. However, unlike the traditional banking system, where credit intermediation is performed "under one roof"—that of a bank—in the shadow banking system, it is performed through a daisy-chain of non-bank financial intermediaries in a multi step process. . . . The shadow banking system performs these steps of shadow credit intermediation in a strict, sequential order with each step performed by a specific type of shadow bank and through a specific funding technique. . . . The intermediation chain always starts with origination and ends with wholesale funding, and each shadow bank appears only once in the process.

The thrust of the above description is the concept of a "chain," with the common theme being the step-by-step maturity/liquidity and credit transformation, often initiated by loan origination.[1] Outside the stark example of the shadow banking system prior to financial crisis, money market mutual funds (MMMFs) often issue daily "debt" to households, but hold commercial papers with maturity of one to six months; and these commercial papers are issued by banks and other non-bank financial institutions to fund even longer-term and riskier projects. More recently, banks lend to private debt funds who then lend to real firms; and loan funds hold tranches of CLOs who then hold baskets of leveraged loans.[2]

[1] As anatomized by Adrian et al. (2012), it is then followed by so-called "loan warehousing," which refers to the act of collecting a significant volume of eligible loans in a special purpose vehicle (SPV), which then issues asset-backed commercial papers (ABCP) to the public, as well as issues loans to the next layer of asset-backed securities (ABS) warehousing. This process might further involve an ABS collateralized-debt-obligation (CDO), but eventually reaches the wholesale funding markets populated by money market investors as well as long-term fixed income investors (say pension funds and insurance companies).

[2] Neil Callanan and Silas Brown, "Banking Crisis Raises Concerns About Hidden Leverage in the System,"

Figure 1: Credit Intermediation Index, 1960–2020



The credit intermediation index is calculated as the ratio of the total liability of all domestic sectors to the total liability of domestic nonfinancial sectors (Greenwood and Scharfstein, 2013). Both series are obtained from the Flow of Funds at the annual level.

Following Greenwood and Scharfstein (2013), we plot the credit intermediation index, which is the ratio of total liabilities of all sectors over the total end-user liability, in Figure 1. Similar to "money multiplier," the credit intermediation index approximates the average credit chain length in the economy, where the total end-user liability is proxied by the total liabilities of domestic *nonfinancial* sector. This ratio grew significantly during the 1990s when securitization became popular, decreased after the 2007-09 financial crisis, and remains at a high level from a historical perspective. During the last decade, each dollar from investors flows through about 2.2 layers of intermediaries before reaching the final borrower. This pattern aligns with the findings of Philippon (2015), who shows that the share of intermediated assets and financial income relative to GDP has grown substantially over time.

Despite extensive literature on banking, it remains unclear why market participants rely on multiple *layers* of intermediaries rather than a single intermediary to channel funds from households to firms, as envisioned by Diamond (1984). Some argue that long credit chains may give unsophisticated investors a false sense of "safety," although professional money market funds often represent these households. Another explanation is regulatory ar-

Bloomberg, March 27, 2023; Laurie DeMarco, Emily Liu, and Tim Schmidt-Eisenlohr, "Who Owns U.S. CLO Securities? An Update by Tranche," *Feds Notes*, June 25, 2020. Recently, Chernenko, Ialenti, and Scharfstein (2024) provide evidence for banks lending to business development companies (BDCs), who then lend to middle-market companies.

bitrage, where financial institutions intentionally create complex financing chains to obscure certain activities. While empirical studies (Acharya, Schnabl, and Suarez, 2013; Karolyi and Taboada, 2015; Demyanyk and Loutskina, 2016) support this view, they do not fully explain the rapid rise of the securitization market in the mid-1980s, which may be better understood as contracting innovation rather than mere regulatory arbitrage (Calomiris and Mason, 2004).

We study the economics of credit chains by considering a dynamic model, in which a long-lived ultimate borrower obtains funds from overlapping generations (OLG) of households. The impatient borrower is endowed with an underlying asset that matures with certain probability each period and only produces random cash-flows upon maturity. Households, on the other hand, are born with endowments and live for two dates. Different from the impatient ultimate borrower, they do not discount their consumption in the second date. The relative impatience wedge implies that the (impatient) ultimate borrower would like to pledge out future cash-flows and borrow from (patient) households.

A group of financial intermediaries—whom we refer to as "experts"—share the same discount rate as the ultimate borrower and manage funds and facilitate liquidation in the secondary market. The ultimate borrower can first borrow from funds run by these experts, who then borrow from OLG households. These layers of funds are linked with each other by debt contracts, forming a credit chain with an endogenous chain length. For tractability, we focus on debt contracts with exogenous face value and contract maturity probability; but each layer can adjust the interest rates to rollover its debt, taking as given other layers' contracts and households' strategies. When contracts mature, the borrower—whether the ultimate borrower or an intermediary fund—needs to rollover its debt. Rollover fails when the cash-flow realization is too low. Creditors liquidate this borrower's assets in the secondary market, where experts serve as buyers who then resell to the next cohort of households. Impatient intermediaries demand compensation, which translates to secondary market transaction costs from the perspective of OLG households.

We assume that the households always hold one-period debt, which captures the growing appetite for money-like assets in recent decades (Greenwood, Hanson, and Stein, 2015; Carlson et al., 2016). However, when cash-flows are uncertain, short-term debt exposes the borrowers to rollover risks frequently (He and Xiong, 2012), limiting their debt capacity. The key insight of this paper is that a credit chain, compared to direct borrowing, can increase the ultimate borrower's capacity to borrow.

We first consider a simplified setting with an exogenous probability of rollover failure in Section II. Compare the debt value in the following two cases: the one-layer case, where the ultimate borrower directly obtains funding from households using one-period debt; and the two-layer case, where the ultimate borrower first borrows using medium-term debt from an intermediary fund, who then borrows from households using one-period debt. In the one-layer case, a negative shock forces the ultimate borrower's *long-term underlying asset* to be liquidated. In contrast, in the two-layer case, following a negative fundamental shock, it is the fund's asset—which is the *medium-term debt* issued by the ultimate borrower—that is being liquidated. Therefore, credit chains effectively shorten the maturity of liquidated assets. Together with secondary market transaction costs, the two-layer case therefore delivers a higher endogenous liquidation value—and hence a larger debt capacity to begin with—by potentially avoiding secondary market frictions if future rollovers succeed.

We highlight that the above insight of "credit chain helps shorten the maturity of *liquidated* asset" not only relies on that intermediary fund holds a shorter-term debt (the medium-term debt has shorter maturity than the underlying asset). More importantly, Section II.B demonstrates that the gains from credit chains also stem from which contracts survive liquidation, reflecting a deeper Coasian principle about firm boundaries. In the one-layer case, all potential future household–borrower short-term debt contracts lie on the liability side of the liquidated entity (the ultimate borrower), and bankruptcy wipes out these future household–borrower contracts. In contrast, in the two-layer case, the medium-term intermediary–borrower debt contracts survive the bankruptcy procedure, since these contracts sit on the asset side of the liquidated intermediary fund (when it defaults on households) and are external to the bankruptcy process.

To summarize, the simplified setting highlights three key ingredients that deliver a greater debt capacity under credit chains in our model. First, we need maturity transformation via debt contracts, so that the ultimate borrower issues a medium-term debt to the intermediary fund with a maturity shorter than that of the underlying asset backing it. Second, when the intermediary fund defaults to households and is liquidated, the credit chain allows the contracts between the fund and the ultimate borrower to remain intact; as a result, a liquidated asset with a credit chain has a shorter effective maturity than one without. Third, given secondary-market trading frictions, the endogenous liquidation value of a shorter-term asset is higher—by saving on transaction costs—than that of an otherwise comparable longer-term asset. Taken together, we thus demonstrate that when impatient borrowers seek to pledge future cash flows but face high secondary-market and liquidation costs, credit chains can lower ex-post liquidation losses and raise ex-ante debt value, thereby supplying more money-like securities. In this way, they mitigate the trade-off between pledging cash flows and minimizing liquidation losses, much like SPVs in practice.

We generalize the model to multiple layers, so that the probability of rollover failure becomes endogenous. In addition, households pay an exogenous (dead-weight) bankruptcy cost per layer. The equilibrium features an endogenous credit length together with a constant rollover threshold, so that rollover fails if realized cash-flows lie below that threshold. Showing that the equilibrium contracts are time-invariant and layer-independent, Section III.C characterizes the equilibrium credit chain and default risks in the general model. Same as in the simplified setting, the benefit of borrowing via layers comes from the fact that a longer chain delivers shorter maturity assets during liquidation, which is desirable in that debt payments can flow toward departing households in a frictionless way if the future fundamental improves. Somewhat surprisingly, the equilibrium chain length emerged in a decentralized market is constrained efficient from the social perspective, despite of various trading frictions in the chain structure. This is because the fund in the last layer, which determines the equilibrium chain length, internalizes the trade-offs of longer chains through the interest rate it pays to the households.

One of the key assumptions of our model is that debt issuance costs in the primary market are lower than both i) liquidation costs and ii) secondary market transaction costs. Two points are noteworthy. First, the assumption of frictional secondary market trading and liquidation is common in the money and banking literature (say, Bryant, 1980), leading to demand for money (which corresponds to short-term debt in our model). Second, although ii) does not apply universally to all markets, it does hold for many instruments observed in the shadow banking system. For example, Friewald, Jankowitsch, and Subrahmanyam (2017) document that the average secondary market transaction costs for asset-backed securities (ABS) and mortgage-backed securities (MBS) are 43 bps and 58 bps respectively. They are much higher compared with ABCP issuance costs which are around 10 bps (Kacperczyk and Schnabl, 2010); in fact, the SPVs that are issuing ABCPs are set up in order to streamline the process of debt rollovers, which minimizes issuance costs.

The main model abstracts away from modeling information frictions, which allows us to focus on the credit chain's role in facilitating maturity transformation, underscoring how our mechanism is distinct from the established literature on asset pooling and tranching (DeMarzo, 2004). We also do not model the benefit of diversification as in Diamond (1984), which, by itself alone, implies a single layer of intermediary is sufficient. Related to shutting off the role of diversification, we also do not consider a general network in which many borrowers interact with one another through the intermediary network. Central to our mechanism are the transaction costs associated with secondary market trading and liquidation, facilitated by experts as financial intermediaries (He and Krishnamurthy, 2013; Brunnermeier and Sannikov, 2014). In the last section of the paper, we introduce information asymmetry among new born household buyers in our dynamic setting, which provides a microfoundation for this secondary market transaction cost.

As detailed in the literature review, our paper differs fundamentally from the literature of asset trading chains. Oftentimes, these papers focus on certain market frictions that prevent the asset seller (with a relatively low valuation) from directly selling to the first-best buyer (with the highest valuation), and an intermediary who holds the asset temporarily ensues. Our focus, instead, is on intermediation credit chains where one agent's liability is another agent's asset, which is missing in the literature of asset trading chains.

Literature review. Our paper belongs to a recent literature that studies the role and frictions of credit chains, motivated by the growing intermediation chain in the modern financial system (Adrian and Shin, 2010; Adrian et al., 2012). Di Maggio and Tahbaz-Salehi (2017) study how the distribution of collateral along the credit chain matters for the intermediation capacity and systemic stability. In Donaldson and Micheler (2018), credit chains arise when banks rely more on non-resaleable debt; as in our paper, liquidation losses are smaller in defaults when the borrowing is done via layers. The difference is that, instead of assuming exemption of automatic stay, we start with a common type of frictions and show that having a layer in the middle—by shortening the maturity of liquidated assets—endogenously mitigates the loss from default.[3]

There is a long literature on the theory of financial intermediation; we focus on the ben-

[3] More recently, Glode and Opp (2021) and Gryglewicz and Mayer (2023) study the externality of economic agents' decisions in an exogenously given intermediation chain, and He, Jiang, and Xu (2025) study the role of information technology when intermediaries as a middleman to facilitate the originate-to-distribute model in the CLO market.

efit of having multiple layers of intermediaries instead of just one. One-layer intermediation is the robust prediction in leading models in this field; for instance, Diamond (1984) shows that banks reduce monitoring cost through diversifying projects' idiosyncratic risks, which are absent in our model.[4] Our paper is closer to Diamond and Rajan (2001) conceptually. There, an intermediary is necessary—but a single layer is enough—as it has specific skill in collecting repayments from firms and can also commit to repaying its creditors by offering demand deposits. Like our paper, Diamond and Rajan (2001) micro-found the continuation game after asset liquidation, and show that intermediaries enhance recovery value if default happens. But inalienable human capital (of entrepreneurs/bankers), which is the backbone of Hart and Moore (1998) and Diamond and Rajan (2001), plays no role in this paper.

On the literature of network and contagion,[5] we focus on a simple form of network, i.e. chains, and study the credit chain length; in other words, we endogenize network formation within the simple chain structure. Relatedly, a recent literature has also investigated asset trading chains, where an asset is bought and re-sold by a sequence of dealers before it reaches the final buyer. Glode and Opp (2016) show trading via a sequence of moderately informed intermediaries can reduce allocation inefficiency caused by asymmetric information.[6] The literature has also examined the length and price dispersion of intermediation chains in an over-the-counter (OTC) market with search frictions (Atkeson, Eisfeldt, and Weill, 2015;

[4]Our analysis is related to the literature on bank runs and instability of short-term debt (Diamond and Dybvig, 1983; Calomiris and Kahn, 1991; Goldstein and Pauzner, 2005; Acharya, Gale, and Yorulmazer, 2011; He and Xiong, 2012), but in an endogenous multi-layer structure. Similarly to Qi (1994), we consider an OLG setup in which intergenerational transfers through financial institutions improve welfare, but could lead to runs. The runs between layers in our model capture the repo market and commercial paper runs by institutional investors during the global financial crisis, which has been well documented (Gorton and Metrick, 2012; Copeland, Martin, and Walker, 2014; Krishnamurthy, Nagel, and Orlov, 2014; He and Manela, 2016; Schmidt, Timmermann, and Wermers, 2016).

[5]To name a few, Allen and Gale (2000) and Elliott, Golub, and Jackson (2014) show how financial networks provide diversification and insurance against liquidity shocks, but on the other hand, lead to fragility and cascades of failures. A similar point is delivered by Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015). Allen, Babus, and Carletti (2012) also consider rollover risks of short-term debt in clustered structures, where banks share common assets. Recently, Donaldson, Piacentino, and Yu (2022) show the usage of long-term debt in financial networks can be stabilizing, as banks hit by liquidity shocks can raise additional funding using interbank long-term debt as collateral and dilute existing long-term creditors. We rule out debt dilution and focus on maturity transformation along the credit chain.

[6]In a follow-up paper, Glode, Opp, and Zhang (2019) show that a sufficient long intermediation chain can also eliminate trading inefficiencies caused by agents with monopoly power screening counterparties. In a general equilibrium context, a recent paper by He, Jiang, and Xu (2025) studies the role of information technology and intermediation in the originate-to-distribute model.

Hugonnier, Lester, and Weill, 2019; Sambalaibat, 2021; Shen, Wei, and Yan, 2021). Our focus is on credit chains where one agent's liability is another agent's asset, which is the key for "credit chains."[7]

II A Simplified Model and Intuition

We consider a discrete-time dynamic model with three types of risk-neutral agents: OLG households, an ultimate borrower who is long-lived, and a group of long-lived experts. For ease of illustration, we first simplify certain aspects of our model (which will be relaxed later in Section III) to facilitate the comparison between the economy with one layer and that with two layers.

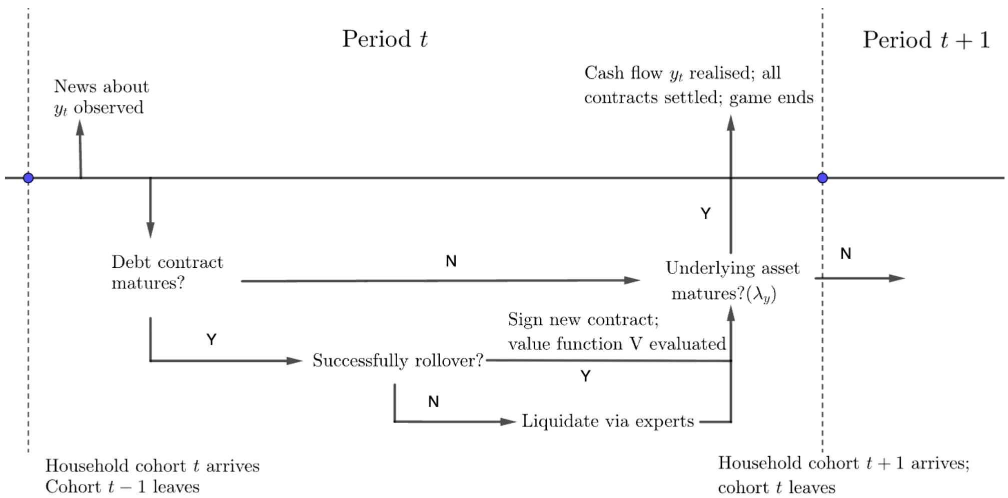
A The Setting

Economic Environment. A long-lived agent (hereafter he) with a discount rate $\alpha \in (0,1)$ has a long term asset that produces nothing before maturity. We refer to him as the ultimate borrower. At the beginning of period t, the public "news" on the (potential) cash-flow $y_t \geq 0$ arrives. We assume y_t is binary and i.i.d. across periods; with probability p (or $1-p$), good news (or bad news) is realized and $y_t = \bar{y}$ (or $y_t = 0$). (The full model features a general distribution of y_t.) At the end of each period, the asset matures with probability $\lambda_y \in (0,1)$, in which event the asset pays off y_t units of consumption good at the end of the period and the game ends. Note that if the asset does not mature in period t, no cash-flow is produced and a new value of the fundamental will be drawn next period. (We will explain the timing in more detail shortly.) Throughout, we refer to this asset as the underlying asset, which could be a pool of loans or mortgages.

There are OLG households in this economy. Cohort-t is born at the beginning of period t and leaves the economy at the beginning of period $t+1$, which occurs right when Cohort-$t+1$ arrives; see Figure 2. Each cohort consists of a measure 1 of representative households, who are endowed with $e > 0$ units of consumption goods when born. They can choose to consume

[7] With a slightly broader interpretation, our model also sheds light on "rehypothecation," i.e., the reuse of collateral in secured financing transactions, which is also called "collateral chains" and is a widespread practice to enhance market functioning between banks and nonbanks (Infante and Saravay, 2020). As most repo transactions in the U.S. are conducted on an "outright" basis with complete ownership transfer at each leg, rehypothecation in a collateral chain is closer to asset trading chains.

Figure 2: Timing



This figure illustrates the timing of events in each period for Section II.

c_t^t in period t or invest in the securities issued by the ultimate borrower or the intermediary funds, and consume c_{t+1}^t in period $t+1$ (and then leave the economy). Household's utility is $c_t^t + c_{t+1}^t$; importantly, there is no discount between periods.

There is a financial intermediary sector which consists of a group of "experts" who are long lived with an exogenous discount rate $\alpha \in (0,1)$. For simplicity we take the experts' discount rate to be the same as that of the ultimate borrower's. Experts can operate some intermediary funds who raise financing from households and in turn provide credit to the ultimate borrower; this is the credit chain we analyze in the paper. Throughout, we refer to the chain length by L; the chain is indexed such that layer-l borrows from layer-$(l+1)$, so that the ultimate borrower is labeled as layer-0 while households are labelled as layer-L.

In our model, the gain of trade comes from the impatient borrower and experts (with discount rate $\alpha < 1$) financing from more patient households (with a discount rate 1). The key is how to sell the underlying asset's cash-flows from the hands of the relatively impatient agent (the ultimate borrower) to the patient but OLG households, and we show that a credit chain could achieve certain efficiency gain via intermediary funds run by experts.

Debt contracts and timing. Borrowers (either the ultimate borrower or the intermediary funds) issue debt contracts with face value e (which binds at households' endowment). We assume that debt contracts issued to intermediary funds mature with exogenous probability λ_d in each period, while the debt contract issued to households matures with probability 1. In addition, all debt contracts mature when the underlying asset matures. We shall explain (in page 23) that this debt maturity structure fixes the total maturity transformation in the system (from the underlying asset's maturity $1/\lambda_y$ to 1), regardless of the number of layers (to be introduced soon).

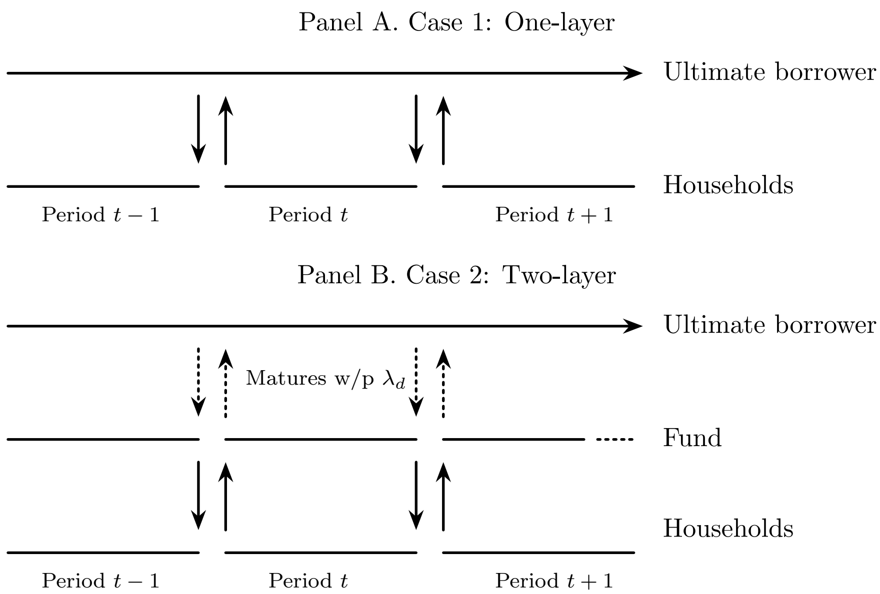
The debt contract promises to pay $F_y \in [0, \bar{y}]$ if the underlying asset matures and pays the face value e if the debt contract matures. Due to limited liability, the actual payment upon asset maturity is $\min(F_y, y_t)$. Without loss of generality we focus on the class of issue-at-par debt contracts, i.e., F_y adjusts such that the price of debt is equal to e at issuance.

As shown in Figure 2, at the beginning of each period, everyone observes the "news" regarding y_t first. Then, whether the debt contracts held by cohort-$(t-1)$ households mature or not is realized. Cohort-t households arrive, and after that, cohort-$(t-1)$ households (who receive the debt payment or liquidation value) leave the economy. At the end of each period, whether the underlying asset matures or not is realized. If it matures, then the underlying asset pays off cash-flow y_t; all debt contracts are paid and the economy ends. Otherwise, the economy continues to period $t+1$.

Why credit chain improves efficiency? We now explain the key economic force that drives the benefit of credit chain. First, Figure 3, Panel A illustrates the one-layer case (i.e., $L = 1$) in which the ultimate borrower directly obtains funding from households using a one-period debt, which must be rolled over every period. Suppose that rollover fails at the beginning of period t when $y_t = 0$. Households therefore as creditors receive the liquidated underlying asset, which will be traded in the secondary market.

Alternatively, the ultimate borrower obtains funding via a credit chain as in the two-layer case (i.e., $L = 2$): he first borrows from an intermediary fund using debt that matures with probability λ_d each period, and the intermediary fund then borrows from households using one-period debt (Figure 3, Panel B). Both the intermediary fund and the ultimate borrower bear some degree of maturity mismatch. Suppose that at the beginning of period t the ultimate borrower's debt has not matured but the intermediary fund fails to roll over its one-period debt. In this scenario, the bankruptcy court liquidates the intermediary fund's asset, which is the debt issued by the ultimate borrower. This debt matures with probability

Figure 3: Illustration of the Credit Chain



This figure provides a simple illustration of the contracts in the credit chain structure. In panel (a), the households directly fund the ultimate borrower using one-period contract. In panel (b), the ultimate borrower first borrows from an intermediary fund, using contract that matures with probability λ_d; the fund then borrows from the households using one-period contract.

λ_d next period; note that it has a maturity shorter than the underlying asset given that it also repays whenever the underlying asset matures with probability λ_y. As we will highlight shortly, that this debt survives the liquidation process plays a crucial role for improving debt capacity ex-ante.

Comparing the above two cases in the event of rollover failure illustrates the key feature of the credit chain: it changes the type of asset that is being liquidated as a result of rollover failure. In Figure 3, Panel A with one-layer direct borrowing, the liquidated asset is the *underlying asset*, while in Figure 3, Panel B with a two-layer credit chain, the liquidated asset is the *asset of an intermediary fund* (i.e., the intermediate-term debt issued by the ultimate borrower). Importantly, the asset of the intermediary fund has shorter maturity compared to the underlying asset. Therefore by shielding the long-term underlying asset from being liquidated, a credit chain shortens the (expected) maturity of the asset that is being liquidated.

If, in addition, the liquidation value is decreasing in the maturity of liquidated assets, then we reach the key take-away of our paper that intermediating via layers improves efficiency by reducing liquidation losses. As we show shortly, we achieve this by placing our model in a setting with (exogenous) intermediation friction during the liquidation process. As this assumption renders shorter-term liquidated assets more valuable, it allows us to endogenously derive that credit chains improve liquidation value, even though i) the assets share the same fundamentals and ii) agents—the ultimate borrower, funds, and households—face the same intermediation frictions. In Section IV.C, we further endogenize the intermediating friction by information asymmetry.

Debt rollover and intermediation friction in liquidation. Consider the borrower in layer-l who needs to refinance/rollover its debt contract.[8] Rollover is successful if the borrower is able to raise enough money from the new-born households to pay back e to its creditors; successful rollover involves no cost. We assume that the positive cashflow realization $\bar{y}$ is large enough such that when good news is realized, rollover is always successful.

However, when bad news realized, rollover fails given the sufficiently low fundamental $y_t = 0$. (As a result, in this simplified setting the rollover failure probability is exogenously given by $1 - p$.) Creditors take over and liquidate the asset held by the borrower in layer l, which could be the underlying asset or the debt issued by layer $l - 1$. We assume that the liquidation is intermediated by the experts who buy the liquidated asset first and then sell it to the next cohort of households. The next cohort of households needs to hold this asset for one period; if the asset does not mature at the beginning of the following period, they have to resell it to the experts (who then sell it to the new cohort). This secondary market trading friction is the same as the liquidation friction, as both processes are intermediated by the experts. Finally, the chain length is restored to its original level at the end of the following period (but before potential maturity of the underlying asset; see Figure 2).

These experts who intermediate liquidation have discount rate $\alpha \in (0, 1)$ like other experts. There are many interpretations for α besides their opportunity costs of time; for instance, in He and Krishnamurthy (2012, 2013), experts need to commit certain equity capital to operate the distressed funds, which is costly. We emphasize that the discount factor α is applied to the liquidated asset's market value, which is then endogenously determined

[8] Throughout the paper we use the word "refinance" and "rollover" interchangeably.

by the credit chain structure. This is similar in spirit to Diamond and Rajan (2000) where the continuation game after asset liquidation is micro-founded.[9] In the baseline model, we take α as an exogenous preference parameter; in Section IV.C, we provide a further micro-foundation for the discount α based on information asymmetry among potential new household buyers.

Denote the value recovered by the creditors as $B_l(L)$ from the liquidation of layer l's asset (intermediated by experts), given the total chain length L. As we will show shortly, the endogenous value $B_l(L)$ of the liquidated asset is decreasing in its (expected) maturity, which is negatively related to l because different layers' assets have different maturity.

B The Benefit of Credit Chain

We now explain in detail the key mechanism via which credit chains create value.

Case 1: One-Layer Financing. Suppose that the ultimate borrower directly issues one-period debt to the households, as in Figure 3, Panel A. Denoting the cohort-t households' debt value by $V(y_t, 1)$, where the first argument is the cash-flow and the second argument is the number of layers (we omit the t-subscript given the stationary environment). Therefore

$$V(y_t, 1) = \lambda_y \min(F_y^1, y_t) + \underbrace{(1-\lambda_y)[pe + (1-p)B_0(1)]}_{\equiv v(1)}. \tag{1}$$

In (1), F_y^1 is set such that when $y = \bar{y}$, $V(y, 1) = e$. The continuation value $v(1)$, which does not depend on today's cash-flow y_t thanks to i.i.d. fundamentals, can be understood as follows. At the beginning of period $t+1$, with probability p, good news is realized with $y_{t+1} = \bar{y}$ and the households receive the face value e; with probability $1-p$ bad news realizes with $y_{t+1} = 0$, and the resulting rollover failure leads the underlying asset to be liquidated at price $B_0(1)$. Here, the subscript "0" indicates that it is layer-0's asset being liquidated and "1" inside parentheses indicates the chain length.

Because of the intermediation friction in liquidation, $B_0(1)$ equals α times the value of the underlying asset from the perspective of the cohort-$(t+1)$ households, given the bad

[9] The difference is that in their framework, a single layer of intermediary is sufficient to increase the recovery value if default happens; whereas in our model, having multiple layers of intermediaries is crucial for enhancing recovery value.

news. To calculate this, note that with probability λ_y, the underlying asset matures in period $t+1$ and yields $y_{t+1} = 0$. With probability $1 - \lambda_y$, the underlying asset does not mature; in this scenario, at the beginning of period $t+2$, cohort-$(t+1)$ households sell the underlying asset to cohort-$(t+2)$ households at a discount α. Cohort-$(t+2)$ values the underlying asset at $V(y_{t+2}, 1)$ because the credit chain will be restored at the beginning of $t+2$. Taken together, the value of $B_0(1)$ is given by

$$B_0(1) = \alpha \left\{ \lambda_y \times 0 + (1 - \lambda_y) \alpha \mathbb{E}[V(y_{t+2}, 1)] \right\}. \tag{2}$$

Case 2: Two-Layer Financing. Now suppose the ultimate borrower first issues debt (which matures with probability λ_d) to an intermediate fund who then issues one-period debt to households. The households' value, denoted by $V(y_t, 2)$, is given by

$$V(y_t, 2) = \lambda_y \min(F_y^2, y_t) + \underbrace{(1 - \lambda_y)\{pe + (1-p)[\lambda_d B_0(2) + (1 - \lambda_d) B_1(2)]\}}_{\equiv v(2)}. \tag{3}$$

As before, F_y^2 is set such that $V(y, 2) = e$ when $y = \bar{y}$ and $v(2)$ is the continuation value. Here, the first and second terms in (3) capture the payout when the underlying asset matures and does not mature, with probability λ_y and $1 - \lambda_y$ respectively. To understand $v(2)$, in period $t+1$, with probability p good news realization leads to a successful rollover. This explains pe in $v(2)$, and it does not matter whether it is the intermediary fund or the ultimate borrower that rolls over the debt.

However, if bad news is realized (with probability $1 - p$), then it matters whether the ultimate borrower's debt (which is held by the intermediary fund) matures or not. With probability λ_d, the ultimate borrower's debt matures and the ultimate borrower cannot rollover his debt. In this scenario, the underlying asset is liquidated at $B_0(2)$, and similarly the subscript "0" indicates that it is layer-0's asset being liquidated and "2" inside parentheses indicates the chain length. We can determine $B_0(2)$ similar to $B_0(1)$ in (2), which gives

$$B_0(2) = \alpha \left[\lambda_y \times 0 + (1 - \lambda_y) \alpha \mathbb{E}[V(y_{t+2}, 2)] \right]. \tag{4}$$

Finally, with probability $1 - \lambda_d$, the ultimate borrower's debt does not mature; in this situation the intermediary fund—who issued a one-period debt to households—has to liquidate its asset at $B_1(2)$, which explains the last term in (3). Note that the intermediary fund's asset, which survives the bankruptcy procedure, is the debt issued by the ultimate borrower that matures with probability λ_d each period; it has shorter maturity in expectation compared to the underlying asset.

Comparison. From the ultimate borrower's perspective, minimizing his financial cost is equivalent to maximizing the continuation value of households. The comparison between the households value of (1) in the one-layer case and the value of (3) in the two-layer case boils down to comparing the liquidation values if rollover fails in period $t+1$:

$$v(2) - v(1) = (1-\lambda_y)(1-p)[B_0(2) - B_0(1) + (1-\lambda_d)(B_1(2) - B_0(2))]. \tag{5}$$

Substituting the expression for $V(y,1)$ and $V(y,2)$ from Eq. (1) and (3) into Eq. (2) and (4), we get that $B_0(2) - B_0(1)$ inside the bracket in (5) is proportional to $v(2) - v(1)$. This allows us to express $v(2) - v(1)$ as

$$v(2) - v(1) = \frac{(1-\lambda_y)(1-p)(1-\lambda_d)}{1-(1-\lambda_y)^2(1-p)^2\alpha^2}[B_1(2) - B_0(2)]. \tag{6}$$

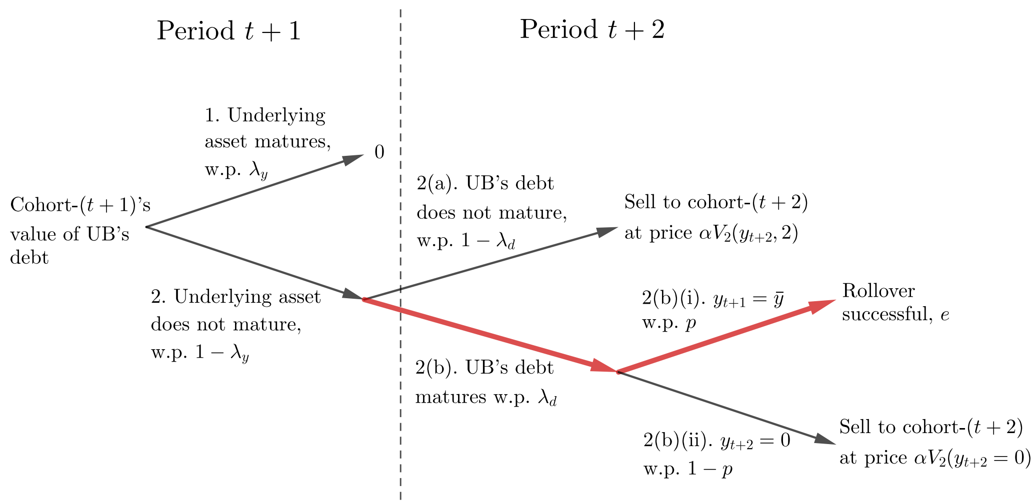
Note that $B_1(2)$ is the liquidating value of the asset held by the intermediary fund, which has shorter maturity than the underlying asset, whose liquidating value is denoted by $B_0(2)$. If a shorter-maturity asset has a higher liquidation value, i.e. $B_1(2) > B_0(2)$, then a positive (6) implies that the value in the two-layer case is greater than that in the one-layer case.

Liquidation value and asset maturity: intermediation frictions. We now derive the expression for $B_1(2)$ and show that $B_1(2) > B_0(2)$ endogenously due to intermediation frictions. We stand at $t+1$ knowing $y_{t+1} = 0$ (which leads to rollover failure of liquidation), but before the realization of whether the underlying asset reaches maturity. Figure 4 summarizes the event tree:

1. If the underlying asset matures in period $t+1$, it yields 0;

2. If the underlying asset does not mature, then there are two scenarios to determine the value received by cohort-$(t+1)$ households at the beginning of period $t+2$:

Figure 4: Event Tree that Determines Value of the Ultimate Borrower's Debt $B_1(2)$



Cohort-$(t+1)$ households' value from holding the ultimate borrower's (UB's) debt, with different future events with the corresponding probabilities. Cohort-$(t+1)$ households are standing at period $t+1$ after fund default, i.e., when $y_{t+1} = 0$ (with probability $1-p$) and the ultimate borrower's debt does not mature (with probability $1-\lambda_d$), as suggested by the last term in Eq. (3).

(a) With probability $1-\lambda_d$, the ultimate borrower's debt does not mature in period $t+2$, and cohort-$(t+1)$ households resell the contract to cohort-$(t+2)$ households at a price $\alpha\mathbb{E}[V(y_{t+2}, 2)]$.

(b) With probability λ_d the ultimate borrower's debt matures in period $t+2$, whose actual payment depends on the realization of y_{t+2}.

   i. If $y_{t+2} = \bar{y}$, which occurs with probability p, then the ultimate borrower can successfully rollover its debt. Cohort-$(t+1)$ households receive face value e. (We highlight this path in Figure 4 as this is the event where the benefit of a two-layer structure comes from!)

   ii. If $y_{t+2} = 0$, which occurs with probability $1-p$, rollover fails and cohort-$(t+1)$ liquidates the asset by selling the underlying asset to cohort-$(t+2)$ at price $\alpha V(y_{t+2} = 0, 2)$.

The following expression takes into account all the possibilities listed above:

$$B_1(2) = \alpha \left\{ \lambda_y \times 0 + (1-\lambda_y)[(1-\lambda_d)\alpha \mathbb{E}[V(y_{t+2}, 2)] + \lambda_d (pe + (1-p)\alpha V(y_{t+2} = 0, 2))] \right\}. \quad (7)$$

Note, at $t+2$ the chain is restored and therefore we have $V(y_{t+2}, 2)$ as cohort-$(t+2)$ households' valuation for this contract.

Using (4) and (7), and substituting in $V(y_{t+2} = \bar{y}, 2) = e$, we get

$$B_1(2) - B_0(2) = \alpha(1-\lambda_y) \underbrace{\lambda_d}_{\substack{\text{ultimate borrower's debt} \\ \text{matures in period } t+2}} \cdot \underbrace{p}_{\substack{\text{good news in} \\ \text{period } t+2}} \cdot (1-\alpha)e > 0. \quad (8)$$

Since the discount α is applied to the market value of the liquidated asset in both $B_1(2)$ and $B_0(2)$, the difference between them ultimately stems from the difference in that endogenous market value. Plugging (8) into (6) yields

$$v(2) - v(1) = \frac{(1-\lambda_y)^2(1-p)\alpha}{1-(1-\lambda_y)^2(1-p)^2\alpha^2}(1-\lambda_d)\lambda_d p(1-\alpha)e > 0, \quad (9)$$

i.e., the debt value in the two-layer credit chain is larger than that in direct borrowing. Because the benefit of the credit chain occurs in the case when the ultimate borrower's debt does not mature in period $t+1$ but matures in period $t+2$, the benefit is proportional to $(1-\lambda_d)\lambda_d$, which is the largest at $\lambda_d = 1/2$. This is independent of λ_y because whether debt matures is only relevant in the case when the underlying asset does not mature.

Key economic intuition. As shown in (6), the improvement in debt value in the two-layer case relative to the one-layer case comes from the higher liquidation value of the intermediary fund's asset compared to that of the underlying asset. When there is no intermediary fund (so one-layer chain), it is the underlying asset that is liquidated. All subsequent short-term contracts between the households and the ultimate borrower, which sit on the liability side of the liquidated entity (the ultimate borrower), are destroyed. As a result, all following cash-flows are subject to the transaction cost no matter what happens in period $t+2$, rendering a lower equilibrium market value of the liquidated asset. In contrast, in the two-layer credit chain with an intermediary fund, after rollover failure in period $t+1$, it is the fund's asset—a debt that matures with probability λ_d—that is being liquidated. In other words, the medium-term borrower-intermediary debt contract, which sits on the asset side of the liquidated entity (the fund), is still preserved after the liquidation.

Consistent with the above explanation, Eq. (8) precisely shows the source of the gain from the credit chain: following liquidation in the subsequent period (period $t+2$), when i) good news regarding the underlying asset is realized with probability p and ii) the medium-term debt contract between the ultimate borrower and the intermediary fund matures with probability λ_d, the ultimate borrower can successfully rollover its debt in period $t+2$ without secondary market trading, saving the $t+2$ trading cost, which is captured by the last term $(1-\alpha)e$ in Eq. (8). This event is captured in the highlighted path in Figure 4.

To summarize, the benefit of the credit chain comes from the fact that it partially preserves the subsequent short-term debt in the case of liquidation, reducing transaction costs incurred and delivering an endogenously lower default cost. This distinction in which contracts survive liquidation is also illustrated in the three-period example of the NBER working paper version (w29632), with the same mechanism operating in our infinite-horizon setting.[10] More broadly, this contrast reflects a deeper Coasian principle about firm boundaries and what contracts survive during bankruptcy: while all future contracts are destroyed in the one-layer case during liquidation, the two-layer structure preserves the claims between the intermediary fund and the ultimate borrower, which are external to the bankruptcy process.

C Mechanism Robustness and Discussions

Generally speaking, the benefit of credit chains arises as long as longer-term assets have larger liquidation costs, which is often the case empirically. As discussed above, the two-layer credit chain dominates the one-layer chain because liquidating the ultimate borrower's asset (which is the long-term underlying asset) is more costly than liquidating the intermediary fund's asset (which is the medium-term debt backed by the underlying asset), and our model with OLG households and secondary market frictions endogenously generates a default cost that is increasing in the maturity of liquidated assets. Given a higher liquidation cost of longer-term assets, it is better to issue short-term debt against medium-term asset (as in

[10]In the three-period example, when the bad news occurs in the first period, rollover fails. Absent the credit chain, households obtain the underlying long-term asset from the bankruptcy process, while with credit chain they receive a short-term debt issued by the ultimate borrower (which is the asset of the intermediary fund) from the bankruptcy process. This is more transparent than our infinite horizon setting because of the restoration of chains.

our two-layer credit chain) than to issue short-term debt against long-term asset (as in our one-layer direct financing). Our result is robust to other micro-foundations that link the default cost to asset maturity.

As evident from Eq. (8), our key mechanism works as long as $\alpha \in (0, 1)$. What we really need is that the liquidation process of the asset is more frictional than the issuance/rollover of debt issued against that asset, which we have assumed to be costless in this paper. This is reasonable in certain context, for example, the special purpose vehicles (SPVs); the credit chain structure, just like SPVs that we observe in practice, supplies more money-like securities by helping insulate interim negative fundamental shocks and protect the underlying assets from being discounted heavily. In the data, the secondary market transaction costs for the securities that SPVs hold, such as MBS and ABS, are around 50 bps, which are likely lower bounds for liquidation costs. However, the issuance cost for shorter term debt for these vehicles, which are purposefully set up to minimize the debt rollover costs, is only around 10 bps (Kacperczyk and Schnabl, 2010). The rollover cost of money market fund shares is also close to zero, which is much smaller compared to the transaction costs of the assets.[11] Note that in the context of our setting, the relevant comparison is indeed the cost incurred each time when transaction happens (α in our model) and the cost incurred each time these SPVs rollover their debt (which has been normalized to zero in our model). By issuing a short-term debt against the long-term asset, in the period when the short-term debt matures (as our short-lived households need liquidity), one saves on the secondary market transaction cost of the asset but incurs issuance (refinancing) cost. Finally, our model applies well to market-based financing such as MBS as it involves little monitoring from creditors, a feature that is also absent in our model.

Remark 1 ***Deterministic debt maturity.*** *The intuition revealed here carries over to general credit chains with multiple layers, as we will show shortly. In addition, the mechanism also works when the debt contract matures deterministically. In Appendix IA.A, we consider the credit chain backed by a long-term underlying asset that matures in L periods. There, the optimal financing structure features an $(L-1)$-layer credit chain, where every layer bears some maturity mismatch. Collapsing the $(L-1)$ layers to one-layer does not yield the same*

[11] The framework applies less well to the equity market, where the creation fees in the ETF primary market is around 10 to 15 bps (State Street Global Advisors, 2024), while the bid-ask spreads ranges from 17 bps for large cap stocks to 148 bps for small cap stocks (Aliyev et al., 2024).

result. This is a key difference from the classic literature on financial intermediation that emphasize the role of diversification (Diamond, 1984): diversification suggests that one layer of intermediary is sufficient and cannot explain multiple layers observed in markets such as ABCPs.

Remark 2 ***State contingent debt maturity.*** *Because our mechanism is related to liquidation, in general some carefully designed debt contract can achieve the same (or even better) outcome than credit chain. In Appendix IA.B, we show that debt with state-contingent maturity, that is, the debt matures only if interim cash-flows news is positive but does not mature otherwise, could improve efficiency even further compared to the two-layer case.*[12] *This is because such contracts avoid all liquidation while still allowing for rollover when feasible, thereby minimizing transaction costs at the same time. The state-contingency helps by essentially altering the state in which bankruptcy occurs. However, this ideal arrangement is difficult to implement in practice because the fundamental of the underlying asset is often not contractable. The layered-structure is a market-based approximation to this ideal case, but an imperfect one due to the random maturity.*

III The Model and Equilibrium Analysis

We now extend the simple setting to the full model. We discuss our model assumptions after presenting the full model in Section III.A. We define the equilibrium in Section III.B, and then analyze the equilibrium in Section III.C.

A The Full Model

General distribution of cash-flows and endogenous default probability. Different from the example with binary cash-flow realizations in Section II.B, where the default probability $1-p$ is exogenously given by the probability of bad news, we now let y_t to be continuously distributed, with $H(\cdot)$ denoting the cumulative distribution function (CDF) and $h(\cdot)$ the corresponding probability density function (PDF). A dynamic coordination problem regarding households rollover decisions akin to "dynamic debt runs" in He and Xiong (2012) arises

[12] In the three-period setting studied in our NBER working paper version (w29632), state-contingent maturity is equivalent to the two-layer case.

here,[13] and we refer to the probability of rollover failures as the run probability. By endogenizing both default probability and chain length, this richer setting allows us to study the connection between financial stability and credit chain network. Finally, the endogenous default probability also allows us to discuss the constrained efficiency of the credit chain length in a meaningful way (see Section IV.B).

Debt-like contracts. We allow for a contract space that is broader than that in the example, but still "debt"-like. Denote by $\mathcal{F}_t$ the information set at the very end of period t, after knowing whether the underying asset matures or not is realized (Figure 2). Let T be the contract termination time (when either the underlying asset or debt matures, which is a stopping time measurable to $\mathcal{F}_t$). Any "debt"-like contract needs to specify i) promised payment upon debt maturity; ii) debt maturity; and iii) promised payment upon maturity of the underlying asset.

We set the promised payment upon debt maturity to be the households endowment e and debt maturity rate to be $\lambda_d \in (0,1)$, while focusing on the third to analyze endogenous rollover decisions. As in Section II, we assume that households hold debt that matures every period.[14]

Therefore, our debt contract takes the form of $\{F_{y,s}\}_{s=t}^{T}$, which specifies the following promised future payments upon the maturity of the underlying asset from the debtor to the creditor (w.p. stands for with probability):

$$\min(F_{y,s}, y_s) \cdot \mathbf{1}_{\text{underlying asset matures at period } s, \text{ w.p. } \lambda_y} + e \cdot \mathbf{1}_{\text{debt contract matures at period } s+1, \text{ w.p. } \tilde{\lambda}_d}. \tag{10}$$

Here, $\{F_{y,s}\}$ is $\mathcal{F}_{s-1}$-measurable for any $s \geq t$; and $\tilde{\lambda}_d = \lambda_d(1)$ for contracts issued to funds (households). One can interpret $F_{y,s}$ as interest payment each period (upon maturity of the underlying asset) and e as the face value to be paid (upon debt maturity). As we discuss shortly, it is the "debtness" of face value e—rather than that of interest payment $F_{y,s}$—that drives our result.

Denote by π_t the sequence of interest payments $\{F_{y,s}\}_{s=t}^{T} \in \Pi \equiv \mathbb{R}_+^{T-t+1}$. Each period

[13] If future cohorts are more likely to roll over their debt, the current cohort of households are less likely to face liquidation, and they will be more willing to roll over their debt as well.

[14] However, it is possible that on the equilibrium path, conditional on rollover failure households are holding liquidated assets that do not mature every period, as in Section II.B. This treatment fixes the total maturity transformation of the system.

t, all funds (and the ultimate borrower) can choose $\pi_t \in \Pi$ if their existing debt contracts mature. A new debt contract is signed after the existing debt matures with y's information in hand, but before knowing whether the underlying asset matures; see Figure 2. We focus on the class of issue-at-par debt contracts as before, and further impose Assumption 1.

Assumption 1 *Issuers with limited liability cannot raise new debt before existing debt matures. However, issuers have the option to prepay their existing debt any time.*

First, we rule out dilution by preventing issuers from raising new debt before their existing debt is repaid. Second, we allow debtors, after knowing the realization of y_t, to renegotiate by "prepaying" the debt—i.e., they can pay e and eliminate all future obligations. As we show later, this assumption implies "stationarity" so that the optimal debt contract chosen at any period along the equilibrium path is independent of history.[15] We suppress the time t index from now on, unless necessary.

Credit chain and prepayment clauses along the chain. The credit chain structure is presented in Figure 5. The "0-layer fund" of a credit chain corresponds to the ultimate borrower and the L-layer corresponds to households—the ultimate lenders. And, we call funds that sit at layer $i < l$ ($i > l$) to be the upper (lower) layers of fund l. The debt contracts between layers could potentially be different. Denote by $\pi_l = \{F_y(l)\}$ the contract issued by a fund in layer l borrowing from layer $l + 1$.

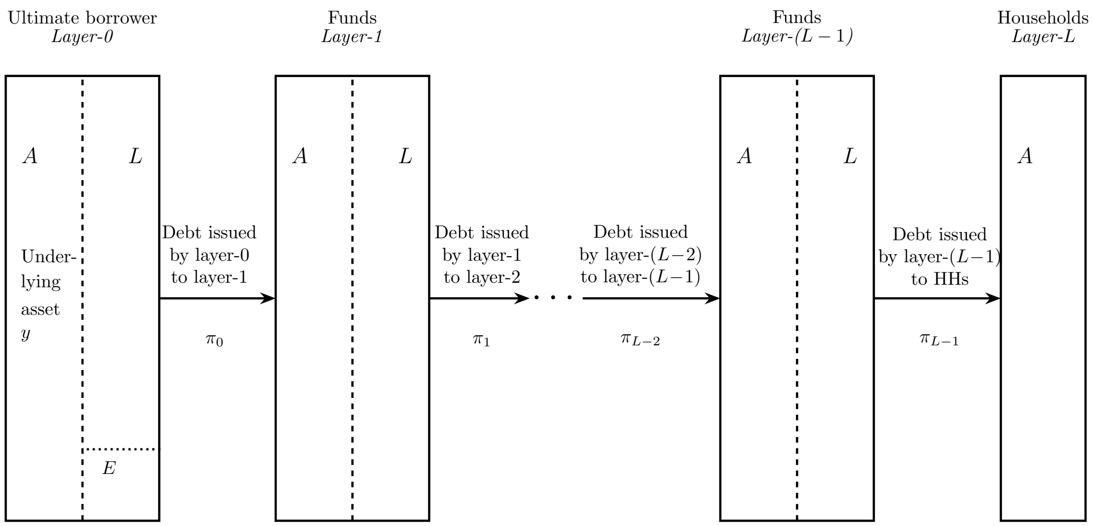
Credit chain features contracting externality. To facilitate analysis, we impose Assumption 2, i.e., "prepayment" clauses regarding other players in the chain; this differs from Assumption 1, which governs prepayment option within each layer over time.

Assumption 2 *When the underlying asset matures, all the debt contracts mature; when debt claim issued by layer l matures, all the debt claim issued by layer l' matures for all $l' \geq l$. And, limited liability implies that $F_y(l) \leq F_y(l-1)$ for $\forall 1 \leq l < L$.*

When $(l + 1)$'s debt claim issued by l matures, all debts issued by lower layers $i \geq l + 1$ mature, so that the payment from $l + 1$—whether l makes it full or gets liquidated—

[15]Because of the stationary structure of the fundamental (i.e., i.i.d. y_t's), the optimal debt contracts would have been stationary if we assume debt contracts to be short-term ($\lambda_d = 1$). We show that this is true even for the case of random maturity debt (so debt effectively is long-term); essentially, the prepayment option (of the lenders) is the minimum element to guarantee the stationarity of optimal contracting in our model.

Figure 5: Credit Chains



This figure illustrates the structure of the credit chain. Layer-0 is the ultimate borrower, holding the underlying asset and issuing debt contract π_0 to layer-1 funds. Funds in layer-l hold the debt issued by layer-0 on the asset side, and issue debt contract π_l to layer-$l+1$. The households hold debt contract π_{L-1} issued by the last layer of funds, layer-$(L-1)$.

trickles down to the ultimate departing households.[16] To simplify expression, we refer to the scenario that "either the underlying asset matures, or any debt contract issued by any fund $i \in \{1, \cdots, l-1\}$ matures" simply as that "layers above l mature." This prepayment clause ensures that the rollover failures of top layers propagate through the credit chain to lower layers, creating contagion and spillover effects.

The random maturity setup together with the prepayment clause conveniently captures "maturity transformation" along the credit credit. That is, each layer's asset has longer maturity than its liability side. Importantly, when the chain length increases, the maturity of the liquidated asset in expectation decreases in the event of rollover failure.[17]

[16] In equilibrium we show $F_y(l) = F_y(l-1)$. And, if multiple contracts mature, only the one with the highest layer (the smallest layer number) matters.

[17] Layer-l's asset matures as long as the debt issued by one of the layers above matures, which occurs with probability $1-(1-\lambda_d)^l$ conditional on the underlying asset does not mature. In fact, prepayment clause says in any layer, its liability matures whenever asset matures but not the other way around, which implies "maturity transformation." As fund-l's asset maturity decreases with l, when the chain length increases, it is more likely that rollover failure occurs at layers with shorter maturity asset. This implies the expected maturity of liquidated asset decreases. Finally, we adopt the random maturity setup for model tractability,

Last but not least, because the layer-$(L-1)$ fund issues short-term debt, viewing the system as a whole, its liability matures with probability 1 while its asset matures with probability $\lambda_y \in (0,1)$. Hence in our model the total maturity transformation is fixed at $1/\lambda_y > 1$ (to 1), regardless of the chain length L.

Debt rollover and restructuring cost. When the debt issued by layer l matures, successful rollover occurs when y exceeds above certain endogenous threshold $\hat{y}_l$. Given i.i.d. cash-flows, the new cohort of households can form a new credit chain with the same optimal length of L.[18] When y is below the endogenous threshold, rollover fails. Liquidation follows the same process (intermediated by experts) as explained in the simplified setting on page 12, and the endogenous liquidation proceeds is denoted by $B_l(y, L)$.[19]

We assume that the chain is then restored in the following period. We essentially need some bankruptcy cost, and a delay of chain length restoration is the simplest way to capture this inefficiency.[20] Further, we impose a restructuring/legal cost $c \geq 0$ for each layer during bankruptcy, so that the liquidation proceeds received by the creditors is $B_l(y, L) - c(L-l)$; this prevents the optimal chain length from being unbounded. It also captures "the spillover costs" to bottom layers when a top layer fails to rollover its debt.

Finally, when the layer-0 ultimate borrower fails to rollover his debt, bankruptcy occurs but experts can locate the original borrower who is the first-best holder to operate the underlying asset.[21] The original chain is restored and the economy is stationary, and this ensures that the private loss in a bankruptcy is the same as the social loss.

though Remark 1 shows that the mechanism goes through when debt maturity is deterministic.

[18]There are many different ways to implement the same outcome, as essentially in this arrangement departing households receive the payment e financed by new-born households. For example, all funds can simply ask for the funds from their corresponding lender for rollover. In the final layer, the new-born households simply replace departing households. The credit chain stays exactly the same going forward.

[19]This liquidation value $B_l(y, L)$ depends on y, which is the cash-flow if the underlying asset matures at the end of period. (In our example with binary y, the relevant y in liquidation is $y = 0$, which has been omitted for brevity.)

[20]Our mechanism goes through as long as the restoration is delayed with a non-zero probability, or in a setting where restoration occurs with a constant probability each period, instead of restoration after one period. In the NBER Working Paper Version (w29632), we consider a more general case where the chain restoration is delayed with some probability.

[21]This can be motivated by project-specific human capital just like in Diamond and Rajan (2000)

Discussion of model assumptions. First, we focus on credit chain length and therefore leave endogenous debt maturity choice to future research.[22] As discussed after Assumption 2, we intentionally set the debt maturity to households to be 1, so that the total maturity transformation in the system is fixed (from the underlying asset maturity $1/\lambda_y$ to 1). In the NBER working paper version (w29632) of this paper (He and Li, 2022), we set debt maturity rate λ_d uniformly across all layers and deliver the same mechanism. In addition, Appendix IA.C.IA.C.1 shows that the $L-1$ fund prefers to issue one-period debt than to issue debt that matures with probability λ_d as long as the per layer bankrupcty cost c is not too large.

Second, we have exogenously fixed the face value of the debt at the household endowment e. Appendix IA.C.IA.C.2 endogenizes the face value sequence and derives the condition for binding at e. A larger face value benefits from the discount rate wedge between the ultimate borrower and households but risks future rollover failures. Due to resource constraints, the face value cannot exceed household endowment, and therefore it binds at e when e is sufficiently low.

Third, the "debt" form of interest payment upon *the maturity of the underlying asset* is not essential; we can show that this is indeed the optimal contract given limited liability. This is because the game ends without inefficient liquidation after the underlying asset matures. (Inefficient liquidation only occurs after a debt contract matures with a sufficiently low y_t.) Nevertheless, the simple debt form on interest payments allows us to make a sharper claim, as we show that in equilibrium $\{F_{y,s}\} = F_y^*$ is stationary (for all layers). Or equivalently, in equilibrium, the optimal $\mathcal{F}_s$-measurable interest payment is $\min(F_y^*, y_s)$.

B Value Functions and Equilibrium Definition

Each period the layer-l fund sets its contract (denoted by π_l), taking the fundamental (y), the total chain length (L), and the contract from the layer above (π_{l-1}) as given. Denote the layer l's value function by $V_l(y, \pi_l; \pi_{l-1}, L)$; this is evaluated after debt maturity but before the underlying asset maturity (in Figure 2). For layer-0, the ultimate borrower's value function depends only on y, π_0, and L. Denote the market price of the debt issued by layer-l under contract π_l by $P_l(\pi_l, y; \pi_{l-1}, L)$. We may write the price of the debt and the value function simply as $P_l(y)$ and $V_l(y)$ whenever there is no risk of confusion.

[22]For models with endogenous debt maturity structure, see He and Milbradt (2016) and Hu, Varas, and Ying (2021).

For notational convenience, we denote $\tilde{F}_y(l) = \min(F_y(l), y)$ and $\tilde{F}_y(-1) = y$. We also denote by m_l the probability that layer l's asset does not mature:

$$m_l \equiv (1-\lambda_d)^l \quad \text{for} \quad 0 \le l \le L-1 \tag{11}$$

which satisfies $1 - m_{l+1} = 1 - m_l + m_l \lambda_d$. Since debt held by households always matures, we can define $m_L \equiv 0$.

Fund managers and ultimate borrower. For $0 \le l < L$, we calculate layer-l's payoff in period 0 to be

$$P_l(\pi_l, y; \pi_{l-1}, L) - P_{l-1}(\pi_{l-1}, y; \pi_{l-2}, L) + V_l(y, \pi_l; \pi_{l-1}, L), \tag{12}$$

where $P_{-1} \equiv 0$. Layer-l issues its debt π_l for a proceed of P_l, and then purchases the debt from layer-$(l-1)$ at a price of P_{l-1} (except for the ultimate borrower), where P_l and P_{l-1} are the market prices of the underlying debt. The last term captures its continuation payoff.

Following the convention of using prime to indicate variables in the next period, we can write $V(y, \pi_l; \pi_{l-1}, l, L)$ for $0 < l < L$ recursively as,

$$V_l(y, \pi_l; \pi_{l-1}, L) = \lambda_y \underbrace{(\tilde{F}_y(l-1) - \tilde{F}_y(l))}_{\text{Underlying asset matures}} \tag{13}$$

$$+ (1-\lambda_y)\alpha \Bigg\{ m_{l+1}\mathbb{E}\Big[\underbrace{V_l(y', \pi_l; \pi_{l-1}, L)}_{\text{Neither debt issued by nor held by layer } l \text{ matures}} \Big] \tag{14}$$

$$+ \sum_{i=0}^{l-1} (m_i - m_{i+1}) \mathbb{E}\Big[\underbrace{\mathbf{1}^i_{rollover}(-P'_{l-1} + \max_{\pi'_l}(P'_l + V_l(y', \pi'_l; \pi'_{l-1}, L)))}_{\text{Debt held by layer } l \text{ matures}} \Big] \tag{15}$$

$$+ (m_l - m_{l+1}) \mathbb{E}\Big[\underbrace{\mathbf{1}^l_{rollover}(-e + \max_{\pi'_l}(P'_l + V_l(y', \pi'_l; \pi_{l-1}, L)))}_{\text{Debt held by layer } l \text{ does not mature but debt issued by layer } l \text{ matures}} \Big] \Bigg\}. \tag{16}$$

In the above expression, (13) captures the payoff to layer-l when the underlying asset matures with probability λ_y; otherwise with probability $1 - \lambda_y$, we have the next three terms.

First, (14) captures the continuation value of layer-l when neither its asset nor liability side matures, which occurs with probability m_{l+1}.[23] Second, (15) captures the payoff if layer-l's asset side matures; this happens whenever debt issued by any layer-i ($i < l$) matures. In this case, layer-l's debt also matures under Assumption 2, and therefore it receives zero if rollover fails. When rollover is successful ($\mathbf{1}^i_{rollover} = 1$), then layer-$l$ receives e from its debtors, and pays e to its creditors — the two terms cancel out. In the refinancing stage, it receives P'_l from its new creditors and gives P'_{l-1} to its debtors. Going forward, layer l's valuation is $V(y', \pi'_l; \pi'_{l-1}, l, L)$, where π'_l is the new contract issued by layer-l and π'_{l-1} is a new contract given to layer-l. Here, fund l optimally chooses π'_l to maximize the sum of new debt proceeds and its continuation payoff $P'_l + V_l(y', \pi'_l; \pi_{l-1}, L)$. Third, (16) captures the event that the debt issued by layer-l matures but layer-l's asset has not matured yet;[24] there, if rollover is successful, layer-l raises P'_l, pays off e to existing creditors and chooses a new contract π'_l. Otherwise, rollover fails and layer-l's payoff is 0.

Note that the ultimate borrower is labeled as layer 0. His value function is similar to that of the fund manager's, except that he returns after the bankruptcy to manage the underlying asset as the first-best holder. For the ultimate borrower's value function, see Appendix A.A.

Households. In equilibrium, new-born households are paying the competitive price $P_{L-1}(y)$ for the debt contract:

$$P_{L-1}(y) = V_L(y; \pi_{L-1}, L) = \lambda_y \underbrace{\tilde{F}_y(L-1)}_{\text{Underlying asset matures, } = \min(F_y(L-1), y)} + (1-\lambda_y)\left\{ \sum_{l=0}^{L-1} (m_l - m_{l+1}) \mathbb{E}[\underbrace{\mathbf{1}^l_{rollover} e + (1 - \mathbf{1}^l_{rollover})[B_l(y, L) - c(L-l)]}_{\text{Rollover happens at layer } l \le L-1}] \right\}. \tag{17}$$

In (17), with probability $1 - \lambda_y$ the underlying asset does not mature, though households' debt matures with probability 1. When debt issued by layer-l matures (with probability $m_l - m_{l+1}$), the repayment trickles down to households due to the prepayment clauses. The departing households get paid by e if rollover is successful, or they receive the liquidation

[23] For the last layer of fund ($l = L-1$) who borrows from households, its liability side always matures (recall $m_L = 0$); and hence this scenario never occurs.

[24] This occurs with probability $m_l - m_{l+1} = \lambda_d m_l$ for $1 \le l < L-1$. For layer-($L-1$), this occurs with probability m_{L-1}, as its liability side always matures (layer-L households hold one-period debt).

proceeds $B_l(y, L) - c(L - l)$ if rollover fails.

As illustrated by Section II.B, the liquidation value $B_l(y, L)$ plays a key role in our model. It equals to the buyer households' valuation for the asset, discounted by α, i.e.,

$$B_l(y,L) = \alpha \Big\{ \underbrace{\lambda_y \tilde{F}_y(l-1)}_{\text{Underlying asset matures}} + (1-\lambda_y)\Big[m_l \underbrace{\mathbb{E}[\alpha V_L(y';L)]}_{\text{Debt does not mature}} \tag{18}$$

$$+ \sum_{i=0}^{l-1} (m_i - m_{i+1}) \mathbb{E} \underbrace{\left[\mathbf{1}^i_{rollover} e + (1 - \mathbf{1}^i_{rollover})(\alpha V_L(y', L) - c(l-i)) \right]}_{\text{Debt matures}} \Big] \Big\}. \tag{19}$$

Recall that the households hold the liquidated asset (debt issued by layer $l - 1$) directly for one period, and the chain is restored to L in the following period. If the underlying asset matures with probability λ_y during this period, then households get paid $\tilde{F}_y(l - 1)$; this is the first term in (18). If neither the underlying asset nor the debt matures, which occurs with probability $(1 - \lambda_y)m_l$, then it is sold at discount α to the next cohort of households (who then hold debt issued by the restored chain with length L); this is the second term in (18). Finally, if the underlying asset does not mature but debt matures (which could occur if any debt issued by layers above l matures), then households either get paid by e given successful rollover or receive the liquidation proceeds $\alpha V_L(y, L) - c(l - i)$ if rollover fails.[25] This is captured by (19).

Equilibrium definition. In equilibrium, the fund managers (and the ultimate borrower) choose contracts π_l to maximize their payoff in (12), subject to limited liability as in Assumption 2. The equilibrium chain length L^* is such that the last layer of fund manager $L^* - 1$ prefers to borrow directly from households than to borrow via other fund managers:

$$P_{L^*-1}(L^*) + V_{L^*-1}(L^*) \geq P_{L^*-1}(L^* + l) + V_{L^*-1}(L^* + l) \quad \text{for} \quad l \geq 1. \tag{20}$$

Furthermore, all the other intermediary layers prefer to borrow via other funds than to borrow from households, i.e. for $0 < l < L^* - 1$,

$$P_l(L^*) + V_l(L^*) \geq P_l(l + 1) + V_l(l + 1). \tag{21}$$

[25] Since the chain will be restored by the end of period, the buyer's valuation for the asset is $V_L(y, L)$, discounted by α. Taking into account of the legal cost $c(l - i)$ gives the liquidation proceeds.

Finally, all intermediary funds earn zero profit in equilibrium because of perfect competition. The equilibrium is defined formally in Appendix A.B.

C Equilibrium Analysis

We now conduct formal analysis on the equilibrium of our economy. After showing that the equilibrium contract features stationarity and layer independence, i.e., the interest payment F_y is the same for all layers and is stationary over time, we formally establish that the liquidation value is increasing in the chain length. Both results allow us to characterize and analyze the equilibrium credit chain length L^*.

Equilibrium contract. We show that the equilibrium contract features stationarity and layer-independence; and for this section we put back time subscript t. Each period t, after y_t is observed but before the underlying asset matures, layer-l chooses a new contract for its creditors when either the debt issued by himself or the debt held by himself matures, i.e., the event $\mathbf{1}_{rollover}$ in Eq. (15) and (16) occurs. The problem of layer-l ($0 < l < L$) is equivalent to:

$$\max_{\pi_{l,t}} \quad P_{l,t} + V_l(y_t, \pi_{l,t}; \pi_{l-1,t}, L) \tag{22}$$

$$s.t. \quad P_{l+1,t} + V_{l+1}(y_t, \pi_{l+1,t}; \pi_{l,t}, L) - P_{l,t} = 0, \tag{23}$$

$$F_{y,s}(l) \leq F_{y,s}(l-1) \quad \forall s \geq t. \tag{24}$$

Eq. (23) says that the equilibrium payoff of layer-$(l+1)$ is 0 given perfect competition, as $P_{l+1,t} + V_{l+1}(y_t, \pi_{l+1,t}; \pi_{l,t}, L)$ is layer-$(l+1)$'s payoff from issuing debt while $P_{l,t}$ is how much he pays to layer-l; and (24) is the limited liability constraint imposed in Assumption 2. We have the following lemma on the equilibrium contract.

Lemma 1 *The interest rate payment in the optimal debt contract is stationary and independent of fund layer l, so that $\tilde{F}_{y,t}(l) = \min(y_t, F_y^*)$.*

Start with stationarity. Recall that successful rollover occurs when y_t exceeds certain endogenous threshold $\hat{y}_{l,t}$ which is measurable to $\mathcal{F}_{t-1}$. By definition, $\hat{y}_{l,t}$ is the payment to period t creditors, so that the present value of the debt contract—i.e., all future promised payments at $t+s$ with $s \geq 1$—equals the debt value e. But Assumption 1 in Section III says that the debtor can always unilaterally prepay his debt; hence in a renegotiation proof contract the funds set $F_{y,t+s}(l) = \hat{y}_{l,t+s}$ for $s \geq 0$, i.e., the interest payment equals the run threshold for all periods. But the face value to be refinanced (which is e, as debts are issued-at-par) is constant over time. As a result, the endogenous rollover threshold $\hat{y}_{l,t}$ is also constant over time, yielding the stationarity of $F_y(l)$.[26]

Next, we argue that $F_y(l)$ has to be the same for all l as well due to perfect competition. In light of limited liability constraint (24), we only need to rule out the case $F_y(l-1) > F_y(l)$. Suppose this is the case; layer-l then earns positive spread when the underlying asset matures, implying strictly positive profit in expectation—contradicting with perfect competition.

Liquidation value. The next proposition formally gives the key property of $B_l(y, L)$ that drives the benefit of a long-chain.

Proposition 1 *Liquidation value $B_{L-j}(y, L)$ is increasing in L for $L \leq L^*$ and any $j \leq L$.*

We show this formally in Appendix B.A. By fixing the distance j between the bankruptcy layer $L - j$ and households while varying the chain length L, Proposition 1 shows that the further away from the ultimate borrower the higher the liquidation value. Intuitively, the asset in liquidation at the breaking point $L - j$ (where rollover fails) can be considered as a collection of debt contracts issued by all layers above; and consistent with the intuition of maturity transformation, the further away the breaking point from the ultimate borrower, the shorter-term the liquidated asset. Just as the key intuition illustrated in Section II.B, these shorter-term claims are desirable in that if favorable fundamental y realizes later then debt payments can flow toward departing households in a frictionless way (i.e., without the discount factor α), leading to a higher liquidation value.

To see the connection between our full model and the simplified setting in Section II, consider the case when $j = 1$ and $c = 0$. The improvement in the liquidation value when the chain length increases from L to $L + 1$ is given by

$$B_L(y, L+1) - B_{L-1}(y, L) \propto \lambda_d(1 - H(F_y^*))(1-\alpha)e > 0, \tag{25}$$

[26]The cutoff $\hat{y}_{l,t}$ is pinned down by $V_L(\hat{y}_{l,t}, \{F_{y,t+j}(l)\}_{j=1}^{\infty}, L) = e$. Since the intermediate layers all have zero payoffs, a successful rollover is determined by promising the households a value equal to the face value e. In renegotiation-proof contracts, $F_{y,t+j}(l) = \hat{y}_{l,t+j}$. With stationarity, $\hat{y}_l$ is pinned down by $V_L(\hat{y}_l, \{\hat{y}_l\}, L) = e$.

where $H(F_y^*) \equiv \Pr(y < F_y^*)$ is the endogenous probability of rollover failure in equilibrium. Eq. (25) is just (8) derived in Section II.B, with the only difference being that the probability of successful rollover is now $1 - H(F_y^*)$ given a general distribution of cash-flows (instead of the exogenous probability p with $y = \bar{y}$ in the simplified setting.) The intuition is exactly the same as in our simplified setting in Section II. Proposition 1 formally states this property in our full model: The more the layers between the point of bankruptcy and the underlying asset, the shorter-term the liquidation asset is, the higher the liquidation value, and the greater the ex-ante debt value.

Characterizing equilibrium. Stationary and layer-independent contracts allow us to simplify the households' value as a function of F_y, L, and y as follows:

$$V_L(F_y, L; y) = \lambda_y \min(F_y, y) \tag{26}$$

$$+ \underbrace{(1-\lambda_y)\Big\{(1-H(F_y))e + H(F_y)\big[\sum_{l=0}^{L-2} m_l \lambda_d(\mathbb{E}[B_l(y,L)|y<F_y] - c(L-l)) + m_{L-1}(\mathbb{E}[B_{L-1}(y,L)|y<F_y] - c)\big]\Big\}}_{v_L(F_y,L)}. \tag{27}$$

Conditional on rollover being successful ($y \geq F_y$), the households' valuation of the debt V_L should equal e, representing households' binding participation constraint in equilibrium. Therefore the following equation pins down equilibrium F_y as a function of L,

$$V_L = e \quad \Rightarrow \quad \lambda_y F_y + v_L(F_y, L) = e. \tag{28}$$

Here, $v_L(F_y, L) \equiv V_L(F_y, L; y) - \lambda_y \min(F_y, y)$ is the continuation value in the event that the underlying asset does not mature in this period, which is independent of the current realization of y.

We impose Assumption 3 so that the solution to (28) as a function of F_y is unique.

Assumption 3 *The following inequality holds for all F_y,*

$$\lambda_y - (0,0,...,0,1)\Psi^{-1}\frac{\partial \Psi}{\partial F_y}\Psi^{-1}\eta + (0,0,...,0,1)\Psi^{-1}\frac{\partial \eta}{\partial F_y} \geq 0, \tag{29}$$

where the exact expressions for $\Psi(F_y)$ and $\frac{\partial \eta(F_y)}{\partial F_y}$ are in Appendix B.C.

Now we are ready to determine L^*. Given the competitive fund sector, the problem faced by the last fund layer is equivalent to maximizing the sum of the fund's payoff and the households' payoff. Because $\lambda_y \min(F_y, y)$ in (26) is a transfer between households and funds, the fund problem is equivalent to maximizing v_L. Proposition 2 follows.

Proposition 2 *Under Assumption 3, the equilibrium (promised) interest payment F_y^* and equilibrium chain length L^* is the unique solution to the following equations*

$$e = \lambda_y F_y^* + \underbrace{\begin{bmatrix} 0 & 0 & ... & 0 & 1 \end{bmatrix}}_{1\times(L^*+1)} (\Psi(F_y^*)^{-1}\eta(F_y^*)), \tag{30}$$

$$0 = \alpha(1-\lambda_y)\lambda_d(1-\lambda_d)^L m_{L-1} e(1-H(F_y^*))(1-\alpha) - [1-(1-\lambda_d)^L + \alpha(1-\lambda_y)(1-\lambda_d)^L(1-(1-\lambda_d)^L)H(F_y^*)]c, \tag{31}$$

where Ψ is a $(L^*+1)\times(L^*+1)$ matrix and η is a $(L^*+1)\times 1$ vector, with both being functions of F_y^*. The exact expressions for Ψ and η are in Appendix B.C.

As explained, the equilibrium chain length L^* is effectively characterized by maximizing households' continuation payoff v_L, with (31) as the first-order condition. (In practice credit chain length L should take an integer value. Although it is straightforward to impose this restriction, for exposition convenience we do not impose this requirement here.) The first term in (31) gives the marginal benefit of a longer chain. To see this, consider the special case when $c = 0$ and compare the difference in households continuation value when the chain length is L versus $L+1$. Appendix C.2 shows that for a given F_y we have

$$v_{L+1} - v_L \propto B_L(y, L+1) - B_{L-1}(y, L). \tag{32}$$

As shown in (25), the difference in liquidation value $B_L(y, L+1) - B_{L-1}(y, L)$ is positive and proportional to the first term in (31). Intuitively, having multiple layers increases the liquidation value because the liquidated asset is of shorter maturity in expectation.

On the cost side, which is the second term in (31), the total bankruptcy cost given rollover failure increases in the number of layers disrupted. Combining this with (30), which captures the households' participation constraint in (28), yields the equilibrium F_y^* and L^*.

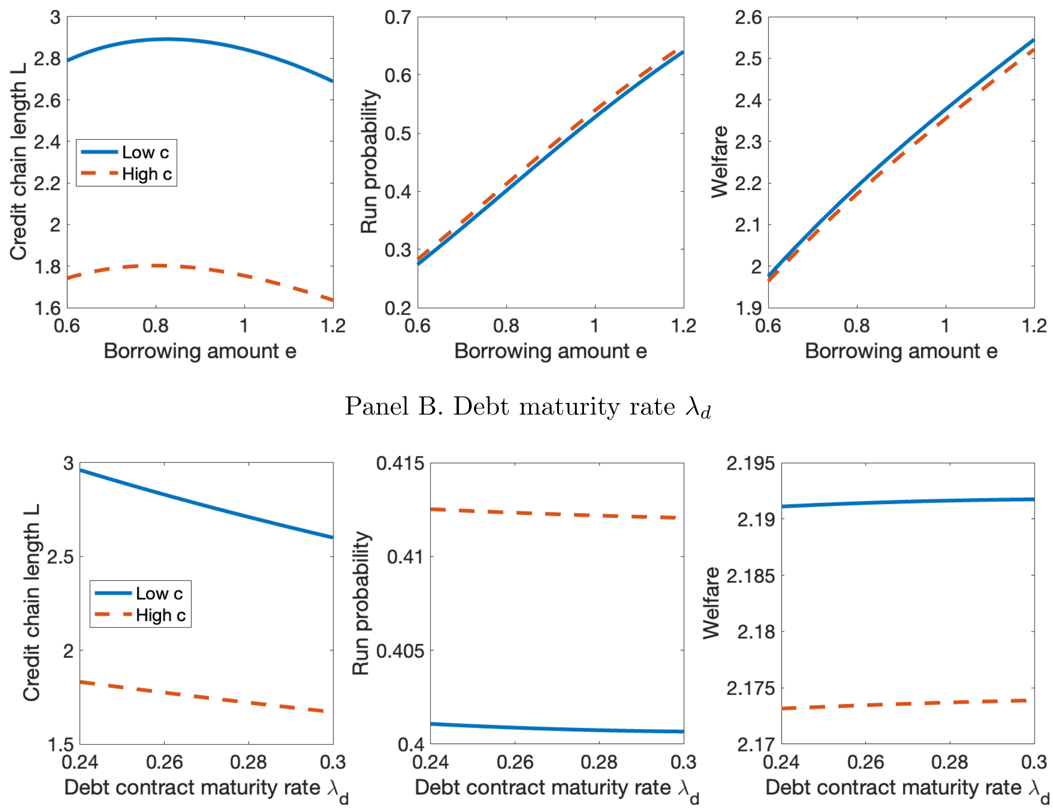
IV Model Implications and Extensions

We now discuss model implications based on the equilibrium characterized in Proposition 2. In Section IV.C we further provide an information-based mechanism to micro-found the exogenous intermediation friction α in the main model.

Figure 6: Comparative Statics with respect to e and λ_d

Panel A. Borrowing amount e

Panel B. Debt maturity rate λ_d



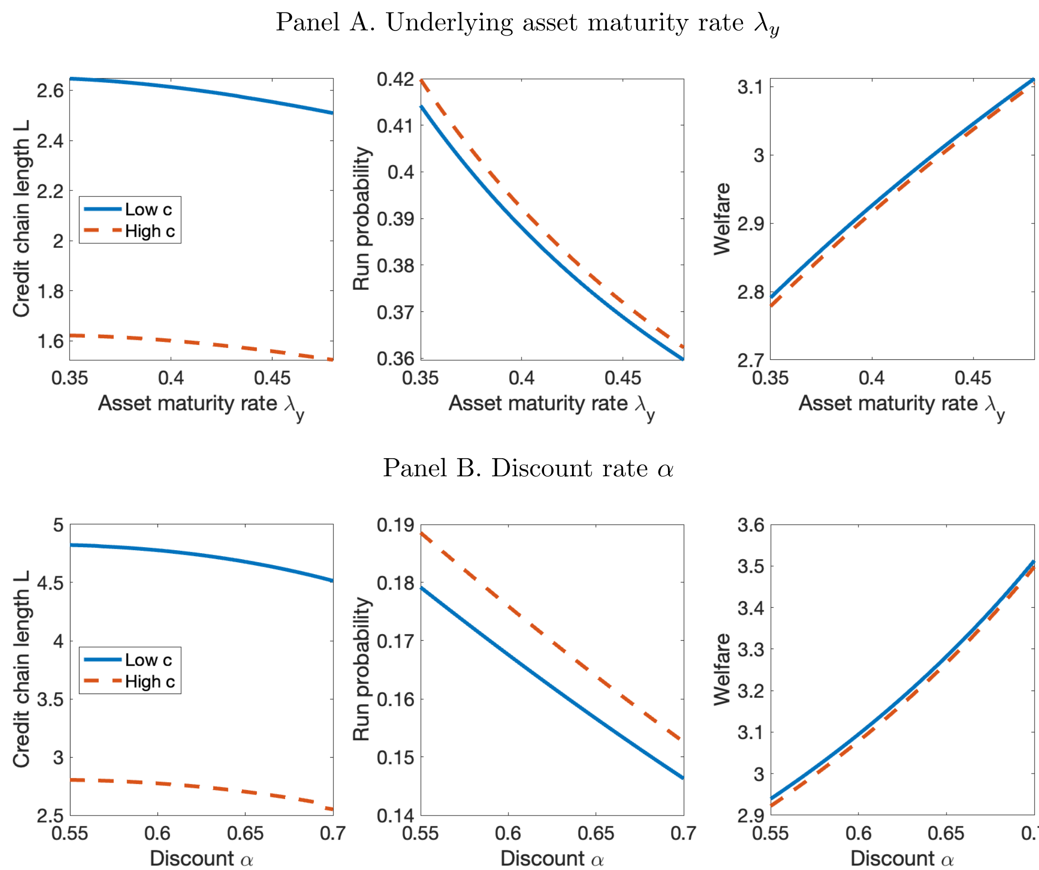
Numerical illustration of comparative statics of equilibrium outcomes. Parameter values (unless specified in the x-axis): $\lambda_d = 0.25$, $\alpha = 0.5$, $\lambda_y = 0.2$, $g(y) = 0.3\exp(-0.3y)$, $e = 0.8$. The blue solid line plots the equilibrium outcome when $c = 0.01$ and the red dotted line plots the equilibrium outcome when $c = 0.025$.

A Comparative Statics

How does credit chain length vary with our parameters? Start with the legal cost c; we have seen in Proposition 2 that the marginal benefit of a longer chain is positive, hence the equilibrium credit chain length L^* diverges to infinite when $c = 0$. When $c > 0$, additional cost in the case of rollover failure increases with L, leading the equilibrium chain length to be finite as suggested by the first-order condition (31). We have the following proposition.

Proposition 3 *The equilibrium credit chain length L^* is decreasing in bankruptcy cost c,*

Figure 7: Comparative Statics with respect to λ_y and α



Numerical illustration of comparative statics of equilibrium outcomes. In panel (a), parameter values are: $\lambda_d = 0.25$, $\alpha = 0.55$, $g(y) = 0.35\exp(-0.35y)$, $e = 1$. In panel (b), parameter values are: $\lambda_d = 0.12$, $\lambda_y = 0.1$, $e = 0.8$, and $g(y) = 0.12\exp(-0.12y)$. The blue solid line plots the equilibrium outcome when $c = 0.012$ and the red dotted line plots the equilibrium outcome when $c = 0.03$.

i.e. $\frac{\partial L^*}{\partial c} \leq 0$. *When* $c = 0$, *we have* $L^* = \infty$.

When the legal cost c is higher, the equilibrium chain length is shorter, the run probability is higher, and the total welfare—which is the sum of all agents' payoff defined later in Section IV.B—is lower. However, for other parameter values, the effects are generally mixed. Figure 6 and 7 plot several numerical illustrations of how equilibrium chain length, run probability and welfare vary with parameter values.

To understand the opposing forces, consider the marginal benefit of extending the chain length when the households' endowment e, i.e., borrowing amount or leverage in the system, is larger. Recall that the benefit of longer chains comes from a higher liquidation value of the debt, which is proportional to $e(1-H(F_y^*))(1-\alpha)$ in (25). The direct effect of higher leverage e increases the marginal benefit of longer chains. However, higher leverage also increases the probability of rollover failures $H(F_y^*)$, as shown in panel (a) of Figure 6; and this indirect effect through the equilibrium rollover threshold reduces the benefit of longer chains. Figure 6 Panel (a) presents a case where the effect of e on the chain length is non-monotone. Finally, welfare naturally increases with e because e is the endowment of households.

In general, the direct effect on chain length and the indirect effect through the equilibrium rollover threshold operate in opposite directions, which also applies to other parameters. For example, when λ_d becomes smaller, the asset side of any given layer has longer maturity in expectation and it is more costly to liquidate. This force pushes more layers in the chain. However, a smaller λ_d leads to a higher run probability, so the indirect effect goes in the opposite direction. In Figure 6 Panel (b), the direct effect dominates and chain length is longer when λ_d is smaller. The comparative statics with respect to the underlying asset maturity rate λ_y is qualitatively similar — a smaller λ_y implies longer expected maturity for the underlying asset and hence a more severe maturity mismatch.

Finally, Figure 7 Panel (b) illustrates the effects of the intermediation discount α. The direct effect of higher α reduces the marginal benefit but the indirect effect via probability of rollover failure counteracts it. When $\alpha = 1$, i.e., without any transaction/liquidation cost, we have $L^* = 1$ as there is no liquidation loss to start with, implying no benefit of using long chains. This implies that L^* decreases in α for α being close to 1. But for general α values the comparative statics is undetermined: a higher α also reduces the probability of rollover failure, and may increase the marginal value of longer chains. In Figure 7 Panel (b), assets with worse liquidity (smaller α) are supported by longer credit chains (greater L^*). This pattern is consistent with the case of MBS where the underlying assets (real estate properties) are with illiquid secondary markets and the intermediation chain is long.

B Welfare Analysis

We now study whether the decentralized equilibrium is constrained-efficient from the social planner's perspective. We ask: Can the social planner improve welfare by restricting the credit chain length, say via a regulation that caps L? The answer is negative.

Consider a constrained planner who chooses L to maximize the sum of all agents' utilities, subject to that the contracts are determined by the decentralized equilibrium as characterized in Section III.C. Since equilibrium contracts are layer-independent and time-invariant, and all the middle layers $1 \leq l \leq L-1$ earn zero profits in market equilibrium, the social welfare $W(F_y(L), L)$ equals:[27]

$$W(F_y(L), L; y) = e + \lambda_y y + \underbrace{v_0(F_y(L)) + v_L(F_y(L), L)}_{\equiv w_L}. \tag{33}$$

Here, we take y as given, and highlight $F_y(L)$ which is the equilibrium interest rate given chain length L. Denote the continuation value of the social welfare by $w_L = v_0 + v_L$. For households (layer L), their value v_L not only depends on the interest rates F_y but also the chain length L (through the liquidation value). But for the ultimate borrower (layer 0), his value only depends on the interest rates F_y, which also captures the probability of rollover failure.[28]

The continuation value of the social welfare w_L can be expressed recursively as

$$w_L = (1-\lambda_y)\Big\{\alpha\lambda_y\mathbb{E}[y] + \alpha w_L + \underbrace{(1-H(F_y))(1-\alpha)e - H(F_y)\sum_{l=0}^{L-1}(m_l - m_{l+1})\left[\alpha(v_L(L) - b_l) + c(L-l)\right]}_{\equiv\Delta}\Big\}. \tag{34}$$

To understand (34), note that the term Δ represents the gain from trade (via endogenous credit chains) in our economy. To see this, consider the case of autarky without lending. There, the households' payoff is e, while the ultimate borrower's payoff $\hat{V}_0 = \lambda_y y + \hat{v}_0$, with $\hat{v}_0$ denoting his continuation value if the underlying asset does not mature. One can express $\hat{v}_0$ recursively as

$$\hat{v}_0 = (1-\lambda_y)\left(\alpha\lambda_y\mathbb{E}[y] + \alpha\hat{v}_0\right), \tag{35}$$

which makes it clear that the difference between (35) and (34) comes from Δ. The first part

[27] Future newborn households' endowments are not included in the welfare calculation.

[28] Strictly speaking, this is only true for $L > 1$. However, when $L = 1$, the ultimate borrower is the one deciding whether to borrow from the households or another fund, hence he internalizes the total welfare.

of Δ captures the impatience wedge between households and the ultimate borrower, and the second part captures the cost in the event of rollover failure.[29]

We now show that the equilibrium chain length L^* emerging from the decentralized market, defined on page 28, is constrained efficient. Consider the impact of varying credit chain length on the total welfare, evaluated at the decentralized equilibrium L^*. Both the ultimate borrower's payoff (v_0) and the households' payoff (v_L) in (33) are affected:

$$\frac{dW}{dL} = \frac{dv_0}{dF_y} \underbrace{\frac{dF_y}{dL}}_{=0}\Big|_{L=L^*} + \underbrace{\frac{dv_L}{dL}}_{=0}\Big|_{L=L^*}. \tag{36}$$

As argued in Section III.C right before Proposition 2, in the decentralized equilibrium, the privately optimal chain length L^* is chosen to maximize households' payoff v_L. Hence the second part of Eq. (36) is equal to 0 at the decentralized equilibrium L^*. Furthermore, as suggested by $v_L = e - \lambda_y F_y$ in (28), maximizing v_L amounts to minimizing F_y. Intuitively, layer-l will only borrow via another layer of funds if extending the credit chain reduces interest F_y; otherwise, layer-l should borrow directly from households. Hence the first part of Eq. (36) is also 0. The following proposition summarizes our key result in this section.

Proposition 4 *The constrained social planner's solution coincides with that in the decentralized equilibrium.*

Why does the first-order condition of L from the social perspective coincide with that in the decentralized market in our model? At a high level, this is because the trade-offs of extending chains, including the additional restructuring cost, is reflected in the interest rate paid by the last layer of funds to OLG households. If the cost of extending the chain outweighs the benefit, fund managers will directly borrow from households for a lower interest rate—in other words, households are willing to accept a lower interest rate to rollover the debt. Thanks to this force, in our model funds internalize the cost and benefit of longer chains through the interest rate they pay. We expect this force to be general in other settings, though we leave it to future research for a more thorough analysis on this issue.

We emphasize that our constrained-efficiency result is conditional on intermediation friction (α) and bankruptcy costs (c) being fixed. Incorporating bankruptcy externalities, in

[29] When $c = 0$, we can show that $\Delta > 0$, representing a gain from trade. See Appendix B.E for details.

which the bankruptcy cost c is endogenous to the number of layers being liquidated, could lead to a chain that is too long in the decentralized equilibrium. For instance, consider an extension where there are limited resources to handle distressed intermediary funds. Individual agents in the decentralized economy take these equilibrium variables as given, while the planner internalizes the effect of chain length on number of layers being liquidated and eventually the bankruptcy costs. Then, the wedge between the social planner and the decentralized economy boils down to the effect of credit chain length on the fraction of funds that go through rollover failures, which leads the decentralized equilibrium chain length to be longer than the socially optimal one.

More broadly, if the overall degree of maturity transformation generated by the system varies with the chain length as in the NBER working paper version (w29632), then there is a wedge between systemic risk (the probability that the credit chain experiences a run) and the rollover risk of a given layer.[30] In this setting, the planner would like to regulate the chain length to reduce systemic risk. Standard fire-sale externalities (Shleifer and Vishny, 1992; Lorenzoni, 2008; He and Kondor, 2016 and many more), in which fire-sale discount gets more severe when more assets are being liquidated, also lead to chains that are too long.

C Micro-Foundation for Intermediation Friction

In this section, we provide a formal micro-foundation for the intermediation friction $\alpha \in (0, 1)$ based on information asymmetry.

Uncertainty in default. Suppose that following default, the resolution process via the court introduces additional uncertainty to the contract payoffs. Specifically, if default happens in period t at layer-l, the asset of layer-l is liquidated and sold to cohort-t households. The court resolution system introduces uncertainty such that the cash-flow received by the asset holders equals the product of the cash-flow that the liquidated asset delivers and a random noise $\tilde{\epsilon}_t > 0$ with CDF $G(\cdot)$. To highlight friction, we assume that $\tilde{\epsilon}_t$ has mean 1 (with positive probability above 1) and is independent of y_t; the assumption of $\mathbb{E}[\tilde{\epsilon}_t] = 1$ sharpens

[30] As explained in "Discussion of model assumptions" in Section III.A, in the NBER version of this paper (He and Li, 2022) we assume that OLG households are holding debt that matures with probability λ_d (instead of 1). This implies that for the system as a whole the debt rollover occurs with probability $1-m_L = 1-(1-\lambda_d)^L$, which is increasing in the credit chain length L. The decentralized market equilibrium minimizes the rollover risk of a given layer, but may generate too much systemic risk overall.

the theoretical result of an endogenous discount.

Furthermore, the noise introduced by the court in the bankruptcy process builds up. If the underlying asset has not matured in period t, then nature draws a new noise, $\tilde{\epsilon}_{t+1}$ (independent of $\tilde{\epsilon}_t$ and y_{t+1}) from the same distribution and it is multiplied to the cash-flow of the liquidated asset. These noises apply only to liquidation as a result of the resolution process; they do not affect the underlying asset payoffs so that future debt contracts that are newly issued by the ultimate borrower or the funds are not subject to this noise. Furthermore, all the noises are resolved in the chain restoration process.

Information structure and trading mechanism. At the beginning of period t, there is a unit measure of households in cohort-t who do not know the value of $\tilde{\epsilon}_t$. We call them uninformed households; they can be viewed as the households in the main model. In addition, there is an exogenous measure $\zeta \in (0, 1)$ of households in cohort-t who know the value of $\tilde{\epsilon}_t$ privately; we call them the informed households.[31] There are still intermediaries in this economy to form funds but they no longer participate in the liquidation and secondary market trading. For tractability reason, we assume that the value of $\tilde{\epsilon}_t$ becomes publicly observed at the beginning of period $t + 1$.[32]

When trading the liquidated asset in the secondary market, all buyers behave competitively based on their information and can bid at most one unit of the asset. Each household submits a bidding price such that he breaks even in expectation. The market clears at the price where the measure of households with bid larger than (or equal to) the price is equal to the total supply of one unit of liquidated asset. The asset is first allocated to bidders with the highest price, and bidders with the same bid get equal allocation randomly. Given that $\zeta < 1$, the market always clears at a price equal to the reservation value of uninformed households. However, uninformed households suffer from standard winner's curse given the presence of informed households a la Rock (1986).

Liquidation value, liquidation price, and endogenous discount factor. Denote by $A_l(y_t, L, \tilde{\epsilon}_t)$ the realized value of the liquidated asset in period t, conditional on $\tilde{\epsilon}_t$. Its corresponding liquidation price is denoted by $B_l(y_t, L)$; just as in the main model it captures the proceeds

[31]We interpret these informed households as distressed funds who only enter the market after a default.

[32]The same information structure then applies to cohort-$(t + 1)$ regarding $\tilde{\epsilon}_{t+1}$, if $\tilde{\epsilon}_{t+1}$ is drawn when the economy continues to period $t + 1$.

(excluding the legal cost) received by departing households. The endogenous price $B_l(y_t, L)$ could potentially depend on $\tilde{\epsilon}_t$ but we will show this is not the case in our setting.

Our goal is to show that $B_l(y_t, L)$ takes exactly the same form as given in (18)-(19), with $\alpha \in (0,1)$ being an endogenous discount factor pinned down by both the distribution of the court resolution uncertainty (G) and the fraction of informed households (ζ). To determine $B_l(y_t, L)$, we keep track of the value of the liquidated asset (debt issued by layer $l-1$) to the next cohort. The realized value of the liquidated asset conditional on $\tilde{\epsilon}_t$ is (recall $\tilde{F}_y = \min(F_y, y)$)

$$\begin{aligned}
A_l(y_t, L, \tilde{\epsilon}_t) =& \lambda_y \tilde{F}_y \tilde{\epsilon}_t + (1-\lambda_y)\Big\{(1-\lambda_d)^l \mathbb{E}[\underbrace{B_L(y_{t+1}, L, \tilde{\epsilon}_t)}_{\substack{\text{Case 1: Debt does not mature,}\\ \text{sold to cohort } t+1}} |\tilde{\epsilon}_t] \\
&+ \sum_{i=0}^{l-1} \lambda_d (1-\lambda_d)^i \Big[(1-H) \underbrace{\mathbb{E}[e\tilde{\epsilon}_t\tilde{\epsilon}_{t+1}|\tilde{\epsilon}_t]}_{\substack{=e\tilde{\epsilon}_t, \text{ Case 2: Debt}\\ \text{matures and successful rollover}}} \\
&+ H\mathbb{E}[\underbrace{B_L(y_{t+1}, L, \tilde{\epsilon}_t)}_{\substack{\text{Case 3: Debt matures}\\ \text{and rollover fails}}} - c(l-i)\tilde{\epsilon}_t\tilde{\epsilon}_{t+1} | y_{t+1} < F_y, \tilde{\epsilon}_t]\Big]\Big\}, \qquad (37)
\end{aligned}$$

where the first term captures the event of the underlying asset maturing. When it does not mature, $B_L(y_{t+1}, L, \tilde{\epsilon}_t)$ is the proceeds from reselling the period-t liquidated asset to cohort-$(t+1)$ when either i) the (liquidated) debt contract does not mature (Case 1) or ii) (liquidated) debt contract matures but default happens again in period $t+1$ (Case 3). We will explain $B_L(y_{t+1}, L, \tilde{\epsilon}_t)$ shortly; and the subscript of B has changed to L from l since the credit chain is restored the period after default. In Case 2, when the debt matures and the rollover is successful, the cohort-t households receive $e\tilde{\epsilon}_t\tilde{\epsilon}_{t+1}$ from the debt issuer, but in expectation it equals $e\tilde{\epsilon}_t$ as $\mathbb{E}[\tilde{\epsilon}_{t+1}|\tilde{\epsilon}_t] = 1$.

We solve for $B_L(y_{t+1}, L, \tilde{\epsilon}_t)$ first. At the beginning of period $t+1$ after the realization of y_{t+1}, if the debt contract does not mature or when it matures but default occurs again, the liquidated asset is sold to cohort $(t+1)$ with additional noise $\tilde{\epsilon}_{t+1}$. Because of chain restoration in $t+1$, the realized valuation of the liquidated asset to cohort-$(t+1)$ buyers is $V_L(y_{t+1}, L)\tilde{\epsilon}_t\tilde{\epsilon}_{t+1}$, with mean $V_L(y_{t+1}, L)\tilde{\epsilon}_t$ conditional on the information set of uninformed households (recall ϵ_t becomes publicly observable at period $t+1$). However, because $\tilde{\epsilon}_{t+1}$ is observed by the informed households only, the uninformed households who face a winner's curse shade their bids, leading the equilibrium price to be below the liquidated asset's expected value.

The break-even condition of the uninformed households implies that we can define an endogenous constant $\alpha \in (0,1)$ so that

$$B_L(y_{t+1}, L, \tilde{\epsilon}_t) = \alpha\tilde{\epsilon}_t V_L(y_{t+1}). \tag{38}$$

As shown in Appendix B.F, the endogenous discount factor α is the unique solution to the following equation:[33]

$$1 - \alpha = \zeta \int_{\alpha}^{\infty} (\tilde{\epsilon}_{t+1} - \alpha) dG(\tilde{\epsilon}_{t+1}). \tag{39}$$

The left hand side is the expected surplus earned by the buyers (including both informed and uninformed), which equals the information rent earned by the informed households who bids only when $\tilde{\epsilon}_{t+1} > \alpha$. Plugging (38) to (37), and factoring out $\tilde{\epsilon}_t$ in $A_l(y_t, L, \tilde{\epsilon}_t)$, we have

$$\begin{aligned} A_l(y_t, L, \tilde{\epsilon}_t) = \tilde{\epsilon}_t \Big\{ \lambda_y \tilde{F}_y + (1-\lambda_y) \Big[m_l \alpha \mathbb{E}[V_L] \\ + \sum_{i=0}^{l-1} (m_i - m_{i+1}) [(1-H)e + H(\alpha \mathbb{E}[V_L | y_{t+1} < F_y] - c(l-i))] \Big] \Big\}. \end{aligned}$$

Define $\hat{A}(y_t, L)$ as the term inside the curly bracket, so that $A_l(y_t, L, \tilde{\epsilon}_t) = \tilde{\epsilon}_t \hat{A}(y_t, L)$.

Once we have solved for $A_l(y_t, L, \tilde{\epsilon}_t)$ conditional on $\tilde{\epsilon}_t$, we move one period back to determine the equilibrium price $B_l(y_t, L)$ of the liquidated asset. When trading occurs in period t, only informed households know $\tilde{\epsilon}_t$. The problem resembles that in period $t+1$ and yields the same solution

$$B_l(y_t, L) = \alpha \mathbb{E}[A_l(y_t, L, \tilde{\epsilon}_t)] = \alpha \hat{A}(y_t, L), \tag{40}$$

with α is the same endogenous discount factor given in Eq. (39). Notice that $\hat{A}(y_t, L)$ is the same as the term inside the bracket of $B_l(y_t, L)$ in (18)-(19) in the main model; this is exactly what we aim to show.[34]

[33] The left hand side of (39) is decreasing in α, and positive (negative) when $\alpha = 0$ ($\alpha = 1$).

[34] In this setup, the liquidation proceed received by the creditors is $B_l(y_t, L) - c(L-l)\tilde{\epsilon}_t$, which in expec-

Long-term debt. We have ruled out the possibility of households holding long-term debt (or equity), motivated by households' preference for money-like securities for various reasons. In the NBER working paper version (w29632) of this paper (He and Li, 2022), we allow for direct issuance of long-term debt which is costly too due to the repeated trading discount (that households pay every period), and Appendix IA.C.IA.C.1 shows that the last layer of fund actually prefers to issue one-period debt when c is small. We also note that the resolution uncertainty in liquidation process modeled in this section does not work in the case of long-term debt (only secondary market trading but without bankruptcy). Nevertheless, one could introduce the noise ϵ_t as coming from the involvement of intermediaries. Potentially buyers have asymmetric information about this noise, which will generate a trading discount in each period similar to the case above.[35] This generates a similar trading discount in all the secondary market transactions.

V Conclusion

By highlighting a feature that we often see in the modern market-based financial system, we study a new dimension of the credit intermediation where one agent's liability is another agent's asset in the credit chain. We develop a new framework to illustrate the novel economic benefit of credit chains, characterize the equilibrium credit chain, and then study welfare implication of the equilibrium credit chain.

Different from existing research that only looks at systemic risk for each part of the financial system one at a time, our paper tries to provide a holistic view of the financial system when analyzing risks and welfare. This is important because regulations that impact one sector of the financial system will induce changes in the whole sector, affecting other institutions that interact with that sector. Without a model that includes the linkages among different institutions, researchers cannot properly assess the impact of any individual institution or policy. We hope future studies can use our model to answer these questions by further incorporating other empirically relevant features.

tation is equal to $B_l(y_t, L) - c(L - l)$.

[35] Alternatively, the asymmetric information among the buyers could be about the fundamental value y_t. While the value of y_t is publicly announced in the primary market due to road shows and prospectus required in the due diligence process, in the secondary market, only the informed buyers know the exact value of y_t.

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Appendix A: More Details on the Full Model

A Ultimate Borrower's Value Function

Recall that the ultimate borrower is labeled as layer 0. Like fund managers, his value is:

$$V_0(y,\pi_0;L) = \underbrace{\lambda_y(y-\tilde{F}_{y,0})}_{\text{underlying asset matures}} + (1-\lambda_y)\alpha\Bigg\{(1-\lambda_d)(1-\mathbf{1}_{L=1})\mathbb{E}\Big[\underbrace{V_0(y',\pi_0;L)}_{\text{Debt issued by layer-0 does not mature}}\Big] \tag{A.1}$$

$$+(\lambda_d(1-\mathbf{1}_{L=1})+\mathbf{1}_{L=1})\mathbb{E}\Big[\underbrace{\mathbf{1}^0_{rollover}(-e+\max_{\pi_0'}(P_0'+V_0(y',\pi_0';L)))}_{\text{Debt issued by layer-0 matures and rollover succeeds}} + \tag{A.2}$$

$$\underbrace{(1-\mathbf{1}^0_{rollover})[(1-\lambda_y)\alpha(-P_{-1}'+\max_{\pi_0'}(P_0'+V_0(y',\pi_0';L)))]}_{\text{Debt issued by layer-0 matures and rollover fails}}\Big]\Bigg\}. \tag{A.3}$$

The term $\mathbb{E}\big[V_0(y',\pi_0;L)\big]$ in the second part of (A.1) captures the continuation value when debt does not mature;[36] (A.2) captures the value if debt matures and rollover is successful.

The main difference between the ultimate borrower's payoff and intermediary funds' payoffs is reflected in the last term in (A.3), when debt matures but rollover fails. Because of the ultimate borrower's unique human capital in managing the underlying asset, he is re-hired back after the bankruptcy if the chain is restored in the next period. Essentially, the expert in the distress fund sells the underlying asset back to the ultimate borrower at price P_{-1}' (one can view the distress fund as layer -1). The ultimate borrower takes price P_{-1}' as given, chooses a new contract π_0' (and hence initializes a new chain) to maximize the sum of proceeds from issuing debt (P_0') and his continuation value (V_0).

We allow the ultimate borrower to be rehired for keeping the contract stationary over time. Since the ultimate borrower has no savings when he is rehired, the price charged by the distress fund P_{-1}' cannot be larger than the debt proceeds that the ultimate borrower can raise P_0'. We assume the distress fund has all the bargaining power so that $P_{-1}'=P_0'$.[37]

[36] In the special one-layer case where the ultimate borrower is directly issuing to households one-period debt (which always matures), this term equals zero since $1-\mathbf{1}_{L=1}=0$.

[37] The impatient borrower with discount rate $\alpha<1$ indeed has no savings, as he would prefer consume his previous debt proceeds immediately. Also recall that we have always assumed that the distress fund has all the bargaining power, so that the liquidation value equals the fair value of the debt when other layers are broken.

B Definition of the Decentralized Equilibrium

Define $\hat{\Pi}$ as the set of feasible contracts that are renegotiation proof and subject to the resource constraint (imposed by limited endowment from OLG households):

$$\hat{\Pi} \equiv \{\pi \in \Pi : V_L(\{F_{y,s}\}_{s=t}^{T}, L) \leq e \quad \text{for} \quad \forall t\}. \tag{A.4}$$

Definition 1 *The equilibrium credit chain is a set of contracts $\{\pi_{l,t}\}_{0 \leq l \leq L-1}$ and credit chain length L^* such that*

1. *For $1 \leq l \leq L-1$, when layer-l's liability matures in period t,*[38]

$$\pi_{l,t} = \underset{\pi \in \hat{\Pi}}{\arg\max} \quad \mathbf{1}^{l}_{rollover}(P_l(y_t, \pi; \pi_{l-1,t}, L^*) + V_l(y_t, \pi; \pi_{l-1,t}, L^*)), \tag{A.5}$$

$$s.t. \quad F_{y,t}(l) \leq F_{y,t}(l-1) \tag{A.6}$$

When layer-0's liability matures,

$$\pi_0 = \underset{\pi \in \hat{\Pi}}{\arg\max} \quad \mathbf{1}^{0}_{rollover}(P_0(y_t, \pi; L^*) + V_0(y_t, \pi; L^*)). \tag{A.7}$$

2. *The equilibrium L^* is such that the last layer of fund manager ($L^* - 1$) prefers to borrow directly from households than to borrow via other fund managers:*

$$P_{L^*-1}(L^*) + V_{L^*-1}(L^*) \geq P_{L^*-1}(L^* + l) + V_{L^*-1}(L^* + l) \quad \textit{for} \quad l \geq 1. \tag{A.8}$$

Furthermore, for all other funds $0 < l < L^* - 1$,

$$P_l(L^*) + V_l(L^*) \geq P_l(l+1) + V_l(l+1). \tag{A.9}$$

In other words, the funds in intermediary layers prefer to borrow via other funds than to borrow from households.

[38] When $t = 0$, $\mathbf{1}^{l}_{rollover} = 1$ for all l.

3. Due to perfect competition,

$$P_l - P_{l-1} + V_l = 0. \tag{A.10}$$

C Definition of Equilibrium in the Planner's Problem

As before, $\hat{\Pi}$ denotes the set of feasible contracts that are renegotiation proof and subject to the resource constraint.

Definition 2 *The planner's solution* L^{**} *solves* $\max_L W(\{\pi_{l,t}(L)\}_{0\leq l\leq L-1}, L)$ *such that*

1. For $1 \leq l \leq L-1$*, when layer-*l*'s liability matures,*

$$\pi_l(L) = \underset{\pi\in\hat{\Pi}}{\arg\max} \quad \mathbf{1}^l_{rollover}(P_l(y,\pi;\pi_{l-1},L) + V_l(y,\pi;\pi_{l-1},L)), \tag{A.11}$$

$$s.t. \quad F_y(l) \leq F_y(l-1). \tag{A.12}$$

When layer-0*'s liability matures,*

$$\pi_0(L) = \underset{\pi\in\hat{\Pi}}{\arg\max} \quad \mathbf{1}^0_{rollover}(P_0(y,\pi;L) + V_0(y,\pi;L)). \tag{A.13}$$

2. Due to perfect competition,

$$P_l - P_{l-1} + V_l = 0. \tag{A.14}$$

Appendix B: Proofs and Derivations

A Proof for Proposition 1

If the total chain length is L, the liquidation value if the chain breaks at l is,

$$\begin{aligned} B_l(y,L) =& \alpha\Big\{\lambda_y \min(y,F_y) + (1-\lambda_y)\Big[(1-\lambda_d)^l \mathbb{E}[\alpha V_L(y',L)] \\ &+ \sum_{i=0}^{l-1} \lambda_d(1-\lambda_d)^i (e(1-H) + H(\alpha\mathbb{E}[V_L(y')|y'<F_y] - c(l-i)))\Big]\Big\} \end{aligned}$$

Define

$$b_l(L) \equiv (1-\lambda_y)\Big[(1-\lambda_d)^l \mathbb{E}[\alpha V_L(y', L)] + \sum_{i=0}^{l-1} \lambda_d (1-\lambda_d)^i (e(1-H) + H(\alpha \mathbb{E}[V_L(y')|y' < F_y] - c(l-i)))\Big] \tag{B.1}$$

As will be shown in Appendix B.C, when $L < L^*$, $\frac{dv_L}{dL} > 0$, where $\frac{dv_L(L)}{dL} \equiv v_{L+1}(L+1) - v_L(L)$. Furthermore, define $\frac{\partial b_l(L)}{\partial L} = b_l(L+1) - b_l(L)$. Using Eq. (B.1),

$$\frac{\partial b_l(L)}{\partial L} = (1-\lambda_y)\Big[(1-\lambda_d)^l \alpha \frac{dv_L(L)}{dL} + \sum_{i=0}^{l-1} \lambda_d m_i \alpha \frac{\partial \mathbb{E}[V_L(y')|y' < F_y]}{\partial L}\Big]$$

where $\frac{\partial \mathbb{E}[V_L(y')|y'<F_y]}{\partial L} = \frac{\partial v_L}{\partial L}$. Hence $\frac{\partial b_l(L)}{\partial L} > 0$ when $\frac{dv_L}{dL} > 0$.

Next, we can write $b_{L-j+1}(L+1) - b_{L-j}(L)$ as

$$b_{L-j+1}(L+1) - b_{L-j}(L) = b_{L-j+1}(L+1) - b_{L-j+1}(L) + b_{L-j+1}(L) - b_{L-j}(L)$$

Since $b_l(L)$ is increasing in L when $L \leq L^*$, $b_{L-j+1}(L+1) - b_{L-j+1}(L) \geq 0$. Furthermore,

$$b_{l+1}(L) - b_l(L) = (1-\lambda_y)[\lambda_d(1-\lambda_d)^l e(1-H)(1-\alpha) - cH(1-m_{1+l})] \tag{B.2}$$

When $L \leq L^*$, the first order condition is positive. As shown in C.1, this implies

$$\alpha(1-\lambda_y)\lambda_d(1-\lambda_d)^L m_{L-1} e(1-H)(1-\alpha) \geq [1-(1-\lambda_d)^L + \alpha(1-\lambda_y)(1-\lambda_d)^L(1-(1-\lambda_d)^L)H]c$$

Furthermore

$$[1-(1-\lambda_d)^L + \alpha(1-\lambda_y)(1-\lambda_d)^L(1-(1-\lambda_d)^L)H]c > \alpha(1-\lambda_y)(1-\lambda_d)^L(1-m_{1+l})Hc$$
$$\alpha(1-\lambda_y)\lambda_d(1-\lambda_d)^L m_{L-1} e(1-H)(1-\alpha) \leq \alpha(1-\lambda_y)\lambda_d(1-\lambda_d)^L m_l e(1-H)(1-\alpha)$$

Hence $\lambda_d(1-\lambda_d)^l e(1-H)(1-\alpha) - cH(1-m_{1+l}) > 0$. Together, we have $b_{L-j+1}(L+1) - b_{L-j}(L) > 0$.

B Proof for Lemma 1

We want to show that $F_{y,t}(l) = \min(F_{y,t}(l-1), \hat{y}_{l,t})$, given the face value of the debt equals to e. At time t, for a given sequence of future payments $\{F_{y,t+j}(l)\}_{j=1}^{\infty}$, there exists $\hat{y}_{l,t}$ such that

$$P_l = V_L(\{\hat{y}_{l,t}, \{F_{y,t+j}(l)\}_{j=1}^{\infty}\}, L) = e$$

$\hat{y}_{l,t}$ is the run threshold. Because y_t is i.i.d. across periods, $\hat{y}_{l,t}$ does not depend on the history of y. If layer-l's debt matures in period t, then it must be the case that

$$F_{y,t}(l) = \min\{\hat{y}_{l,t}, F_{y,t}(l-1)\}$$

Otherwise, layer-l would not be able to rollover. Since layer-$(L-1)$ issues one period debt, this is always the case, i.e.

$$F_{y,t}(L-1) = \min\{\hat{y}_{L-1,t}, F_{y,t}(L-2)\} \qquad \forall t$$

For layer-$l < L-1$, consider the case when debt is issued in period s, where $s < t$. Since the fund manager can always renegotiate with the households, it must be the case that

$$F_{y,t}(l) \leq \min\{\hat{y}_{l,t}, F_{y,t}(l-1)\}$$

If $F_{y,t}(L-1) < \min(\hat{y}_{L-1,t}, F_{y,t}(L-2))$, then by setting $\tilde{F}_{y,t}(L-1) = \min(\hat{y}_{L-1,t}, F_{y,t}(L-2))$ and setting $\hat{F}_{y,t+1}(L-1) = F_{y,t+1}(L-1) - \alpha(\min(\hat{y}_t(L-1), F_{y,t}(L-2)) - F_{y,t}(L-1))$, both the borrowing fund and the lending fund remain indifferent. So without loss of generality, we can assume

$$F_{y,t}(l) = \min(\hat{y}_{l,t}, F_{y,t}(l-1))$$

Next, we proceed to show $\hat{y}_{l,t}$ must be a constant.

For layer-0, since y_t is i.i.d., $\hat{y}_{0,t} = \hat{y}_0$ is a constant over time and the distribution of $F_{y,t}(0)$ is stationary. Suppose for any layer-l where $1 \leq l \leq L-1$, $\hat{y}_{l-1,t} = \hat{y}_{l-1}$ is a constant, then $F_{y,t}(l-1)$ is stationary. If $\hat{y}_{l,t} < \hat{y}_{l,t+1}$, then it must exist j, such that

$$\begin{aligned}
&\mathbb{E}_t[F_{y,t+j}(l)] > \mathbb{E}_{t+1}[F_{y,t+j+1}(l)] \\
\Rightarrow &\mathbb{E}_t[\min(\hat{y}_{l,t+j}, F_{y,t+j}(l-1))] > \mathbb{E}_{t+1}[\min(\hat{y}_{l,t+j+1}, F_{y,t+j+1}(l-1))] \\
\Rightarrow &\hat{y}_{l,t+j} > \hat{y}_{l,t+j+1} \qquad \text{(B.3)}
\end{aligned}$$

However, at time $t+j$, the problem faced by the fund is exactly the same as at time t because of stationarity: at both point t and $t+j$, the manager is trying to find the best subsequent of payment such the debt is worth e to households. The two problems are identical. Hence it must be the case that $\hat{y}_{l,t+j} < \hat{y}_{l,t+j+1}$. This contradicts Eq. (B.3). So $\hat{y}_{l,t} = \hat{y}_l$, i.e. it must be a constant over time. By induction, this is true for all $0 \leq l \leq L-1$.

We have now established stationarity. We move on to show $F_y(l) = F_y$, i.e. layer-independence. By the definition of $F_y(l)$,

$$e = P_{l+1} + V_{l+1}(F_y(l+1); F_y(l)) \qquad \text{(B.4)}$$

In perfect competition, $P_{l+1} = e$ and $V_{l+1} = 0$. From the HJB of V_{l+1} (Eq. (13)-(16)), we can see that it is proportional to $F_y(l) - F_y(l+1)$. Hence for $V_{l+1} = 0$, it must be the case that $F_y(l) = F_y(l+1) = F_y$.

C Proof for Proposition 2

Existence and Uniqueness of F_y. A given cohort of household's strategy (run threshold) is $F_y = \frac{e - v_L(F_y')}{\lambda_y}$, where F_y' is other cohort's strategy. A symmetric equilibrium is where $F_y = F_y'$. Moreover, $\frac{d\frac{e-v_L(F_y')}{\lambda_y}}{dF_y'} \leq 1$ at the equilibrium point.

Given $\frac{e-v_L(0)}{\lambda_y} > 0$ and $\lim_{x\to\infty} \frac{e-v_L(x)}{\lambda_y} - x < 0$, there exists at least one intersection of $y = \frac{e-v_L(x)}{\lambda_y}$ with $y = x$ from above. So equilibrium exists.

We next solve for F_y. We write the equations defining $b_l(L)$ and v_L in matrix form

$$\Psi \begin{bmatrix} b_0(L) \\ b_1(L) \\ ... \\ b_{L-1}(L) \\ v_L(L) \end{bmatrix} = \eta \tag{B.5}$$

where

$$\Psi = \begin{bmatrix} 1 & 0 & 0 & ... & 0 & -(1-\lambda_y)\alpha[m_0 + H(F_y)(1-m_0)] \\ 0 & 1 & 0 & ... & 0 & -(1-\lambda_y)\alpha[m_1 + H(F_y)(1-m_1)] \\ ... \\ 0 & 0 & 0 & ... & 1 & -(1-\lambda_y)\alpha[m_{L-1} + H(F_y)(1-m_{L-1})] \\ 0 & 0 & 0 & ... & 0 & 1 \end{bmatrix}$$
$$- (1-\lambda_y)H(F_y)\alpha\lambda_d \begin{bmatrix} 0 & 0 & ... & 0 & 0 & \\ 0 & 0 & ... & 0 & 0 & \\ ... \\ m_0 & m_1 & m_2 & ... & m_{L-2} & \frac{m_{L-1}}{\lambda_d} & 0 \end{bmatrix}$$

and

$$\eta = (1-\lambda_y)[\alpha\lambda_y H(F_y)\underline{X}(F_y) + (1-H(F_y))e] \begin{bmatrix} 0 \\ 1 \\ ... \\ 1 \\ 1 \end{bmatrix} + (1-\lambda_y) \begin{bmatrix} \alpha\lambda_y(H\underline{X}(F_y) + F_y(1-H(F_y))) \\ cH\frac{1-\lambda_d}{\lambda_d} \\ ... \\ cH\frac{1-\lambda_d}{\lambda_d} \\ 0 \end{bmatrix}$$
$$+ (1-\lambda_y)[\alpha\lambda_y F_y(1-H(F_y)) - e(1-H(F_y)) + cH(F_y)(1-\frac{1}{\lambda_d})] \begin{bmatrix} 0 \\ m_1 \\ ... \\ m_{L-1} \\ 0 \end{bmatrix} - (1-\lambda_y)cH(F_y) \begin{bmatrix} 0 \\ 1 \\ ... \\ (L-1) \\ L - \frac{1-\lambda_d}{\lambda_d} - (1-\frac{1}{\lambda_d})m_{L-1} \end{bmatrix}$$

where

$$\underline{X}(F_y) \equiv \mathbb{E}[y \mid y < F_y]$$

Next, to argue uniqueness, we just need to show that $\frac{d\frac{e-v_L(F_y)}{\lambda_y}}{dF_y} \leq 1 \Leftrightarrow \lambda_y + \frac{dv_L(F_y)}{dF_y} \geq 0$. We can express

$$\begin{aligned}
&v_L = (0,0,...,0,1)\Psi^{-1}\eta \\
&\frac{dv_L}{dF_y} = -(0,0,...,0,1)\Psi^{-1}\frac{\partial\Psi}{\partial F_y}\Psi^{-1}\eta + (0,0,...,0,1)\Psi^{-1}\frac{\partial\eta}{\partial F_y}
\end{aligned}$$

We need

$$\lambda_y - (0,0,...,0,1)\Psi^{-1}\frac{\partial\Psi}{\partial F_y}\Psi^{-1}\eta + (0,0,...,0,1)\Psi^{-1}\frac{\partial\eta}{\partial F_y} \geq 0$$

which is satisfied by Assumption 3.

C.1 Characterizing Equilibrium Chain Length

In equilibrium, $F_y(L)$ is determined by $e = \lambda_y F_y + v_L(L)$. In the following proof, unless specified otherwise, $F_y = F_y(L)$ and $H = H(F_y)$. The expression of v_L is given by,

$$\begin{aligned}
v_L =& (1-\lambda_y)(1-H)e + (1-\lambda_y)H \times \Big[\sum_{l=0}^{L-2} m_l\lambda_d(\mathbb{E}[B_l(y,L)|y<F_y] - c(L-l)) \\
&+ m_{L-1}(\mathbb{E}[B_{L-1}(y,L)|y<F_y] - c)\Big]
\end{aligned} \tag{B.6}$$

In equilibrium, $L^* = \arg\max_L v_L(L)$. Taking difference with respect to L ($dv_L(L) = v_L(L) - v_{L-1}(L-1)$ and $\frac{\partial b_l(L)}{\partial L} = b_l(L+1) - b_l(L)$),

$$\frac{dv_L(L)}{dL} = (1-\lambda_y)H\Big[\sum_{l=0}^{L-1} m_l\lambda_d\alpha\frac{\partial b_l(L)}{\partial L} - (1-(1-\lambda_d)^L)c + (1-\lambda_d)^L\alpha(b_L(L+1) - b_{L-1}(L))\Big] \tag{B.7}$$

To examine $\frac{\partial b_l(L)}{\partial L}$,

$$\frac{\partial b_l(L)}{\partial L} = (1-\lambda_y)\Big[(1-\lambda_d)^l\alpha\frac{dv_L(L)}{dL} + \sum_{i=0}^{l-1}\lambda_d m_i\alpha\frac{\partial\mathbb{E}[V_L(y,L)|y<F_y]}{\partial L}\Big]$$

At $\frac{dv_L(L)}{dL}=0$, $\frac{\partial b_l(L)}{\partial L}=0$ for all $1 \le l \le L-1$. Furthermore,

$$\begin{aligned}
b_L(L+1) - b_{L-1}(L) = (1-\lambda_y)\Big[&(1-\lambda_d)^L\alpha(v_{L+1}-v_L) + \lambda_d(1-\lambda_d)^{L-1}e(1-H)(1-\alpha) \\
&+ (1-(1-\lambda_d)^L)H\alpha(v_{L+1}-v_L) - (1-(1-\lambda_d)^L)Hc\Big]
\end{aligned} \tag{B.8}$$

Plug $b_L(L+1) - b_{L-1}(L)$ back into Eq. (B.7) and set $\frac{dv_L}{dL}=0$ at the optimal point. We get,

$$-(1-(1-\lambda_d)^L)c + (1-\lambda_d)^L\alpha(1-\lambda_y)\Big\{\lambda_d m_{L-1}e(1-H)(1-\alpha) - (1-(1-\lambda_d)^L)Hc\Big\} = 0$$

Rearranging terms, we get

$$FOC_{L,prv} = \alpha(1-\lambda_y)\lambda_d(1-\lambda_d)^L m_{L-1}e(1-H)(1-\alpha) - [1-(1-\lambda_d)^L + \alpha(1-\lambda_y)(1-\lambda_d)^L(1-(1-\lambda_d)^L)H]c \tag{B.9}$$

To show the second order condition is satisfied, take derivative with respect to L in Eq. (B.9), we get

$$\begin{aligned}
&2log(1-\lambda_d)\alpha(1-\lambda_y)\lambda_d(1-\lambda_d)^{2L-1}e(1-H)(1-\alpha) \\
&-[-log(1-\lambda_d)(1-\lambda_d)^L + log(1-\lambda_d)\alpha(1-\lambda_y)(1-\lambda_d)^L(1-(1-\lambda_d)^L)H - log(1-\lambda_d)\alpha(1-\lambda_y)(1-\lambda_d)^{2L}H]c \\
=&log(1-\lambda_d)(1-\lambda_d)^L\Big\{2\alpha(1-\lambda_y)\lambda_d(1-\lambda_d)^{L-1}e(1-H)(1-\alpha) - [-1+\alpha(1-\lambda_y)(1-2(1-\lambda_d)^L)H]c\Big\}
\end{aligned}$$

Use the first order condition to substitute out $\alpha(1-\lambda_y)\lambda_d(1-\lambda_d)^{L-1}e(1-H)(1-\alpha)$, the above expression can be rewritten as

$$\begin{aligned}
&c \times log(1-\lambda_d)\Big\{2[1-(1-\lambda_d)^L + \alpha(1-\lambda_y)(1-\lambda_d)^L(1-(1-\lambda_d)^L)H] - (1-\lambda_d)^L[-1+\alpha(1-\lambda_y)(1-2(1-\lambda_d)^L)H]\Big\} \\
&= c \times log(1-\lambda_d)\Big\{2-(1-\lambda_d)^L + \alpha(1-\lambda_y)(1-\lambda_d)^L H\Big\} < 0
\end{aligned}$$

Since $log(1-\lambda_d) < 0$, the second order condition is satisfied.

C.2 Special Case: $c = 0$

We derive the marginal benefit of a longer chain in the special case when $c=0$. Using the expression for v_L in Eq. (B.6), we can compare the debt value when the chain length is L and $L+1$,

$$
\begin{aligned}
&V_{L+1} - V_L = v_{L+1} - v_L \\
=&(1-\lambda_y)H\Big[\sum_{l=0}^{L-1} m_l\lambda_d \mathbb{E}[B_l(y, L+1)|y < F_y] + (1-\lambda_d)^L \mathbb{E}[B_L(y, L+1)|y < F_y] \\
&- \sum_{l=0}^{L-2} m_l\lambda_d \mathbb{E}[B_l(y, L)|y < F_y] - m_{L-1}\mathbb{E}[B_{L-1}(y, L)|y < F_y]\Big] \\
=&(1-\lambda_y)H\Big[\sum_{l=0}^{L-1} m_l\lambda_d (B_l(L+1) - B_l(L)) + (1-\lambda_d)^L (B_L(L+1) - B_{L-1}(L))\Big] \quad \text{(B.10)}
\end{aligned}
$$

It is clear that the difference in debt value purely comes from the differences in liquidation values. Furthermore,First of all,

$$
\begin{aligned}
B_l(L+1) - B_l(L) &= (1-\lambda_y)\alpha[m_l\alpha(v_{L+1} - v_L) + \alpha H \sum_{i=0}^{l-1} \lambda_d m_i (v_{L+1} - v_L)] \\
&= (1-\lambda_y)\alpha^2[m_l + H(1-m_l)](v_{L+1} - v_L)
\end{aligned}
$$

Define $K_L = (1-\lambda_y)\alpha^2 \sum_{l=0}^{L-1} m_l\lambda_d[m_l + H(1-m_l)]$. It is straightforward to

$$
v_{L+1}(L+1) - v_L(L) = \frac{(1-\lambda_y)H(1-\lambda_d)^L(B_L(L+1) - B_{L-1}(L))}{1-(1-\lambda_y)HK_L}. \quad \text{(B.11)}
$$

It is straightforward to show that $K_L < 1$, and $1-(1-\lambda_y)HK_L > 0$. Hence we have (32).

D Proof for Proposition 3

Comparative statics with respect to c. We first consider the comparative statics with respect to the per layer bankruptcy cost c,

$$
\frac{\partial FOC_{L,prv}}{\partial c} = \frac{\partial FOC_{L,prv}}{\partial F_y}\frac{\partial F_y}{\partial c} - [1-(1-\lambda_d)^L + \alpha(1-\lambda_y)(1-\lambda_d)^L(1-(1-\lambda_d)^L)H]
$$

where $\frac{\partial F_y}{\partial c} = -\frac{\frac{\partial v_L(L)}{\partial c}}{\lambda_y + \frac{\partial v_L(L)}{\partial F_y}}$. Since $\frac{\partial v_L(L)}{\partial c} < 0$, $\frac{\partial F_y}{\partial c} > 0$. Furthermore,

$$\frac{\partial FOC_{L,prv}}{\partial F_y} = \frac{\partial FOC_{L,prv}}{\partial H} h(F_y) < 0$$

Hence $\frac{\partial FOC_{L,prv}}{\partial c} < 0$. By implicit function theorem, we have $\frac{\partial L^*}{\partial c} < 0$.

When $c = 0$, the first-order condition for L becomes

$$\alpha(1-\lambda_y)\lambda_d(1-\lambda_d)^L m_{L-1} e(1-H)(1-\alpha) > 0$$

for any positive L. Hence the equilibrium chain length is infinity.

E *Proof for Proposition 4*

We show that the social first order condition is the same as the private first order condition with respect to L. The social planner's problem is formally defined in Appendix A.C. Given w_L is defined in $W(F_y(L), L) = e + w_L + \lambda_y y$, we have

$$\begin{aligned} w_L = (1-\lambda_y)\Big\{ &(1-H(F_y))((1-\alpha)e + \alpha(\lambda_y \mathbb{E}[y|y \geq F_y] + w_L)) \\ &+ \lambda_d H(F_y) \sum_{l=0}^{L-2} m_l \left[\alpha(\lambda_y \mathbb{E}[y|y < F_y] + w_L) - \alpha(v_L(L) - b_l) - c(L-l)\right] \\ &+ m_{L-1} H(F_y) \left[\alpha(\lambda_y \mathbb{E}[y|y < F_y] + w_L) - \alpha(v_L(L) - b_{L-1}) - c\right] \Big\} \end{aligned}$$

Consider $w_{L+1} - w_L$ for a given F_y,

$$\begin{aligned} w_{L+1} - w_L = (1-\lambda_y)\Big\{ &(1-H(F_y))(w_{L+1} - w_L) - (1-(1-\lambda_d)^L) H(F_y) c \\ &+ \lambda_d H(F_y) \sum_{l=0}^{L-1} m_l \Big[\alpha(w_{L+1} - w_L) - \alpha(v_{L+1} - v_L) + \alpha \frac{\partial b_l(L)}{\partial L}\Big] \\ &+ (1-\lambda_d)^L H(F_y) \left[\alpha(w_{L+1} - w_L) - \alpha(v_{L+1} - v_L) + \alpha(b_L(L+1) - b_{L-1}(L))\right] \Big\} \end{aligned}$$

Moving $w_{L+1} - w_L$ to the left hand side, we get

$$w_{L+1} - w_L \propto (1-\lambda_y)H(F_y)\Big\{ \underbrace{\lambda_d \sum_{l=0}^{L-1} m_l \alpha \frac{\partial b_l(L)}{\partial L} - (1-(1-\lambda_d)^L)c + (1-\lambda_d)^L \alpha(b_L(L+1) - b_{L-1}(L))}_{\propto (v_{L+1}-v_L)}$$
$$- \alpha(v_{L+1} - v_L)\Big\}$$

Finally, we have

$$v_{L+1} - v_L \propto \lambda_d \sum_{l=0}^{L-1} m_l \alpha \frac{\partial b_l(L)}{\partial L} - (1-(1-\lambda_d)^L)c + (1-\lambda_d)^L \alpha(b_L(L+1) - b_{L-1}(L))$$

Hence at the decentralized equilibrium where $v_{L+1} - v_L = 0$, we have $w_{L+1} - w_L = 0$. In other words, the decentralised condition coincides with the social planner's first order condition with respect to L.

F Derivations for the Micro-Foundation in Section IV.C

Suppose default happens in period t, we first derive the re-sell price for the liquidated asset in period $t+1$, $B_L(y_{t+1}, L, \tilde{\epsilon}_t)$. After the asset is sold to cohort-$(t+1)$ by cohort-t, the credit chain is restored and $\tilde{\epsilon}_{t+1}$ is resolved. Hence the valuation of the liquidated asset from the perspective of buyers, who are cohort-$(t+1)$'s households, is $V_L(y_{t+1}, L)\tilde{\epsilon}_t\tilde{\epsilon}_{t+1}$, where $\tilde{\epsilon}_t$ is observed by all households while $\tilde{\epsilon}_{t+1}$ is observed by the informed households only. The equilibrium price $B_L(y_{t+1}, L, \tilde{\epsilon}_t)$ solves the break-even condition of uninformed households:

$$\begin{aligned}&(1-\zeta)\mathbb{E}[\tilde{\epsilon}_t\tilde{\epsilon}_{t+1}V_L - B_L | \tilde{\epsilon}_t\tilde{\epsilon}_{t+1}V_L > B_L, \tilde{\epsilon}_t] \Pr(\tilde{\epsilon}_t\tilde{\epsilon}_{t+1}V_L > B_L | \tilde{\epsilon}_t) \\ &+ \mathbb{E}[\tilde{\epsilon}_t\tilde{\epsilon}_{t+1}V_L - B_L | \tilde{\epsilon}_t\tilde{\epsilon}_{t+1}V_L \le B_L, \tilde{\epsilon}_t] \Pr(\tilde{\epsilon}_t\tilde{\epsilon}_{t+1}V_L \le B_L | \tilde{\epsilon}_t) = 0,\end{aligned} \tag{B.12}$$

where the expectation is conditional on $\tilde{\epsilon}_t$ (so taken over the potential realizations of $\tilde{\epsilon}_{t+1}$). Simplifying Eq. (B.12) we get,

$$\tilde{\epsilon}_t V_L - B_L - \zeta \mathbb{E}[\tilde{\epsilon}_t \tilde{\epsilon}_{t+1} V_L - B_L | \tilde{\epsilon}_t \tilde{\epsilon}_{t+1} V_L > B_L, \tilde{\epsilon}_t] \Pr(\tilde{\epsilon}_t \tilde{\epsilon}_{t+1} V_L > B_L | \tilde{\epsilon}_t) = 0$$
$$1 - \frac{B_L}{\tilde{\epsilon}_t V_L} - \zeta \int_{\tilde{\epsilon}_{t+1} > \frac{B_L}{\tilde{\epsilon}_t V_L}} \left(\tilde{\epsilon}_{t+1} - \frac{B_L}{\tilde{\epsilon}_t V_L} \right) dG(\tilde{\epsilon}_{t+1}) = 0$$

Define $\alpha \equiv \frac{B_L}{\tilde{\epsilon}_t V_L}$. α solves the following equation

$$f(x) = 1 - x - \zeta \int_x (\tilde{\epsilon}_{t+1} - x) dG(\tilde{\epsilon}_{t+1}) = 0. \tag{B.13}$$

Furthermore

$$f(0) > 0 \qquad f(1) < 0$$
$$f'(x) = -1 + \zeta \int_x dG(\tilde{\epsilon}_{t+1}) < 0$$

Hence $\alpha \in (0, 1)$. The equilibrium price B_L is given by

$$B_L(y_{t+1}, L, \tilde{\epsilon}_t) = \alpha \tilde{\epsilon}_t V_L(y_{t+1}).$$

The determination of $B_l(y_t, L)$ follows a similar approach.

Internet Appendix for "Intermediation via Credit Chains"

Appendix IA.A: Deterministic Maturity Structure

In this section, we consider a case where the underlying asset and debt contracts have deterministic maturity and show that the mechanism works exactly the same. Suppose the underlying asset matures in T periods. To simplify the analysis, we assume the underlying asset only produces cash-flows $\tilde{y} \geq 0$ only at the end of period $t = T$. Good news could arrive with probability $p \in (0, 1)$ in period $t \in \{1, 2...T - 1\}$. If good news arrives in any period, then $\tilde{y} = 1$; otherwise, $\tilde{y} = 0$. The arrival of good news is independent across periods.

The underlying asset is owned by an impatient borrower (he); the borrower and the underlying asset are used interchangeably in this section. He leaves the economy at the end of period 0, implying that he maximizes the payment of cohort-0 households by pledging out his entire cash-flows to households. Cohort t households are born at the beginning of period t, endowed with 1 unit of consumption goods, and have access to a storage technology with zero net return. This cohort can consume $c_t^t > 0$ or invest in financial markets (storage technology or securities issued by the ultimate borrower or funds, as explained shortly), and leave the economy in period $t + 1$ after consuming $c_{t+1}^t > 0$, with a utility of $c_t^t + c_{t+1}^t$.

The debt contract has face value 1 with no coupon payment. When debt contract matures, the borrower needs to rollover the debt by raising money from the new cohort of households. When rollover fails, then the liquidation and secondary market discount is α, as in the main model. In this setting, the rollover is successful only if good news has arrived.

We show that the structure that generates the highest debt value in period 0 is the following: there are $T - 2$ layers of intermediaries. We label the ultimate borrower as layer 0, and the households as layer $T - 1$. The ultimate borrower issues debt with maturity T. Funds in layer l $(0 < l \leq T - 2)$ hold debt with maturity $T - l$. They first issue debt with maturity $T - l - 1$. When such debt matures, they try to rollover and issue debt with maturity that has only one period. In other words, they only bear rollover risk for one period (the last period) before their asset side matures. Once the fund's asset side matures, it leaves the economy, and the existing households hold debt directly issued by the layer above. The structure is illustrated in Figure IA.A.1, where the arrows indicate cash-flow exchanges.

Figure IA.A.1: Illustration of the Layer-Structure in the General Case

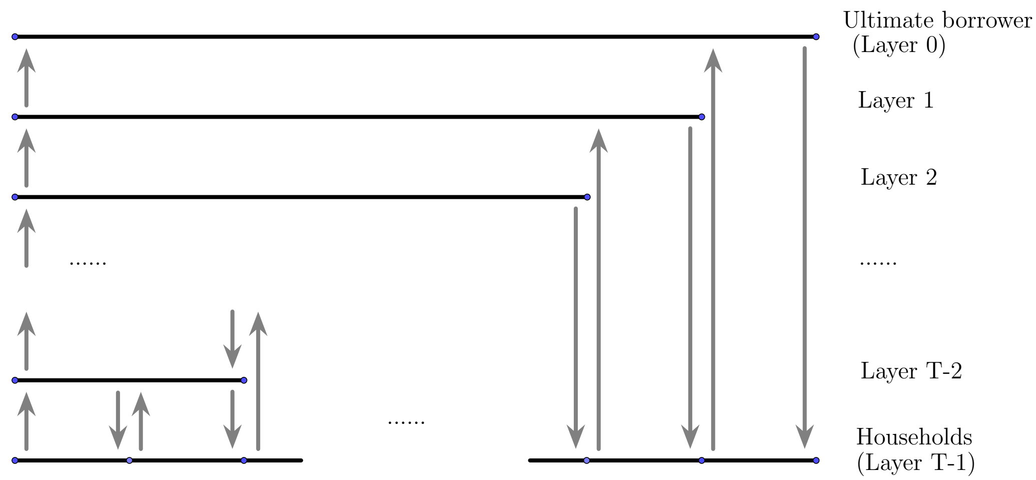
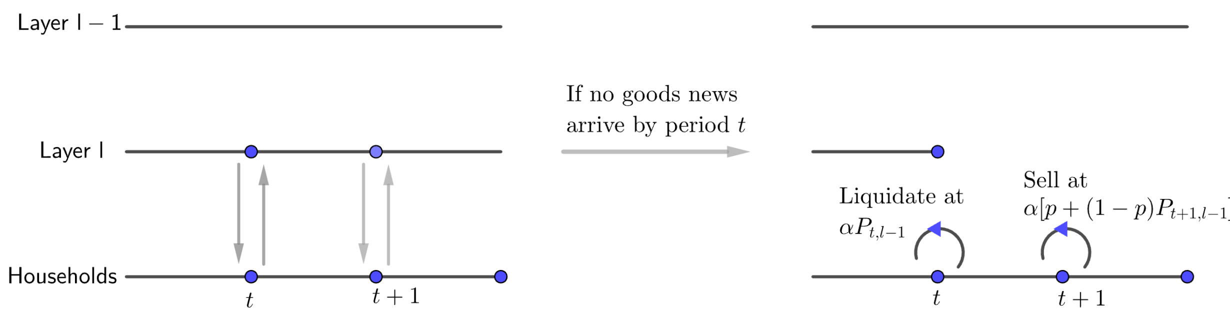


Figure IA.A.2: Alternative Substructure

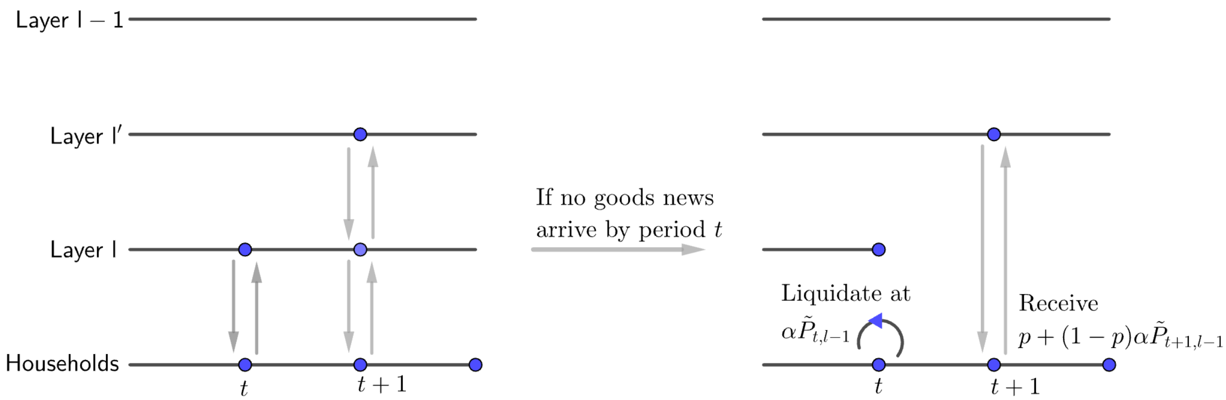


We assume the intermediary funds leave the economy as their asset side matures. Notice that from the households perspective, they are always holding one-period debt, same as in the main model.

Consider an alternative structure in which layer l bears more than one period maturity mismatch risk, i.e. layer l's asset matures in T_l period, but it's liability matures in $T_l - \Delta$, where $\Delta \geq 2$. See the illustration in Figure IA.A.2. We show that debt value can be improved by adding a layer (moving to the structure in Figure IA.A.3) and shortening layer l's maturity mismatch.

Suppose layer l's liability matures in period t. We focus the analysis on the amount of money that households receive at time t. Given the iterative structure, the initial debt value

Figure IA.A.3: Improvement on the Alternative Substructure



P_0 is increasing in the expected payment to the households in any given period. If good news has arrived by time t, then layer l can rollover its debt successfully and so will all the subsequent debt. Hence households receive 1. However, when good news has not arrived by time t, the rollover fails and layer l has to liquidate its asset at $\alpha P_{t,l-1}$, where $P_{t,l-1}$ is the new cohort of households' valuation of debt issued by layer $l-1$ at time t conditional on no good news has arrived by time t. In the structure in Figure IA.A.2,

$$P_{t,l-1} = p \underbrace{\alpha \times 1}_{\text{If goods news arrive in } t+1} + (1-p) \underbrace{\alpha P_{t+1,l-1}}_{\text{If no good news has arrived by } t+1}$$

However, consider the structure in Figure IA.A.3, denote households' valuation of debt issued by layer $l-1$ as $\tilde{P}_{t,l-1}$ (conditional on no good news arrive by time t),

$$\tilde{P}_{t,l-1} = p \times \underbrace{1}_{\text{If goods news arrive in } t+1} + (1-p) \underbrace{\alpha \tilde{P}_{t+1,l-1}}_{\text{If no good news has arrived by } t+1}$$

Notice that from period $t+1$ onward, the two structures are exactly the same. Hence $\tilde{V}_{t+1,l-1} = V_{t+1,l-1}$.

$$\tilde{P}_{t,l-1} - P_{t,l-1} = p(1-\alpha) > 0$$

Hence if there is any layer bearing more than 1 period maturity mismatch, then adding a layer and reducing the maturity mismatch strictly increases debt value. As a result, the structure that yields the highest debt value must be the one in Figure IA.A.1. The mechanism is the same as in the 3-period example: adding layers increases the liquidation value (here it is $P_{t,l-1}$) in the bad states of the world.

Appendix IA.B: Contract with State-Contingent Maturity

In this section, we consider a case where the ultimate borrower issues debt contract with state-contingent maturity directly to households. Specifically, the contract only matures when $y_t = \bar{y}$ in period t. Denote the value of this debt contract as $\tilde{V}$, and the promised payment when the underlying asset matures is $\tilde{F}_y$ such that $\tilde{V}(\tilde{F}_y) = e$.

$$\tilde{V}(y_t) = \lambda_y \min(\tilde{F}_y, y_t) + \underbrace{(1-\lambda_y)(pe + (1-p)\tilde{B})}_{=\tilde{v}}. \tag{IA.B.1}$$

At the beginning of period $t+1$, if $y_{t+1} = \bar{y}$, which occurs with probability p, then the contract matures and households receive e. If $y_{t+1} = 0$, which occurs with probability $(1-p)$, then the contract does not mature and the departing households receives $\tilde{B}$ by selling this contract to the new cohort of households. As before, we assume any secondary market transaction happens through experts, incurring a discount α. Hence the proceeds $\tilde{B}$ is equal to α multiplied by the endogenous market value of this contract.

Next, we determine the contract value perceived by the cohort-$(t+1)$ households, given the bad news. Note that with probability λ_y, the underlying asset matures in period $t+1$, yielding $y_{t+1} = 0$. With probability $(1-\lambda_y)$, the underlying asset does not mature. In this scenario, if $y_{t+2} = e$, then the contract matures and pays off e to cohort-$(t+1)$; otherwise, the contract is resold to the next cohort of households via the secondary market.

$$\tilde{B} = \alpha \left[\lambda_y \times 0 + (1-\lambda_y)(pe + (1-p)\alpha\tilde{V}(y_{t+2} = 0))\right] \tag{IA.B.2}$$

Comparing Eq. (IA.B.1) with the debt value in the two-layer case in Eq. (3), the difference in debt value boils down to the following

$$\tilde{v} - v(2) = (1 - \lambda_y)(1 - p)[\lambda_d(\tilde{B} - B_0(2)) + (1 - \lambda_d)(\tilde{B} - B_1(2))] \tag{IA.B.3}$$

where

$$\tilde{B} - B_0(2) = \alpha(1 - \lambda_y)[pe(1 - \alpha) + (1 - p)\alpha(\tilde{v} - v(2))] \tag{IA.B.4}$$

and

$$\tilde{B} - B_1(2) = \alpha(1 - \lambda_y)[(1 - \lambda_d)pe(1 - \alpha) + (1 - p)\alpha(\tilde{v} - v(2))]. \tag{IA.B.5}$$

Hence the state-contingent maturity contract has an even higher value than the two-layer case because the debt always matures when fundamental is strong, hence it avoids the transaction costs whenever possible; in addition, it also avoids the cost of liquidating long-term assets as the debt never matures when the fundamental is weak. The two-layer structure approximates this state-contingent maturity contract in an imperfect way because of the random maturity structure.

Appendix IA.C: Contract Optimality

IA.C.1 Maturity

In this section we consider whether the last layer of fund prefers to issue debt that matures with probability λ_d to the households or to issue short-term debt that always matures. In the case when the departing households' are holding debt contract that has not matured, they have to sell the debt to the new households on the secondary market at a discount α. Conceptually, assets being liquidated are eventually sold on the secondary market as well, hence we assume this discount is the same as in the liquidation process. Denote the debt value in the former case as $\check{V}_L$, and the corresponding liquidation value in that case as

$\check{B}_l(y, L)$

$$\begin{aligned}\check{V}_L(y, L) =&\lambda_y \min(y, F_y) + (1-\lambda_y)\Bigg\{ \sum_{l=0}^{L-2} (m_l - m_{l+1})[(1-H(F_y))e + H(F_y)\mathbb{E}[\check{B}_l(y, L) - c(L-l)|y < F_y]] \\ &+ \lambda_d(1-\lambda_d)^{L-1}[(1-H(F_y))e + H(F_y)\mathbb{E}[\check{B}_{L-1}(y, L) - c|y < F_y]] \\ &+ (1-\lambda_d)^L \alpha \mathbb{E}[\check{V}_L(y)] \Bigg\}\end{aligned}$$

Denote the continuation value as $\check{v}_L$, where $\check{V}_L(y, L) = \lambda_y \min(y, F_y) + \check{v}_L$. To determine which contracts the borrower prefers, we just have to compare the continuation value of the debt contracts: the contract that gives the households higher continuation value implies that the borrower can promise lower interest rate for the households to break even. Comparing $\check{v}_L$ with v_L in Eq.(B.6), we get

$$\begin{aligned}v_L - \check{v}_L =&(1-\lambda_y)\Bigg\{ \sum_{l=0}^{L-1} \lambda_d(1-\lambda_d)^l [H(F_y)\mathbb{E}[B_l(y, L) - \check{B}_l(y, L)|y < F_y]] \\ &+ (1-\lambda_d)^L[(1-H)(1-\alpha)e + H\mathbb{E}[B_{L-1}(y, L) - c - \alpha V_L(y)|y < F_y] + \alpha(v_L - \check{v}_L)] \Bigg\}\end{aligned} \tag{IA.C.1}$$

Furthermore

$$B_l(y, L) - \check{B}_l(y, L) = \alpha(1-\lambda_y)\left[(1-\lambda_d)^l \alpha(v_L - \check{v}_L) + \sum_{i=0}^{l-1} \lambda_d(1-\lambda_d)^i H\alpha(v_L - \check{v}_L)\right]$$

Plugging this into Eq. (IA.C.1), it is straightforward to show that $v_L - \check{v}_L$ has the same sign as $(1-H)(1-\alpha)e + H\mathbb{E}[B_{L-1}(y,L) - c - \alpha V_L(y)|y < F_y]$, which we investigate next. where

$$
\begin{aligned}
&\mathbb{E}[B_{L-1}(y,L) - \alpha V_L(y)|y < F_y] \\
=&\alpha(1-\lambda_y)\Big\{(1-\lambda_d)^{L-1}\alpha\mathbb{E}[V_L] + \sum_{i=0}^{L-2}\lambda_d(1-\lambda_d)^i[e(1-H) + H(\alpha\mathbb{E}[V_L|y<F_y] - c(L-1-i))] \\
&- (1-H)e - H\sum_{i=0}^{L-2}\lambda_d(1-\lambda_d)^i(E[B_i(y,L)|y<F_y] - c(L-i)) \\
&- H(1-\lambda_d)^{L-1}(E[B_{L-1}(y,L)|y<F_y] - c)\Big\} \\
>&(1-\lambda_y)\alpha\left[-(1-\lambda_d)^{L-1}(1-H)(1-\alpha)e - H\mathbb{E}[B_{L-1}(y,L) - \alpha V_L(y)|y<F_y] + Hc\right] \\
&\mathbb{E}[B_{L-1}(y,L) - \alpha V_L(y)|y<F_y] > \frac{(1-\lambda_y)\alpha[-(1-\lambda_d)^{L-1}(1-H)(1-\alpha)e + Hc]}{1+(1-\lambda_y)\alpha H}
\end{aligned}
$$

Plug this back into $(1-H)(1-\alpha)e + H\mathbb{E}[B_{L-1}(y,L) - c - \alpha V_L(y)|y < F_y]$ we get

$$
\begin{aligned}
&(1-H)(1-\alpha)e + H\mathbb{E}[B_{L-1}(y,L) - c - \alpha V_L(y)|y<F_y] \\
>&(1-H)(1-\alpha)e - H\frac{(1-\lambda_y)\alpha(1-\lambda_d)^{L-1}(1-H)(1-\alpha)e + c}{1+(1-\lambda_y)\alpha H} \\
=&\frac{(1-H)(1-\alpha)e + (1-\lambda_y)(1-\alpha)\alpha H(1-H)e(1-(1-\lambda_d)^{L-1}) - Hc}{1+(1-\lambda_y)\alpha H}
\end{aligned}
$$

As long as c is not too large, this is positive, and $v_L > \check{v}_L$. In other words, the last layer of fund prefers to issue one-period debt than to issue debt that matures with probability λ_d.

IA.C.2 Extension: Endogenizing Face Value

IA.C.2.1 Setting

In this section, we endogenize the face value of the debt contract as well and derive the condition under which the face value equals e. Denote by π_t a generic debt contract; we assume that it takes the form of $\pi_t = \{\tilde{F}_{y,s}, F_{d,s+1}\}_{s=t}^T$, with an exogenously given debt maturity parameter λ_d as in the main text.

$$\tilde{F}_{y,s} \cdot \mathbf{1}_{\text{underlying asset matures at period } s, \text{ w.p. } \lambda_y} + F_{d,s+1} \cdot \mathbf{1}_{\text{debt matures at period } s+1, \text{ w.p. } \lambda_d},$$

where $\{F_{d,s+1}\}$ is $\mathcal{F}_s$-measurable for any $s \geq t$. Importantly, it cannot depend on tomorrow's fundamental y_{s+1}. The space of debt contracts is now $\Pi \equiv \mathbb{R}_+^{T-t+1} \times \mathbb{R}_+^{T-t}$. Same as before, we allow debtors, after knowing the realization of y_t, to renegotiate by "prepaying" the debt contract. In other words, they can pay the lender F_d and eliminate all future obligations.

Because the layer-l fund is essentially using its asset holding with a market value of $F_d(l-1)$ to back its debt issuance with a market value of $F_d(l)$, and fund managers have no initial wealth, we impose the following condition throughout the paper:

$$F_d(l) \leq F_d(l-1) \leq e \text{ for } \forall l. \tag{IA.C.2}$$

The first part of the condition (IA.C.2) essentially rules out the "Ponzi" scheme by any fund in which a fund maintains a debt that is underfunded relative to its asset holdings but keeps rolling over this debt from OLG households. A side benefit of this assumption is that it simplifies the prepayment process, as the cash-flows trickle down to the bottom. The second part $F_d(l) \leq e$ in condition (IA.C.2) captures the fact that households can only afford to pay e.

Similar to the main text, if rollover fails at layer-l, the departing households recover

$$\min(B_l(y, L), F_d(l)) - c \cdot (L - l).$$

The value functions are similar to before except that the face value is $F_{d,t}(l)$, instead of e. We adjust the feasible contract space to account for the endogenous face values.

$$\hat{\Pi} \equiv \{\pi \in \Pi : V_L(\{F_{y,s}, F_{d,s+1}\}_{s=t}^{T}, L) \leq F_{d,t} \leq e \quad \text{for} \quad \forall t\}. \tag{IA.C.3}$$

Finally, the definition of equilibrium now includes the optimal design of F_d's

Definition 3 *The equilibrium credit chain is a set of contracts* $\{\pi_{l,t}\}_{0 \leq l \leq L-1}$ *and credit chain length* L^* *such that*

1. *When layer-l's liability matures,*[39]

$$\pi_l = \arg\max_{\pi \in \hat{\Pi}} \quad \mathbf{1}^l_{rollover}(P_l(y, \pi; \pi_{l-1}, L^*) + V_l(y, \pi; \pi_{l-1}, L^*)), \tag{IA.C.4}$$

$$s.t. \quad F_y(l) \leq F_y(l-1) \leq y \qquad F_d(l) \leq F_d(l-1) \leq e \text{ in (IA.C.2)}. \tag{IA.C.5}$$

2. *The equilibrium L^* is such that the final layer of fund manager $(L^* - 1)$ prefers to borrow directly from households than to borrow via other fund managers:*

$$P_{L^*-1}(L^*) + V_{L^*-1}(L^*) \geq P_{L^*-1}(L^* + l) + V_{L^*-1}(L^* + l) \quad \text{for} \quad l \geq 1. \tag{IA.C.6}$$

Furthermore, for all other funds $0 < l < L^* - 1$,

$$P_l(L^*) + V_l(L^*) \geq P_l(l+1) + V_l(l+1). \tag{IA.C.7}$$

In other words, the funds in intermediary layers prefer to borrow via other funds than to borrow from the households.

3. *Due to perfect competition,*

$$P_l - P_{l-1} + V_l = 0. \tag{IA.C.8}$$

IA.C.2.2 Optimal Contracts

In this section, we derive conditions under which in equilibrium, $F_{d,t}(l) = e$.

Assumption 4 *Denote L^* as the equilibrium chain length. The primitives of our model satisfy:*

$$(1-\alpha)(1 - H(F_y)) - \frac{h(F_y)}{\lambda_y} e - \sum_{l=0}^{L^*-2} m_l \lambda_d h(F_y) \frac{1}{\lambda_y} c(L^* - l) - m_{L^*-1} h(F_y) \frac{1}{\lambda_y} c \geq 0$$

Under Assumption 4, there exists an equilibrium in which the contract is independent of history and $F_{d,t}(l) = e$. Assumption 4 guarantees that inequality (IA.C.2) always binds (so

[39]When $t = 0$, $\mathbf{1}^l_{rollover} = 1$ for all l.

that in the optimal contract $F_{d,t}(l) = e$), and it is more likely to be true when e is relatively small.

We first show that $F_{d,t}(l) = F_d(l)$, i.e. the optimal F_d for each layer is constant over time if the managers do not face rollover issues in this period. We start from the problem between layer $(L-1)$ and the households. Layer $(L-1)$ is given a contract π_{L-2} by layer $(L-2)$; the contract specifies a sequence of payments if debt matures $\{F_{d,t}(L-2)\}_{t=0}^{T}$ and a payment if the underlying asset matures $F_y(L-2)$. T is the stopping time, either when the contract or when the underlying matures. Plugging in P_{L-1}, layer $L-1$ maximizes the following,

$$\max_{F_d(L-1)} -P_{L-2} + \lambda_y F_y(L-2) + (1-\lambda_y)\mathbb{E}\Big[\sum_{i=0}^{L-2}(1-\lambda_d)^i[\lambda_d \mathbf{1}^i_{rollover}(\alpha V_{L-1}(y, \pi'_{L-1}; \pi'_{L-2}, L) + \alpha F_d(L-2) + (1-\alpha)F_d(L-$$

$$+ (1-\mathbf{1}^i_{rollover})(B_i(y,L) - c(L-i-1))] + (1-\lambda_d)^{L-1}\mathbf{1}^{L-1}_{rollover}(\alpha V_{L-1}(y, \pi'_{L-1}; \pi_{L-2}, L) + (1-\alpha)F_{d,L-1})$$

$$+ (1-\lambda_d)^{L-1}(1-\mathbf{1}^{L-1}_{rollover})(B_{L-1}(y,L) - c)\Big]$$

$$s.t. \quad F_d(L-1) \leq F_d(L-2)$$

The first order condition with respect to $F_{d,t}(L-1)$ is

$$0 = -\mu^{\lambda_d}_{L-1,t} + (1-\alpha)\mathbb{E}[\sum_{i=0}^{L-2}(1-\lambda_d)^i \lambda_d \mathbf{1}^i_{rollover} + (1-\lambda_d)^{L-1}\mathbf{1}^{L-1}_{rollover}]$$
$$+ (1-\lambda_d)^{L-1}\frac{d\text{Pr(rollover at layer } L-1)}{dF_{d,t}(L-1)}(F_{d,t}(L-1) - B_{L-1}(y,L) + c)$$

where $\mu^{\lambda_d}_{L-1,t}$ is the Lagrangian Multiplier in front of $F_{d,t}(L-2) - F_{d,t}(L-1) \geq 0$.

If $\pi_{L-2} = \pi^*_{L-2}$ is stationary and $F_{d,t}(L-2)$ is constant over time, then $F^*_{d,t}(L-1) = F_d(L-1)$.

The same logic applies to $F^*_{d,t}(l) = F_d(l)$ for all $0 \leq l \leq L-1$. For $0 \leq l < L-1$, its objective can be written as

$$\max_{F_d(l)} -P_{l-1} + \lambda_y F_y(l-1) + (1-\lambda_y)\alpha\Big\{(1-\lambda_d)^{l+2}\mathbb{E}V_l(y', \pi_l; \pi_{l-1}, L) + (1-\lambda_d)^{l+1}\lambda_d\mathbb{E}(1-\mathbf{1}^{l+1}_{rollover})V_l(y', \pi'_l; \pi_{l-1}, L)$$

$$+ \sum_{i=0}^{l-1}(1-\lambda_d)^i\lambda_d\mathbb{E}[\mathbf{1}^i_{rollover}(F_d(l-1) - F_d(l+1) - P'_{l-1} - P'_l + \max_{\pi'_l}(P'_l + V_l(y', \pi'_l; \pi'_{l-1}, L)) + \max_{\pi'_{l+1}}(P'_{l+1} + V_{l+1}(y', \pi'_{l+1}; \pi_l, L)))]$$

$$+ (1-\lambda_d)^l\lambda_d\mathbb{E}[\mathbf{1}^l_{rollover}(-F_{d,l+1} - P'_l + \max_{\pi'_l}(P'_l + V_l(y', \pi'_l; \pi_{l-1}, L))) + \max_{\pi'_{l+1}}(P'_{l+1} + V_{l+1}(y', \pi'_{l+1}; \pi_l, L))]$$

$$+ (1-\lambda_d)^{l+2}\mathbb{E}V_{l+1}(y', \pi_{l+1}; \pi_l, L) + (1-\lambda_d)^{l+1}\lambda_d\mathbb{E}[\mathbf{1}^{l+1}_{rollover}(-F_d(l+1) + \max_{\pi'_{l+1}} V_{l+1}(y', \pi'_{l+1}; \pi_l, L))]\Big\} + P_{l+1}$$

we know in equilibrium $P'_{l-1} = \max_{\pi'_l}(P'_l + V_l(y', \pi'_l; \pi'_{l-1}, L))$ and $P'_l = \max_{\pi'_{l+1}}(P'_{l+1} +$

$V_{l+1}(y', \pi'_{l+1}; \pi'_l, L))$, so the above can be simplified as

$$
\begin{aligned}
&\max_{F_d(l)} -P_{l-1} + \lambda_y F_y(l-1) + (1-\lambda_y)\alpha\Big\{(1-\lambda_d)^{l+2}\mathbb{E}V_l(y', \pi_l; \pi_{l-1}, L) + (1-\lambda_d)^{l+1}\lambda_d\mathbb{E}(1-\mathbf{1}^{l+1}_{rollover})V_l(y', \pi'_l; \pi_{l-1}, L) \\
&+ \sum_{i=0}^{l-1}(1-\lambda_d)^i\lambda_d\mathbb{E}[\mathbf{1}^i_{rollover}(F_d(l-1) - F_d(l+1))] + (1-\lambda_d)^l\lambda_d\mathbb{E}[\mathbf{1}^l_{rollover}(-F_d(l+1) + V_l(y', \pi'_l; \pi_{l-1}, L) + V_{l+1}(y', \pi'_{l+1}; \pi'_l, L) + P'_{l+1})] \\
&+ (1-\lambda_d)^{l+2}\mathbb{E}V_{l+1}(y', \pi_{l+1}; \pi_l, L) + (1-\lambda_d)^{l+1}\lambda_d\mathbb{E}[\mathbf{1}^{l+1}_{rollover}(-F_{d,l+1} + P'_{l+1} + V_{l+1}(y', \pi'_{l+1}; \pi_l, L))]\Big\} + P_{l+1}
\end{aligned}
$$

subject to $F_{d,t}(l) \leq F_{d,t}(l-1)$. Denote the Lagrangian multiplier as $\mu^{\lambda_d}_{l,t}$. The first order condition with respect to $F_{d,t}(l)$ is

$$
\begin{aligned}
0 &= -\mu^{\lambda_d}_{l,t} + \mu^{\lambda_d}_{l+1,t} + \frac{dP_{l+1}}{dF_{d,l,t}} \\
&= -\mu^{\lambda_d}_{l,t} + \mu^{\lambda_d}_{l+1,t} + (1-\lambda_d)^l\lambda_d\frac{d\text{Pr(rollover at layer } l)}{dF_{d,t}(l)}(F_d(l) - B_l(y, L) + c(L-l))
\end{aligned}
$$

If π^*_{l-1} does not depend on history and is stationary, then it is straightforward that $F^*_{d,t}(l) = F_d(l)$.

Next, we show that $F_d(l) = F_d$ across layers. Since the problem is identical over time, we loose the time subscript. The first order condition with respect to $F_d(L-1)$ in equilibrium is

$$
\begin{aligned}
0 = &-\mu^{\lambda_d}_{L-1} + (1-\alpha)\sum_{l=0}^{L-2}(1-\lambda_d)^l\lambda_d\text{Pr(rollover at layer } l) + (1-\alpha)(1-\lambda_d)^{L-1}\text{Pr(rollover at layer } L-1) \\
&+ (1-\lambda_d)^{L-1}\frac{d\text{Pr(rollover at layer } L-1)}{dF_d(L-1)}[F_d(L-1) - B_{L-1}(y, L) + c]
\end{aligned}
$$

The first order condition with respect to $F_d(l)$ for $0 < l < L-1$ is,

$$
0 = -\mu^{\lambda_d}_l + \mu^{\lambda_d}_{l+1} + (1-\lambda_d)^l\lambda_d\frac{d\text{Pr(rollover at layer l)}}{dF_d(l)}(F_d(l) - B_l(y, L) + c(L-l))
$$

For $l = 0$, the constraint is $F_d(0) \leq e$. Denote its Lagrangian multiplier as $\mu^{\lambda_d}_0$. The first order condition is

$$
0 = -\mu^{\lambda_d}_0 + \mu^{\lambda_d}_1 + \lambda_d\frac{d\text{Pr(rollover at layer 0)}}{dF_d(0)}(F_d(0) - B_0(y, L) + cL)
$$

Substituting in all the Lagrangian multipliers.

$$0 = -\mu_0^{\lambda_d} + (1-\alpha)\sum_{l=0}^{L-2}(1-\lambda_d)^l \lambda_d \Pr(\text{rollover at layer } l) + (1-\alpha)(1-\lambda_d)^{L-1}\Pr(\text{rollover at layer } L-1)$$
$$+ \sum_{l=0}^{L-2}(1-\lambda_d)^l \lambda_d \frac{d\Pr(\text{rollover at layer } l)}{dF_{d,l}}(F_d(l) - B_l(y,L) + c(L-l))$$
$$+ (1-\lambda_d)^{L-1}\frac{d\Pr(\text{rollover at layer } L-1)}{dF_d(L-1)}(F_d(L-1) - B_{L-1}(y,L) + c) \tag{IA.C.9}$$

Denote layer-0's choice as $F_d(0) = F_d$, satisfying equation (IA.C.9). If $\mu_0^{\lambda_d} > 0$, then $F_d = e$, and since $\mu_{L-1}^{\lambda_d} \geq \mu_{L-2}^{\lambda_d} \geq ... \geq \mu_0^{\lambda_d} > 0$, all the constraints are binding, i.e. $F_d(L-1) = F_d(L-2) = ... = F_d$.

If $\mu_0^{\lambda_d} = 0$, then $F_d < e$, it must be the case that $\frac{d\Pr(\text{rollover at layer } l)}{dF_d(l)} < 0$ holds for at least one l. Denote $\hat{l}$ as the smallest l such that $\frac{d\Pr(\text{rollover at layer } l)}{dF_d(l)} < 0$. This implies that for $l < \hat{l}$, $\frac{d\Pr(\text{rollover at layer l})}{dF_d(l)} = 0$, so the first order conditions for $F_d(l)$ $(l \geq \hat{l})$ are the same as that for $F_d(0)$. In other words, $F_d(l) = F_d$. For $l < \hat{l}$, we have $\mu_l^{\lambda_d} > 0$, so the constraint is binding, i.e. $F_d(L-1) = F_d(L-2) = ... = F_d(\hat{l}-1) = F_d$.

So far we have shown that when there is no rollover concerns, we have $F_{d,t}(l) = F_d$ being constant over time and across layers. Now we just to show when y is small, and when the money raised from the unconstrained optimal contract is smaller than the amount owed, the managers cannot deviate and set higher F_d. For managers in layer 1 to layer $L-1$, because $F_d(l) \leq F_d(l-1)$ is binding, they cannot set higher F_d. For layer 0, we will next show that Assumption 4 ensures that $F_d(0) \leq e$ is binding. Hence the ultimate borrower at layer 0 cannot deviate and set higher F_d either. As a result, $F_{d,t}(l) = F_d$ for all layer l and time t.

The proof for $F_{y,t}(l) = F_y$ is the same as in Appendix B.B. In equilibrium, F_y is the minimal payment if the underlying asset matures such that the new households are willing to rollover debt, for a given F_d. By definition

$$F_d = V_L(\{F_y, F_d\}, L) \qquad \text{for} \quad y \geq F_y$$
$$\Rightarrow F_d = \lambda_y F_y + v_L(\{F_y, F_d\}, L)$$

Since all layers have the same F_y and rollover fails when $y < F_y$, we have $\Pr(\text{rollover at layer } l) =$

$1 - H(F_y)$. Plug this expression in the first order condition of F_d, we get

$$
\begin{aligned}
&-\mu_0^{\lambda_d} + (1-\alpha)(1-H(F_y)) - \sum_{l=0}^{L-2} m_l \lambda_d h(F_y) \frac{dF_y}{dF_d}(F_d - B_l(F_y, L) + c(L-l)) \\
&-(1-\lambda_d)^{L-1} h(F_y) \frac{dF_y}{dF_d}(F_d - B_{L-1}(F_y, L) + c) = 0 \\
&\mu_0^{\lambda_d}(e - F_d) = 0 \qquad \mu_0^{\lambda_d} \geq 0
\end{aligned}
$$

When $F_d \leq e$ is binding, $\frac{dF_y}{dF_d} = \frac{1}{\lambda_y}$. Hence

$$
\begin{aligned}
&(1-\alpha)(1-H(F_y)) - \sum_{l=0}^{L-2} m_l \lambda_d h(F_y) \frac{1}{\lambda_y}(F_d - B_l(F_y, L) + c(L-l)) - m_{L-1} h(F_y) \frac{1}{\lambda_y}(F_d - B_{L-1}(F_y, L) + c) \\
\geq &(1-\alpha)(1-H(F_y)) - \sum_{l=0}^{L-2} m_l \lambda_d h(F_y) \frac{1}{\lambda_y}(F_d + c(L-l)) - m_{L-1} h(F_y) \frac{1}{\lambda_y}(F_d + c)
\end{aligned}
$$

Under Assumption 4, the above equation is greater than or equal to 0. Hence $F_d = e$.