Information Span in Credit Market Competition[*]

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Abstract

Recent technological change in lending converts previously subjective assessments into structured, easily accessible data. We study this transformation in a credit market competition model that distinguishes between information span (breadth) and signal precision (quality). Borrower quality depends on multidimensional fundamentals, assessed through hard or soft signals. Two banks observe private hard signals, but only the specialized bank receives a soft signal. Expanding the span of hard information allows the non-specialized bank to evaluate characteristics previously only available to the specialist, and reducing its winners curse. By contrast, greater precision of hard signals strengthens the specialized banks informational advantage.

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Recent technological progress has fundamentally altered how information is produced and used in credit markets. Advances in big data, machine learning, and digital platforms, for example, expand lenders' ability to assess and process borrower characteristics at scale. While these developments are often associated with more accurate screening, their impact extends beyond improvements in signal precision. Modern technologies also expand the *span* of hard information—the set of borrower characteristics that can be captured by structured data—by converting previously qualitative, relationship-based assessments into hard signals that are standardized and relatively easy to process by non-specialists.

This distinction is especially important for banks who rely on information to screen, price, and allocate credit. Traditionally, lending combined hard data—such as financial statements and credit histories—with soft information derived from borrower relationships, site visits, or managerial judgment. Recent technologies not only increase the precision of screening over a fixed set of traits, but also, by hardening soft information, expand the range of borrower characteristics that can be incorporated into hard signals.

Agricultural lending provides a concrete illustration of hardening soft information. Historically, lending to farmers required site visits by specialized loan officers, who relied on experience and local knowledge to assess risks tied to farming techniques, land quality, or infrastructurefeatures conventionally categorized as soft information. Today, satellite imagery and AI-based analytics allow lenders to infer some of these characteristics remotely, using digitized indicators such as vegetation indices or soil quality measures. While these technologies do not eliminate the role of human expertise, they expand the *range* of borrower traits that can be evaluated through hard signals. Crucially, this change reflects an increase in information span—not simply an increase in the precision of existing metrics—and it is this distinction that lies at the core of our analysis.

This remarkable technological advancement has the potential to disrupt the industrial landscape of the banking sector, which motivates us to develop a model that incorporates *information span* in an otherwise standard credit market competition setting. Borrower quality depends on two types of states: hard states, which can be assessed using structured data and modern analytics; and soft states, which require more subjective or specialized knowledge. We adopt a multiplicative structure in which repayment occurs only if both types of states are favorable. Banks receive private signals about these states: all lenders observe a binary hard signal, while a specialized lender additionally observes a continuous soft signal. Allowing hard and soft states to overlap enables us to isolate information span as a distinct margin of technological change and to separate its competitive implications from those of changes in signal precision.

Our framework highlights a sharp distinction between *breadth* (span) and *quality* (precision) of information. Expanding the span of hard information means that hard signals cover a larger subset of borrower fundamentals, including dimensions previously accessible only through soft information. Improving precision, by contrast, increases the accuracy of signals over a fixed set of characteristics. While both forms of technological progress reduce uncertainty, we show that they have fundamentally different—and sometimes opposite—effects on credit market competition.

Unlike improvements in precision, expansions in information span change which borrower fundamentals enter lenders' information sets and, therefore, cannot be represented as Blackwell improvements of existing signals. While one might expect improvements in hard information to uniformly intensify competition, we show that this intuition hinges critically on whether technology improves the span or the precision of information. In our setting, expansions in information span tend to intensify competition, leveling the playing field between specialized and non-specialized lenders, whereas increases in precision reinforce existing informational advantages. As we will explain shortly, this difference is central to understanding recent empirical patterns in credit markets.

In our model of credit market competition outlined in Section 1, a specialized bank endowed with a binary hard signal and a continuous soft signal competes with a non-specialized bank that has a hard binary signal only. We assume that the hard signal is decisive in that each lender makes an offer only if it receives a positive realization. The soft signal—which differentiates our paper from existing models such as Broecker (1990) and Marquez (2002)—is continuous and guides the fine-tuned interest rate offer of the specialized bank; and when the soft signal realization is sufficiently low, the specialized bank rejects the borrower.

We characterize the unique credit market equilibrium in closed form in Section 2. The equilibrium falls into one of two regimes depending on whether the non-specialized bank makes zero profits. In the "zero-weak" regime, the Winner's Curse (in competition) faced by the non-specialized "weak" bank is so severe that it randomly withdraws after a positive hard signal, earning zero profits. In the "positive-weak" regime, the Winner's Curse is less severe, and the non-specialized bank always participates upon receiving a positive hard signal and earns positive profits.

In our setting, the non-specialized (therefore relatively weak) bank faces a Winners Curse, which arises because the specialized lender has exclusive access to the soft signal. The effect operates through two channels. First, the exclusive soft signal gives the specialized bank more precise information about states covered by both hard and soft signals. Second, and novel to the literature, the specialized bank is the only lender with information about states covered exclusively by the soft signalthat is, the "only-soft" fundamental states. This latter component drives the distinct effects of span and precision on equilibrium outcomes through three key elements:

1. *Probability of facing competition*: the non-specialized bank (which competes only upon a positive hard signal) is concerned about the only-soft fundamental only when the specialized bank also competes (if the opponent also receives a positive hard signal);
2. *Beliefs about soft signal upon competition:* When hard and soft signals are correlated, the event of competition itself conveys information about the soft signal's distribution, leading to a more accurate screening of the soft state; and
3. *Inference from winning:* If the non-specialized bank wins the borrower, it infers that the its opponent's soft signal—and thus the only-soft fundamental—is relatively weak.

Section 3 shows that although both the span and precision of hard information improve screening, they can have opposite effects on the non-specialized banks beliefs about competition and the quality of the only-soft state, and thus on credit market competitiveness. To start with, an increase in the span of hard information levels the playing field and intensifies competition. First, a greater span of hard information decreases the probability of competition, as there are more characteristics that need to be positive for a positive hard signal. Second, it leads to an increase in the correlation between hard and soft signals, improving beliefs about the soft signal received by the specialized bank upon competition. Finally, an increase in span improves the expected quality of the only-soft fundamental, as there are fewer fundamentals that need to be favorable for it to be successful. These effects benefit the non-specialized bank, encouraging participation and increasing competition.

In contrast, a higher precision can increase the Winners Curse faced by the non-specialized bank, strengthening the specialized bank's advantage. An increase in the precision of hard information leads to a higher correlation between hard signals, making them "more public" and hence increasing the probability of competition. The higher correlation also implies that the non-specialized bank's inference about the overlapping states is stronger; hence, for a given soft signal received by the specialized bank, the inference on the only-soft fundamental is weaker. These two effects increase the Winner's Curse and can outweigh the improvement in the updating of beliefs about the soft signal upon competition, thereby increasing the informational asymmetries and decreasing credit market competitiveness.

It is worth clarifying two conceptual point regarding the distinction between information span (breadth) and precision (quality). First, theoretically, in our credit market competition model with specialized lending,[1] multidimensional information is crucial in delivering this

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[1] Specialized lending is a practically relevant to match the empirical patterns of lower loan pricing and lower non-default rate among loans granted by specialized lenders (Blickle, He, Huang, and Parlatore, 2025).

distinction. Second, empirically, this distinction between different types of advancements in information technology matter for our understanding of the world. In principle, recent innovations should enhance hard-information-based screening across the board—specialized incumbent banks can adopt these tools (He, Jiang, Xu, and Yin, 2025) just as effectively as fintech challengers and non-specialized lenders. Yet empirical evidence (see, e.g. Berg, Fuster, and Puri, 2022) suggests that technological change has disproportionately benefited non-specialized, weaker lenders, enabling them to close the gap and intensify competition. Our model, which features asymmetric lenders but symmetric technological improvements, offers an explanation: expanding the information span can robustly generate such outcomes, whereas simply increasing the precision of existing signals cannot. As illustrated by our motivating example in agricultural lending, Big-Data technologies enlarge the set of measurable borrower characteristics, granting non-specialized lenders access to "hardened soft information" that was once the exclusive domain of specialized expertise.

This process of "hardening soft information," which expands the span of hard information, has important implications for credit allocation and welfare. We formally show that total welfare is monotonically increasing in the span of hard information. Moreover, when hard-signal precision is relatively low, a broader span can also improve the specialized bank's payoff within the positive-weak equilibrium region. This result highlights that a greater information span—interpretable as industry-wide improvements in information technology—can generate gains for both specialized and non-specialized lenders.

Finally, we consider two important extensions in Section 4. First, motivated by open banking initiatives, we examine a model with correlated hard signals. We show that an increase in this correlation is qualitatively similar to increasing the hard signal precision and has opposite effects compared to increasing the hard information span. Second, we introduce an additional signal about the overlapping states as an alternative model for the hardening of soft information and discuss the robustness of our main takeaway in this alternative setting.

Literature Review

Fintech. Our paper connects to the growing literature on fintech disruption (see Berg, Fuster, and Puri, 2022; Vives, 2019). Empirical studies document the use of alternative and transaction-based data in fintech lending, which enlarges the span of hard information available for credit assessment (Huang, Zhang, Li, Qiu, Sun, and Wang, 2020; Ghosh, Vallee, and Zeng, 2021). For recent theory work, building on canonical credit competition models (Broecker, 1990), He, Huang, and Zhou (2023) analyze the welfare effects of open-banking regulation, while Goldstein, Huang, and Yang (2022) study banks' endogenous liability-side responses to loan-market competition under open banking. While these Broecker-type models measure information technology by signal precision, our multidimensional information structure allows technology to vary along both span and precision.[2] Vives and Ye (2021) develop a spatial model where bank monitoring deteriorates with distance, and the "hardening of soft information" lowers monitoring costs and distance-related monitoring penalties.

The nature of soft/hard information in bank lending. The literature on soft vs. hard information (e.g., Stein, 2002; Liberti and Petersen, 2019) emphasizes that hard information is verifiable and thus transferable within organizations, while soft information is often nonverifiable and modeled as cheap talk (e.g., Bertomeu and Marinovic, 2016; Corrao, 2023; Crawford and Sobel, 1982).[3] Verifiability is not central to our mechanism as we emphasize how technological advances convert soft information into hard, verifiable data. Nevertheless, consistent with verifiability, only the specialized lender access the soft signal: soft information must be collected and interpreted by specialists and is therefore not readily shareable, whereas hard data can be processed mechanically and potentially shared.[4]

Lending market competition and common-value auctions. Canonical frameworks Broecker (1990); Hauswald and Marquez (2003) capture information technology as signal precision within the binary setting. In contrast, our multi-dimensional information structure embeds different aspects of information technology—such as span and precision—and we emphasize their distinct implications for competition. Our companion paper Blickle, He, Huang, and Parlatore (2025) use a similar structure to introduce pricing based on private signals and thereby deliver the empirical patterns documented in Blickle, Parlatore, and Saunders (2023). A key modeling distinction is that lenders' signals are conditionally independent that companion paper, whereas we allow hard and soft signals to be correlated, capturing the hardening of previously soft information.

We also contribute to the literature on common-value auctions by providing a tractable framework with asymmetrically informed, non-Blackwell-ordered bidders. Much of this literature relies on nested or Blackwell-ordered signals (Milgrom and Weber, 1982; Kim, 2008; Hendricks and Porter, 1988) or binary signals Abraham, Athey, Babaioff, and Grubb (2020).[5] Notable exceptions that allow for non-Blackwell information include Arnosti, Beck, and Mil-

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[2] Along the line of different dimensions of information, Huang (2023) highlights that the value of information varies between collateral-backed bank lending and revenue-based fintech lending such as Square.

[3] For related empirical studies, see Liberti and Mian (2009), Paravisini and Schoar (2016). He, Jiang, Xu, and Yin (2025) document a significant rise in IT investment among U.S. banks and show that investments in communication technologies enhance banks ability to generate and transmit soft information.

[4] In Karapetyan and Stacescu (2014), the incumbent bank's incentive to further acquire "soft" information once a borrower's "hard" information is publicly shared. In contrast, our non-Blackwell signal structure allows for differentiating span and precision, and we emphasize their distinct effects on competition.

[5] For finance applications, see Gorbenko and Malenko (2024) where bidder asymmetry emerges endogenously from auction initiation, Povel and Singh (2006) on takeover contests and Chen and Wang (2023) on mergers and acquisitions.

grom (2016), who study auction design for internet display advertising, and Ernst, Spatt, and Sun (2024), who analyze order-by-order auctions for retail order flow with heterogeneous information about order toxicity.

1 Model

We present the model in this section, and highlight the informational structure that is central to our analysis.

1.1 Environment

Consider a credit market competition model with two dates. There are two ex-ante symmetric lenders (banks), indexed by $j \in \{A, B\}$ and one borrower firm; everyone is risk neutral.

Project. At $t = 0$, the firm needs to borrow one dollar to invest in a (fixed-scale) risky project that pays a random cash flow $\tilde{y}$ at $t = 1$. The cash flow realization y depends on the project's quality denoted by $\theta \in \{0, 1\}$. For simplicity, we assume that

$$\tilde{y} = \begin{cases} 1 + \overline{r} & \text{when } \theta = 1, \\ 0 & \text{when } \theta = 0, \end{cases} \tag{1}$$

where $\overline{r} > 0$ is given exogenously, so only the good project pays off. We refer to $\overline{r}$ as the interest rate cap or the return of a good project. The project quality θ is unobservable and the prior probability of a good project is $q \equiv \mathbb{P}(\theta = 1)$. We use "project success," "good project" and/or "good borrower" interchangeably to refer to $\theta = 1$.

Hard and soft states. The project quality $\theta \in \{0, 1\}$ depends on two potentially correlated fundamental states: a "$h$ard" state denoted by θ_h and a "$s$oft" state denoted by θ_s. We assume that both fundamental states are binary so that $\theta_h \in \{0, 1\}$ and $\theta_s \in \{0, 1\}$, with

$$q_h \equiv \mathbb{P}(\theta_h = 1), \text{ and } q_s \equiv \mathbb{P}(\theta_s = 1).$$

Multi-dimensional fundamental states and information span. Following the O-ring theory of economic development by Kremer (1993), we model the hard and soft states in a setting with multidimensional fundamental states. This modeling choice offers a novel way to study the "span" of the information available to banks. More specifically, suppose that the project quality θ depends on N characteristics in the following multiplicative way:

$$\theta = \prod_{n=1}^{N} \theta_n = \overbrace{\prod_{n=1}^{N_h^h} \theta_n \cdot \rlap{$\underbrace{\phantom{\prod_{n=N_h^h+1}^{N_h^h+N_s^h} \theta_n \cdot \prod_{n=N_h^h+N_s^h+1}^{N} \theta_n}}_{\theta_s}$} \prod_{n=N_h^h+1}^{N_h^h+N_s^h} \theta_n}^{\theta_h} \cdot \prod_{n=N_h^h+N_s^h+1}^{N} \theta_n. \tag{2}$$

We assume that $\{\theta_n\}$ follow independent Bernoulli distributions, that is, $\theta_n = 1$ with probability $q_n \in [0, 1]$ for $n = 1, ..., N$, and capture (unobservable) characteristics that are critical to the ultimate success of the project, such as product quality, market and funding conditions, and regulatory environment. As shown in (2), the hard state θ_h covers the first $N^h \equiv N_h^h + N_s^h$ characteristics, while the soft state covers the last $N - N_h^h$. Importantly, the hard and soft states can overlap in the middle characteristics N_s^h, leading to correlated fundamental states.

Since the order of characteristics plays no role in the analysis, it is without loss of generality to analyze a simplified setting with three independent fundamental states as follows:

$$\theta = \overbrace{\theta_h^h \cdot \rlap{$\underbrace{}_{\theta_s}$} \theta_s^h}^{\theta_h} \cdot \theta_s^s, \tag{3}$$

with priors denoted by the following:[6]

$$q_h^h \equiv \mathbb{P}\left(\theta_h^h = 1\right), \quad q_s^h \equiv \mathbb{P}\left(\theta_s^h = 1\right), \quad \text{and} \quad q_s^s \equiv \mathbb{P}\left(\theta_s^s = 1\right).$$

Within this framework, θ_h^h in (3) captures those fundamentals only covered by the hard state ("only-hard"), θ_s^s captures the ones that are only covered by the soft state ("only-soft"), and θ_s^h captures the characteristics that are covered by hard and soft states ("overlapping").

Credit market competition. At date $t = 0$, given its private information about the quality of the borrower's project, each bank j makes a take-it-or-leave-it offer to the borrower firm, or makes no offer (exits the lending market). An offer consists of a fixed loan amount of one and an interest rate r. If the borrower firm receives multiple offers, it accepts the offer with the lowest rate.

1.2 Information Technologies

Although project quality is unobservable, lenders have access to information technologies that generate signals about it. We model information technologies as mappings from some

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[6] Here, $q_h^h = \prod_{n=1}^{N_h^h} q_n$, $q_h^s = \prod_{n=N_h^h+1}^{N_h^h+N_s^h} q_n$, and $q_s^s = \prod_{n=N_h^h+N_s^h+1}^{N} q_n$ given the i.i.d. assumption of $\{\theta_n\}$.

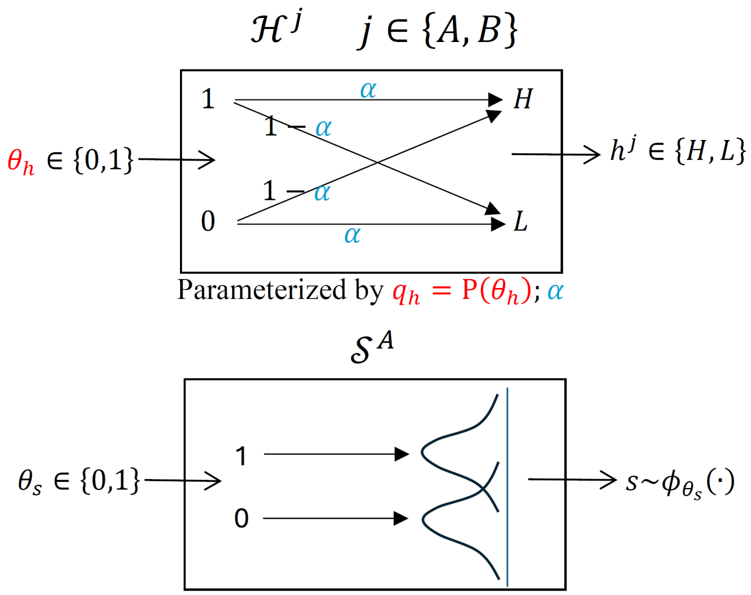

Figure 1: Hard and Soft Information Technologies. On the top panel, θ_h is a binary random variable capturing the fundamental *h*ard state, and the hard information technology $\mathcal{H}^j$ for $j \in \{A, B\}$ treats θ_h as input and produce a hard signal $h^j \in \{0, 1\}$ as the output; $\alpha \in [0, 1]$ is the precision of hard signal. On the bottom panel, θ_s is a binary random variable capturing the fundamental *s*oft state, and the soft information technology $\mathcal{S}^A$ of Bank A treats θ_s as input and produces a soft signal $s \sim \phi_{\theta_s}(\cdot)$ as the output.

fundamental states to signals. More specifically, consider two types of technologies modeled as mappings from the states θ_h or θ_s to bank-specific signal realizations. To capture specialized lending, we assume that both lenders $j \in \{A, B\}$ have a *hard-information*-based private signal h^j about θ_h, while only the specialized bank A has the *soft-information*-based private signal s about θ_s. Figure 1 provides a visual summary of information technology.

1.2.1 Hard Signals

Both lenders have access to "hard" data (including past financial and operating data, as well as "alternative data" that became available following the "Big Data" technology), which they can process to produce a *hard-information*-based private signal h^j about the fundamental state θ_h. We call them "hard" signals. Conditional on the state, hard signals are independent across lenders (Section 4.1 considers correlated hard signals).

Hard signal technology For tractability, we assume hard signals are binary, that is, $h^j \in \{H, L\}$, with H (L) being a positive (negative) signal of θ_h. Specifically, the hard signal technology $\mathcal{H}^j$ takes the binary fundamental hard state $\theta_h \in \{0, 1\}$ as input and generates a binary signal $h^j \in \{H, L\}$ as output. Following most of the literature with exogenous symmetric information technologies (e.g., Broecker, 1990; Marquez, 2002), we assume that

$$\mathbb{P}\left(h^j = H\,|\theta_h = 1\right) = \mathbb{P}\left(h^j = L\,|\theta_h = 0\right) = \alpha \text{ for } j \in \{A, B\} \text{ with } \alpha \in (0.5, 1)\,. \tag{4}$$

As illustrated in the top panel of Figure 1, α measures the precision of the hard signal and governs the (equal) probabilities of Type I and Type II errors. Given the binary fundamental state θ_h, the hard signal technology $\mathcal{H}^j$ can be summarized by two parameters: the prior $q_h = \Pr(\theta_h)$ of the input θ_h, and the signal's precision α given in (4).

Span of (hard) information The input of the hard information technology, θ_h, is determined by the span of hard information. As the span of hard information increases, the hard state covers more characteristics, corresponding to a larger N_s^h in (2) (or θ_s^h becoming more important in (3)). Hence, an expansion of the coverage of θ_h leads to a smaller q_s^h, as there are more characteristics that have to be one for θ_s^h to be successful. With this in mind, we can define the information span (of hard signals) as

$$\eta \equiv 1 - \Pr(\theta_s^h = 1) = 1 - q_s^h > 0. \tag{5}$$

The span of hard information η controls the input θ_h of the hard information technology $\mathcal{H}^j$. Before soft information gets hardened (that is, $\theta_h^s = 1$ is degenerate), this input is $\theta_h = \theta_h^h$ with a prior of $q_h = q_h^h$. As the span of hard information increases, the input becomes $\theta_h = \theta_h^h\theta_s^h$ with a prior of $q_h = q_h^h q_s^h = (1-\eta)q_h^h$; see (3). From the perspective of the hard signal technology $\mathcal{H}^j$, an increase in the span of hard information only changes the prior of θ_h, i.e., $q_h(\eta) = (1-\eta)\,q_h^h$, while keeping its precision α constant.[7]

1.2.2 Soft Signal

We further endow Bank A with an additional signal s to capture the idea that the bank is "specialized" in the borrower. Our preferred interpretation is that s represents a *soft-information*-based private signal, acquired through due diligence, on-site visits, or face-to-face interactions with the borrower. We assume that s is continuously distributed. Beyond mathematical convenience, a continuous distribution captures better the nature of soft information, which typically arises from borrower-specific research and allows for a more granular assessment of borrower quality.

As in the bottom panel of Figure 1, the soft signal technology is also a mapping $\mathcal{S}^A$ from the

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[7] Many existing papers that adopt the binary-fundamental-binary-signal structure, including Marquez (2002), conduct the comparative statics on the prior of the project quality, with the implicit assumption that the signal precision can be kept at a constant.

soft fundamental state $\theta_s \in \{0,1\}$ to a variable s that is correlated with θ_s. It is without loss of generality to work directly with the posterior probability of the soft state being favorable given the realization of the soft signal, that is,

$$s \equiv \Pr[\theta_s = 1|s] \in [0,1]. \tag{6}$$

Let $\phi(s)ds \equiv \mathbb{P}(s \in (s, s+ds))$ with $\int_0^1 \phi\left(s\right)ds = 1$ be the density function of s, which satisfies prior consistency $\int_0^1 s\phi\left(s\right)ds = q_s$. In our numerical examples, we consider $s = \Pr[\theta_s = 1|\theta_s + \epsilon]$ where $\epsilon \sim \mathcal{N}\left(0, 1/\tau\right)$. Here, τ, which is the precision of soft signal (hence analogous to α for the hard signal), captures the signal-to-noise ratio of Bank A's soft information technology.

Denote by $\phi_1\left(s\right) \equiv \phi\left(s|\,\theta_s = 1\right)$ the density of s conditional on $\theta_s = 1$. Using the shorthand notation $s \in ds$ for $s \in (s, s+ds)$, we have (using (6))

$$\phi_1\left(s\right) \equiv \frac{1}{ds}\mathbb{P}(s \in ds|\theta_s = 1) = \frac{\mathbb{P}(\theta_s = 1|s \in ds) \cdot \frac{1}{ds}\mathbb{P}(s \in ds)}{\mathbb{P}(\theta_s = 1)} = \frac{s \cdot \phi(s)}{q_s}. \tag{7}$$

Similarly, we can calculate

$$\phi_0\left(s\right) \equiv \phi\left(s|\,\theta_s = 0\right) = \frac{(1-s)\phi(s)}{1-q_s}. \tag{8}$$

As the soft signal s is the posterior expectation of θ_s and a higher value of s is "good news," $\phi_1(\cdot)$ and $\phi_0(\cdot)$ satisfy the strict Monotone Likelihood Ratio Property in Milgrom (1981).

1.2.3 Decisive Hard Signals and Parametric Assumptions.

For tractability, we assume that the hard signal is "decisive" for participation: Bank j participates only if it receives $h^j = H$. For the specialized Bank A, the hard signal serves as "pre-screening," in the sense that it rejects the borrower upon receiving an L signal, while upon an H signal it makes a pricing decision based on its soft signal s.[8] We therefore impose the following parameter restrictions to ensure that the hard signal is decisive.

Assumption 1. ***(Decisive Hard Signals)***

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[8] Bank A may also rejects the borrower by quoting $r = \infty$ when the soft signal s is sufficiently low. One could interpret the h-signal as "principal", determining whether to lend, and the s-signal as "supplementary" determining loan pricing. Alternatively, the principal signal reflects a credit screening result, while the supplementary signal resembles internal borrower ratings. This ranking highlights the key role of hard information for large banks in assessing new borrowers. As noted by Crawford, Pavanini, and Schivardi (2018), Italian large banks prioritize: (i) central bank data, (ii) Credit Register data, (iii) statistical methods, (iv) bank-specific codified soft information, (v) guarantees, and (vi) branch-level soft information (p. 1677).

1. *Bank A rejects the borrower upon an L hard signal, regardless of any soft signal s:*

$$q_h(1-\alpha)\overline{r} < (1-q_h)\alpha. \tag{9}$$

2. *Bank B is willing to participate if and only if its hard signal $h^B = H$:*

$$q\alpha\overline{r} > (q_h - q)\alpha + (1-q_h)(1-\alpha). \tag{10}$$

Assumption 1 says that the hard signal is sufficiently strong (informative) to serve as prescreening of loan applications for both lenders. Condition (9) states that it is not profitable for Bank A to lend upon receiving a hard signal L, even when it is the monopolist and quotes the highest possible interest rate $\overline{r}$, and the soft signal reveals that the soft fundamental θ_s is good with certainty. This implies that Bank B, which only has a hard signal, also chooses not to compete upon $h^B = L$. Analogously, Condition (10) states that upon $h^B = H$, Bank B is willing to lend at $\overline{r}$ if it is the monopolist lender. This condition implies that Bank A, with an additional soft signal, is willing to lend at $\overline{r}$ if it is the monopolist lender when $h^A = H$ and the realization of the soft signal is favorable enough.

For results in in Section 3.2 we also assume that there are more borrowers with favorable hard fundamentals in the population. This corresponds to the empirically relevant case where a better hard information technology leads to more loan application approvals.

Assumption 2. *The prior probability of the hard state being favorable satisfies $q_h > \frac{1}{2}$.*

1.3 Discussion of Assumptions

Multidimensional information, span, and precision. By incorporating multidimensional information, our model distinguishes span from precision—two dimensions of information quality with distinct economic consequences for credit market competition. The information span η, the main innovation in our analysis, captures the breadth of hard information in contrast to its precision, measured by α for the h-signal and τ for the s-signal. Recent technological advances have improved both. For example, early computing increased the precision of information (e.g. faster processing of bank statements) without expanding its scope. In contrast, Big Data and machine learning have increased the precision of information while, at the same time, broadening what qualifies as hard information by converting subjective or qualitative (soft) data into more objective or quantifiable (hard) metrics (e.g., Amazon's prediction of preferences). For recent evidence of hardening the soft information in the banking industry, see, for example, Hardik (2023).

Hard and soft information. Throughout the paper, we use the hardening of soft information as an example of technological change that can increase the span of hard information. We do this for two reasons: first, to fix ideas and provide a concrete setting in which our model applies; and second, this application is practically relevant in the context of the current "Big Data" environment. However, in the context of Stein (2002), who emphasizes the "verifiability" of hard information relative to soft information, verifiability plays no role in our framework. Instead, our results are broader and apply to any circumstance in which access to information is democratized and characteristics previously accessible only to a monopolist are now "learnable" by all market participants. In particular, we could relabel our analysis in terms of general and specialized information, rather than hard and soft.

Binary and symmetric hard signals. The binary structure of the hard signal reflects the coarse way hard information is often used in practice, e.g., credit scores are grouped into five bins despite being calculated on a 300–850 scale. However, our key insight—that information span differs from signal precision—holds under more general settings. We also assume both lenders share the same hard information technology, focusing on how different aspects of technological improvement affect relative market power under symmetric adoption.[9]

Endogenous information structure. Throughout, we take lenders information technologies as given in order to isolate how technological changes—rather than endogenous acquisition choices—shape competition and inference in credit markets. Our focus is on how the *structure* of available information, in particular the distinction between the span and precision, affects equilibrium outcomes. Endogenizing banks information acquisition would introduce additional margins without altering the mechanisms through which information span and precision operate in our setting. Accordingly, we abstract from acquisition choices to keep attention on the competitive effects of technological change.

2 Credit Market Equilibrium

We now define and solve for the credit market equilibrium, highlighting the economic mechanisms that arise from overlapping information spans and that are absent when information dimensions are disjoint.[10]

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[9] Blickle, He, Huang, and Parlatore (2025) consider a general (binary) information technology that is potentially asymmetric between lenders.

[10] The approach is related to methods developed in Blickle, He, Huang, and Parlatore (2025), who focus on the special case in which $\eta = 0$ to analyze information-based pricing with asymmetric information.

2.1 Credit Market Equilibrium Definition

Define the space of interest rate offers as $\mathcal{R} \equiv [0, \overline{r}] \cup \{\infty\}$; recall $\overline{r}$ is the exogenous return for the good project in (1) and ∞ captures not making an offer.[11]

For Bank A, we denote its pure strategy by $r^A(s) : [0,1] \to \mathcal{R}$,[12] which induces a distribution of its interest rate offers, denoted by $F^A(r) \equiv \Pr\left(r^A \le r\right)$ according to the underlying distribution of the soft signal. We refer to the endogenous support of interest rates when making an offer as the "support" of the interest rate distribution, even though loan rejection $(r = \infty)$ could also occur along the equilibrium path.

Conditional on a positive hard signal, Bank B randomizes its interest rate offers drawing from an endogenous distribution $F^B(r) \equiv \Pr\left(r^B \le r\right)$. Since the domain of offers includes $r = \infty$ (i.e., rejection), it is possible that $F^B(\overline{r}) = \mathbb{P}\left(r^B < \infty | h^B = H\right) < 1$.

The borrower picks the lower rate offered.[13] This implies that, conditional on $h^A = h^B = H$, if Bank B quotes $r^B < \infty$ its winning probability is $1 - F^A\left(r^B\right)$, which equals the probability that Bank A offers a rate that is higher than r^B. Note that this includes the event that Bank A rejects the borrower $(r^A(s) = \infty)$, presumably because of an unfavorable soft signal. If $r^A = r^B = \infty$, the borrower receives no loan.

Definition 1. (**Credit market equilibrium**) A competitive equilibrium in the credit market (with decisive hard signals) consists of the following strategies:

1. A lender j rejects the borrower or $r^j = \infty$ upon $h^j = L$ for $j \in \{A, B\}$; upon $h^j = H$,
    - i) Bank A offers $r^A(s) : [0,1] \to \mathcal{R} \equiv [0, \overline{r}] \cup \{\infty\}$ to maximize its expected profits given $h^A = H$ and s, which induces a distribution function $F^A(r) : \mathcal{R} \to [0,1]$;
    - ii) Bank B offers $r^B \in \mathcal{R}$ to maximize its expected profits given $g^B = H$, which induces a distribution function $F^B(r) : \mathcal{R} \to [0,1]$;
2. The borrower chooses the lowest offer $\min\{r^A, r^B\}$.

As is standard (e.g., Broecker, 1990), there exists an endogenous lower bound on interest rates $\underline{r} > 0$, so that the two distributions $F^j(\cdot)$, $j \in \{A, B\}$ share a common support $[\underline{r}, \overline{r}] \cup \{\infty\}$. The following lemma shows that the equilibrium strategies in our setting are well-behaved.

Lemma 1. ***(Well-behaved Equilibrium Strategies)*** *In any credit market equilibrium:*

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[11] Alternatively, $\overline{r}$ can also be interpreted as exogenous maximum (usury) interest rate. For instance, in Illinois the usury rate for most consumer loans is capped at 36% APR.

[12] We formally prove that in equilibrium Bank A uses pure strategies in Proposition 1.

[13] If two rates offered are equal then the borrow chooses one of them with equal probability, but the exact tie-breaking rule in these zero-measure events plays no role in the analysis.

a. *The two lenders' interest rate distributions* $F^j(\cdot)$, $j \in \{A, B\}$ *are smooth over* $[\underline{r}, \overline{r})$, *that is, no gaps and atomless, so that they admit well-defined density functions;*

b. *At most only one lender can have a mass point at* $\overline{r}$.

Proof. See Online Appendix B.1. □

2.2 Bank Profits and Optimal Strategies

Before computing the banks' profits and optimal strategies, we define the relevant probabilities and posterior beliefs, which are key elements in the equilibrium characterization.

2.2.1 Joint Distributions of Signals and Posterior

Denote by the ordered subscript $\{h^A h^B\} \in \{HH, HL, LH, LL\}$ the events of the corresponding hard signal realizations, where HL stands for Bank A's hard signal being H and Bank B's hard signal being L. Denote by $\overline{p}_{h^A h^B}$ the joint probability of any collection of hard signal realizations; here, the "bar" indicates taking the average over all possible soft signal realizations. For instance,

$$\overline{p}_{HH} \equiv \mathbb{P}\left(h^A = H, h^B = H\right) = q_h \alpha^2 + (1 - q_h)(1 - \alpha)^2. \tag{11}$$

Denote by $\overline{\mu}_{h^A h^B}$ the posterior probability of project success conditional on $h^A h^B$; for instance

$$\overline{\mu}_{HH} \equiv \frac{\mathbb{P}\left(h^A = H, h^B = H, \theta = 1\right)}{\mathbb{P}\left(h^A = H, h^B = H\right)} = \frac{q_h \alpha^2}{q_h \alpha^2 + (1 - q_h)(1 - \alpha)^2} q_s^s. \tag{12}$$

Upon competition, lenders also need to assess the joint probabilities of the hard and soft signals. Denote by $p_{h^A h^B}(s)\, ds \equiv \mathbb{P}\left(h^A, h^B, s \in ds\right)$ the joint probability of the two hard signals being $h^A h^B$ and $s \in ds$, that is, the soft signal s falls in the interval $(s, s + ds)$. Similarly, $\mu_{h^A h^B}(s)$ denotes the posterior probability of project success $\theta = 1$, conditional on the realizations of all signals:

$$\mu_{h^A h^B}(s) = \mathbb{P}\left(\theta = 1 | \, h^A, h^B, s\right) = \frac{\mathbb{P}\left(\theta = 1, h^A, h^B, s \in ds\right)}{\mathbb{P}\left(h^A, h^B, s \in ds\right)}. \tag{13}$$

Under the multiplicative structure in (3), project success $\theta = 1$ implies that $\theta_h = \theta_s = 1$, which allows us to derive the joint probability of $\mathbb{P}\left(\theta = 1, h^A, h^B, s \in ds\right)$ as

$$p_{h^A h^B}(s)\, \mu_{h^A h^B}(s) = \mathbb{P}(\theta = 1) \cdot \mathbb{P}\left(h^A \,|\theta_h = 1\right) \cdot \mathbb{P}\left(h^B \,|\theta_h = 1\right) \cdot \phi\left(s | \, \theta_s = 1\right). \tag{14}$$

2.2.2 Bank A's Strategy

Suppose that Bank A observes a positive hard signal $h^A = H$ and a soft signal s. If it exits the lending market by quoting $r = \infty$, its expected profits are $\pi^A(r = \infty, s) = 0$. If it offers a rate $r \in [\underline{r}, \overline{r}]$, Bank A's expected profits are

$$\pi^A(r, s) \equiv p_{HH}(s)\left[1 - F^B(r)\right][\mu_{HH}(s)(1 + r) - 1] + p_{HL}(s)[\mu_{HL}(s)(1 + r) - 1]. \tag{15}$$

The first term captures the case where both banks receive positive signals, and Bank A wins with probability $1 - F^B(r)$; the second term accounts for the case where Bank B receives a negative hard signal and exits. Since Bank B randomizes its strategy upon $h^B = H$, from Bank A's perspective, winning the price competition is not informative about the borrower's quality (so the belief about borrower quality $\mu_{HH}(s)$ in the first term in (15) is unaffected by $1 - F^B(r)$). However, whether B participates or not informs A's expected quality of the borrower, as captured by $\mu_{HH}(s)$ in the first term and $\mu_{HL}(s)$ in the second term in (15).

Given the profit function defined above, Bank A's optimal interest rate offer is $r^A(s) \equiv \arg\max_{r \in \mathcal{R}} \pi^A(r, s)$, which is decreasing in s (see Proposition 1), hits the interest rate cap $\overline{r}$ when the soft signal worsens (at some threshold $\hat{s}$), and in general will jump to ∞ for sufficiently low s (at another threshold x). In the interior range where $r^A(s) \in [\underline{r}, \overline{r})$, that is for $s \in (\hat{s}, 1]$, we define $s^A(r) \equiv r^{A(-1)}(r)$ as the realization of the soft signal s that induces bank A to offer a rate r. This mapping plays a crucial role in Bank B's beliefs about the soft fundamental state.

2.2.3 Bank B's Strategy

For Bank B, the rate offered by Bank A conveys information about the soft signal realization, subjecting Bank B to an additional Winner's Curse. More specifically, besides the possibility of the specialized lender A's unfavorable hard signal, the non-specialized lender B who wins the price competition also infers that $r^A(s) > r^B$, which implies $s < s^A\left(r^B\right)$ (recall $r^A(s)$ is decreasing). Taking these unfavorable inferences into account, Bank B's lending profits when quoting r are

$$\pi^B(r) \equiv \int_0^{s^A(r)} p_{HH}(t)[\mu_{HH}(t)(r + 1) - 1]\,dt + \overline{p}_{LH}[\overline{\mu}_{LH}(r + 1) - 1]. \tag{16}$$

The first term in (16) captures the event of Bank A seeing $h^A = H$ and competing, while the second term considers $h^A = L$ and Bank A not participating. Bank B infers the project's quality based on the event of "winning the borrower"—which occurs when Bank A receives an unfavorable soft signal realization $s < s^A(r)$. Importantly, this inference, which is informative about θ_s and θ_h when the spans of hard and soft information overlap, is at the heart of our analysis of how different information technologies affect the credit market equilibrium.

To see this more clearly, using (14) one can write Bank B's profits in (16) as

$$\pi^B(r) = \underbrace{(1+r)\cdot\int_0^{s^A(r)} q\alpha^2\phi_1(t)\,dt}_{\text{expected revenue upon competition}} - \underbrace{\int_0^{s^A(r)} p_{HH}(t)\,dt}_{\text{expected cost upon competition}} + \underbrace{\overline{p}_{LH}\left[\overline{\mu}_{LH}(\overline{r}+1)-1\right]}_{\text{expected payoff when } A \text{ exits}}. \tag{17}$$

The first term reflects the expected revenue of lending at r, which is loan repayment $1+r$ multiplying the updated probability of a good project $\int_0^{s^A(r)} q\alpha^2\phi_1(t)\,dt$. As we discuss in the next section, the span η only affects the expected revenue through the upper integration limit $s^A(r)$, whereas α also affects revenue directly through screening $(q\alpha^2\phi_1(t))$.

The second term is the expected cost of lending to borrowers with low soft signal realizations $(s < s^A(r))$. This cost—derived largely from the residual uncertainty Bank B has about θ_s^s—represents the Winner's Curse in competition. Define Bank A's expected quality of the only-soft state, conditional on competition, to be

$$z_s^s(s) \equiv \mathbb{E}\left[\theta_s^s \middle| HHs\right].$$

Bank B is concerned of a low z_s^s, reflecting a low soft signal realization (one can show that $z_s^s(s)$ strictly increases in s). Thus, the Winner's Curse depends on the left tail of Bank B's perceived distribution of z_s^s; specifically, given a cutoff $\hat{z}$, the left tail distribution is given by

$$\Pr\left(z_s^s(t) < \hat{z}\right) = \Pr\left(t < z_s^{s(-1)}(\hat{z})\right) = \int_0^{z_s^{s(-1)}(\hat{z})} p_{HH}(t)\,dt. \tag{18}$$

Here, $z_s^{s(-1)}(\hat{z})$ is the realization of the soft signal that induces a belief $\hat{z}$ about θ_s^s for Bank A. Since (18) has the same structure as the second term in (17), this confirms that the cost reflects the residual uncertainty that Bank B faces toward the only-soft state θ_s^s.

Finally, the last term in (17) is the payoff when Bank A exits upon $h^A = L$. In sum, after observing $h^B = H$, Bank B chooses its strategy $F^B(\cdot)$ to maximize its expected payoff

$$\max_{F^B(\cdot)} \int_{\mathcal{R}} \pi^B(r)\,dF^B(r). \tag{19}$$

Since profit-maximizing Bank B plays mixed strategies, $\pi^B = \pi^B(r)$ for all $r \in [\underline{r}, \overline{r}]$.

2.3 Credit Market Equilibrium Characterization

We first take Bank B's equilibrium profits π^B as given to derive lenders' strategies. Similar to Milgrom and Weber (1982), it is relatively easy to solve for Bank A's equilibrium strategy by invoking Bank B's indifference condition, i.e., Bank B makes the same profit across all rates on the support $[\underline{r}, \overline{r}]$. Plugging in $r = r^A(s)$ in Bank B's profit in (16), and using $\pi^B(r) = \pi^B, \forall r \in [\underline{r}, \overline{r}]$, we have

$$\pi^B = \underbrace{\left[\int_0^s p_{HH}(t)\mu_{HH}(t) + \overline{p}_{LH}\overline{\mu}_{LH}\right]}_{\text{borrowers who repay}} \left(1 + r^A(s)\right) - \underbrace{\left(\int_0^s p_{HH}(t)dt + \overline{p}_{LH}\right)}_{\text{lending cost}}. \tag{20}$$

Solving for $r^A(s)$ yields (26) in Proposition 1 below, which further takes into account the interest rate cap $\overline{r}$.

Although the derivation of Bank B's equilibrium strategy is more involved, the underlying logic is simple: Bank B's equilibrium strategy needs to support $r^A(\cdot)$ in (26) as Bank A's optimal strategy. We summarize the key steps below, drawing on arguments from Blickle, He, Huang, and Parlatore (2025), and modifying them to accommodate overlapping hard and soft states.

Let $Q^A(r;s)$ and $Q^B(r)$ denote the total "effective" borrower (who can repay) of lenders A and B, respectively, when they offer an interest rate r. Note, $Q^A(r;s)$ depends on s because Bank A also knows the soft signal s (while Bank B does not):

$$Q^A(r;s) = p_{HH}(s)\mu_{HH}(s)\left[1 - F^B(r)\right] + p_{HL}(s)\mu_{HL}(s), \tag{21}$$

$$Q^B(r) = \int_0^{s^A(r)} p_{HH}(t)\mu_{HH}(t)dt + \overline{p}_{LH}\overline{\mu}_{LH}. \tag{22}$$

Maximizing (15) one can show that Bank A's first-order condition (FOC) balances the higher probability of winning ($Q^{A\prime}(r;s)dr$) when cutting its rate against a lower payoff from served borrowers ($Q^A(r;s)dr$):

$$\underbrace{Q^{A\prime}(r;s) \cdot \left(1 + r - \frac{1}{\mu_{HH}(s)}\right)}_{\text{MB on marginal borrower type}} = \underbrace{-Q^A(r;s)}_{\text{MC on existing borrower types}}. \tag{23}$$

The term inside the parentheses on the left-hand side in (23) concerns the marginal borrower with quality $\mu_{HH}(s)$. Due to imperfect screening, A incurs a total lending cost of $\frac{1}{\mu_{HH}(s)}$ to serve each good borrower who repays $1 + r$. Similarly, to maximize (16), B's FOC balances the change in its borrowers ($Q^{B\prime}(r)$) against the gain from existing borrowers ($-Q^B(r)$):

$$\underbrace{Q^{B\prime}(r) \cdot \left(1 + r - \frac{1}{\mu_{HH}(s^A(r))}\right)}_{\text{MB on marginal borrower type}} = \underbrace{-Q^B(r)}_{\text{MC on existing borrower types}} . \tag{24}$$

Here, Bank B who quotes r infers the quality of the marginal borrower $\mu_{HH}(s^A(r))$ based on Bank A's equilibrium strategy; see Appendix B.2 for detailed derivations of (23) and (24).

Importantly, both lenders are competing for the same marginal borrower (type) at any interest rate $r \in [\underline{r}, \overline{r})$, that is, $1 + r - \frac{1}{\mu_{HH}(s^A(r))}$. In fact, evaluating (23) at the equilibrium borrower type $s = s^A(r)$ and combining it with (24), we arrive at the following:

$$\frac{Q^{A\prime}\left(r; s^A(r)\right)}{Q^A\left(r; s^A(r)\right)} = \frac{Q^{B\prime}(r)}{Q^B(r)} \Leftrightarrow \frac{d}{dr}\left[\frac{Q^A(r; s)}{Q^B(r)}\right]\Bigg|_{s=s^A(r)} = 0. \tag{25}$$

As lenders balance the same marginal borrower's payoff with the payoff gain from existing customers, in equilibrium, their existing effective customers should change proportionally, as shown in (25). Using this, one can solve for Bank B's equilibrium strategy in Proposition 1.

Lastly, Bank B's equilibrium profit π^B depends on which lender first breaks even when quoting $\overline{r}$ as s decreases: either Bank B breaks even with $\pi^B = 0$, or Bank A breaks even upon $s = \hat{s}$, which renders $\pi^B > 0$. The next proposition characterizes the credit market equilibrium in closed form.

Proposition 1. ***(Credit Market Equilibrium)*** *In the credit market equilibrium, Bank A follows a pure strategy as in Definition 1. In this unique equilibrium, lenders reject borrowers upon a negative hard signal realization $h^j = L$ for $j \in \{A, B\}$. Otherwise (i.e., when $h^j = H$), their strategies are characterized as follows:*

1. *Bank A with soft signal s offers*

$$r^A(s) = \begin{cases} \min\left\{\frac{\pi^B + \int_0^s p_{HH}(t)dt + \overline{p}_{LH}}{\int_0^s p_{HH}(t)\cdot\mu_{HH}(t)dt + \overline{p}_{LH}\overline{\mu}_{LH}} - 1, \overline{r}\right\}, & \text{for} \quad s \in [x, 1] \\ \infty, & \text{for} \quad s \in [0, x). \end{cases} \tag{26}$$

The equation pins down $\underline{r} = r^A(1)$. For $s \in (\hat{s}, 1]$ where $\hat{s} = \sup s^A(\overline{r})$, $r^A(\cdot)$ is strictly decreasing with its inverse function $s^A(\cdot) = r^{A(-1)}(\cdot)$.

2. *Bank B makes an offer with cumulative probability given by* ($\mathbf{1}_{\{X\}} = 1$ *if X holds)*

$$F^B(r) = \begin{cases} 1 - \frac{\int_0^{s^A(r)} t\phi(t)dt}{q_s}, & \text{for} \quad r \in [\underline{r}, \overline{r}), \\ 1 - \mathbf{1}_{\{\pi^B=0\}} \cdot \frac{\int_0^{\hat{s}} t\phi(t)dt}{q_s}, & \text{for} \quad r = \overline{r}. \end{cases} \tag{27}$$

When $\pi^B = 0$, $F^B(\overline{r}) = F^B(\overline{r}^-) \leq 1$ *is the probability that Bank B makes the offer (and with probability* $\frac{1}{q_s}\int_0^{\hat{s}} t\phi(t)\,dt$ *it withdraws by quoting* $r^B = \infty$*); when* $\pi^B > 0$, $F^B(\overline{r}) = 1$ *and there is a point mass of* $\frac{1}{q_s}\int_0^{\hat{s}} t\phi(t)\,dt$ *at* $\overline{r}$.

3. *The equilibrium Bank B's profit is given by*

$$\pi^B = \max\left(\hat{\pi}^B\left(\overline{r}; s^A(\overline{r}) = s_A^{be}\right), 0\right), \tag{28}$$

where s_A^{be} *is the unique solution to* $\hat{\pi}^A\left(\overline{r}|s_A^{be}; F^B(\overline{r}) = \int_{s_A^{be}}^1 \frac{s\phi(s)dt}{q_s}ds\right) = 0$ *with auxiliary functions* $\hat{\pi}^B(\cdot;\cdot)$ *and* $\hat{\pi}^A(\cdot|\cdot;\cdot)$ *defined in Online Appendix B.2.*

Proof. See Appendix A.1 for proof outline, and Online Appendix B.2 for proof. ☐

In Proposition 1, point 1) shows that Bank A offers a higher interest rate as the soft signal deteriorates. Threshold $\hat{s}$ is the highest soft signal where Bank A offers the interest rate cap $\overline{r}$, while threshold $x \leq \hat{s}$ is where Bank A breaks even when offering $\overline{r}$—$\pi^A(\overline{r}, x) = 0$. If $\hat{s} > x$, Bank A holds some monopolistic power as it makes positive profits when offering the monopolistic rate $\overline{r}$. If $\hat{s} = x$, Bank A breaks even when offering $\overline{r}$.

Bank B's equilibrium strategy is characterized in point 2). If $\pi^B = 0$, Bank B randomly chooses to exit the credit market upon receiving a positive hard signal, reflected by $F^B(\overline{r}) < 1$. If $\pi^B > 0$, Bank B always participates upon receiving a positive hard signal ($F^B(\overline{r}) = 1$) and places some mass at the interest rate cap $\overline{r}$. In the first case, the Winner's Curse is strong enough to deter Bank B's participation, granting Bank A monopolistic power. In the second, the Winner's Curse is weaker, and Bank B can make a profit by offering $\overline{r}$ and winning the borrower only when Bank A receives a soft signal $s < x$.

The equilibrium strategies in Proposition 1 depend on the equilibrium profits π^B, and point 3) shows that π^B is pinned down by model primitives (subject to solving for one endogenous constant s_A^{be}). In the *zero-weak* equilibrium $\pi^B = 0$ and only Bank A puts mass on $\overline{r}$. In the *positive-weak* equilibrium $\pi^B > 0$, and only Bank B does so. These outcomes are consistent with point b) in Lemma 1—otherwise, lenders would undercut each other at the interest rate cap $\overline{r}$.

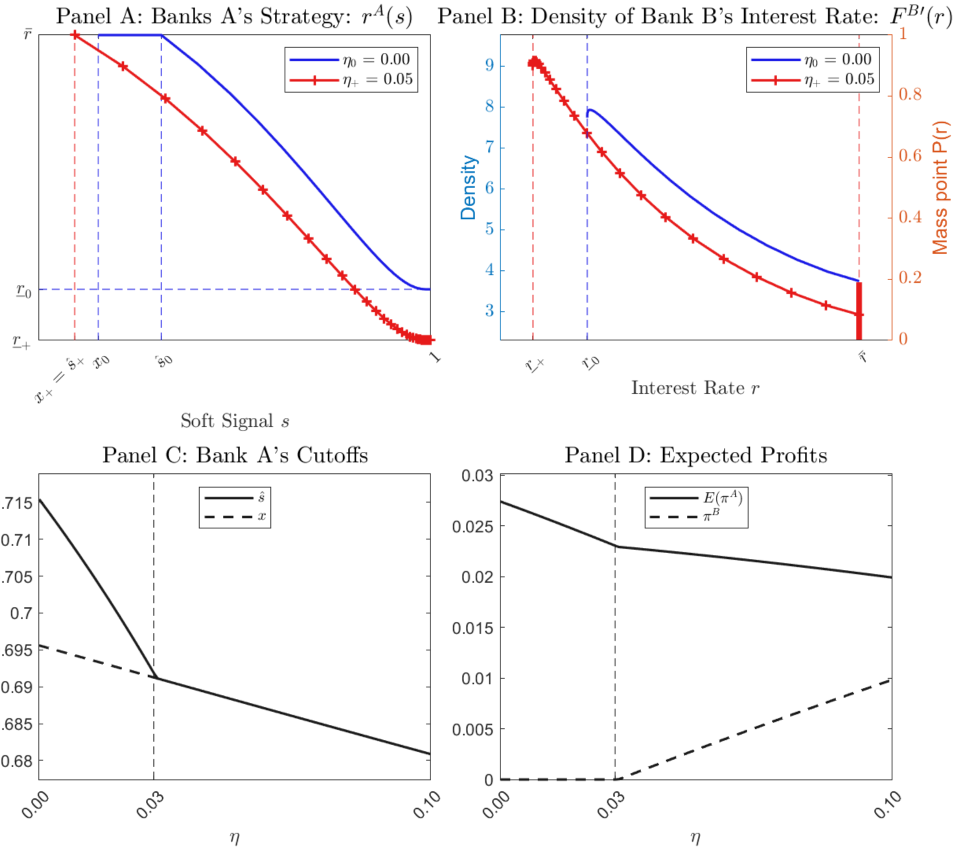


Figure 2: **Equilibrium strategies and profits for information span η.** Panel A depicts $r^A(s)$ as a function of s and Panel B plots $F^{B\prime}(r)$ as a function of r; strategies for $\eta_+ = 0.05$ are depicted in red with markers while strategies with $\eta_0 = 0$ are depicted in blue. Panel C depicts Bank A's thresholds $\hat{s} = \sup s^A(\bar{r})$ and $x = \sup s^A(\infty)$, and Panel D depicts the expected profits for two lenders, both as a function of η. Parameters: $\bar{r} = 0.36$, $q = 0.72$, $q_s = 0.9$, $\alpha = 0.7$, and $\tau = 1$.

2.4 Credit Market Equilibrium under Hardening Soft Information

To fix ideas, we illustrate numerically how the information span η affects the credit market competition equilibrium in Figure 2. We interpret this increase in information span as the outcome of hardening soft information, which makes information—once held only specialists—accessible to non-specialists too. For ease of exposition, we assume that Bank A's soft signal s is obtained from observing a noisy version of θ_s, i.e., $\theta_s + \epsilon$, so that $s = \mathbb{E}[\theta_s | \theta_s + \epsilon]$. Here, $\epsilon \sim \mathcal{N}(0, 1/\tau)$ indicates white noise, with the precision parameter τ capturing the signal-to-noise ratio of Bank A's soft information technology.

The top two panels in Figure 2 plot both lenders' pricing strategies conditional on making an offer, with Panel A plotting $r^A(s)$ as a function of s for Bank A and Panel B the density $F^{B\prime}(r)$ as a function of r for Bank B. We plot the equilibrium pricing strategies for two levels of information span η: the baseline $\eta_0 = 0$, and a higher $\eta_+ = 0.05$. A positive (zero) weak equilibrium arises when η is relatively high (low), hence the subscript "+" for the higher η.

As hardening soft information (a higher η) reduces the informational asymmetries, Bank B becomes more aggressive as its distribution of offered rates shifts downward (panel B), resulting in a smaller equilibrium lower bound $\underline{r}_+ < \underline{r}_0$. In response to the more aggressive bidding by Bank B, we see that the entire curve $r^A(s)$ shifts downward.

Panel C plots the two soft signal cut-offs for the specialized Bank A, i.e., $\hat{s} \equiv \sup s^A(\overline{r})$ at which it starts quoting $\overline{r}$ and $x \equiv \sup s^A(\infty)$ at which it starts rejecting the borrower. For a sufficiently large η, $\hat{s}$ and x coincide, reflecting a zero probability mass on the interest rate cap $\overline{r}$. Finally, Panel D plots the expected profits—$\mathbb{E}(\pi^A)$ and π^B—for both lenders; when η increases, the non-specialized lender becomes relatively stronger, leading to a strictly positive π^B as shown in Panel D.

These panels show how hardening soft information "levels the playing field." Intuitively, for a small span η, the Winner's Curse is too strong as to deter full participation by Bank B and the equilibrium is zero-weak, where the specialized Bank A places a point mass on $\overline{r}$ (when $s \in (x, \hat{s})$, as shown in Panel C). In contrast, as soft information gets hardened and η is large enough, the Winner's Curse faced by the non-specialized Bank B due to the opponent's soft signals becomes relatively minor. This intensifies competition, and leads to a positive-weak equilibrium, where the non-specialized Bank B becomes profitable—so that it enjoys some "local monopoly power" by placing a point mass on $\overline{r}$. We explore these mechanisms formally in the next section.

3 Span vs. Precision

A key advantage of our model is that it allows us to distinguish between aspects of information technology. Our model isolates the span and precision of information, allowing us to examine their distinct effects on the credit market equilibrium.

3.1 Screening Technology

First, we show that both an increase in the span of hard information and in its precision are indeed technological improvements in the sense that they increase the hard signals' screening quality for the project quality θ. To see this, let

$$z(H) \equiv \mathbb{E}\left[\theta = 1 \middle| h^B = H\right] = \frac{\mathbb{P}\left(\theta = 1, h^B = H\right)}{\mathbb{P}\left(h^B = H\right)}. \tag{29}$$

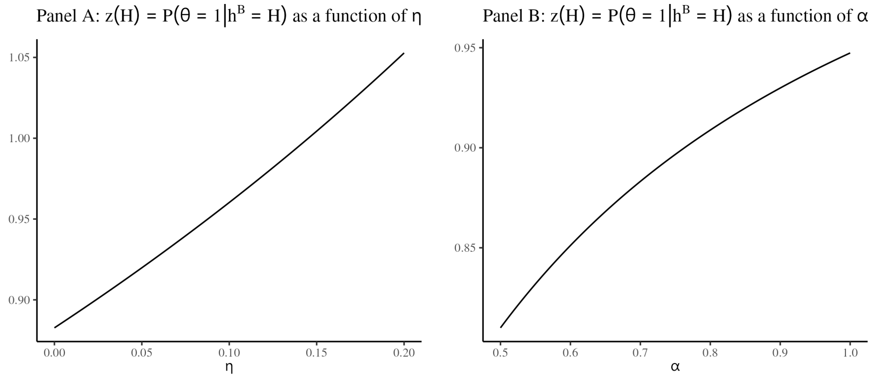


Figure 3: $z(H)$ **and information technology.** Panel A plots $z(H)$ as a function of η and Panel B plots $z(H)$ as a function of α. Parameters: $\alpha = 0.85, \eta = 0.05, q_h^h = 0.9$, and $q_s = 0.9$.

This expression represents the expected project quality for Bank B upon receiving a positive hard signal.[14] As Lemma 2 below shows, $z\left(H\right)$ is increasing in both η and α. On one hand, a larger span η implies that a positive hard signal indicates more fundamental states to be favorable, and hence increases the expected project quality. On the other hand, a higher precision makes the realized high hard signal more informative about the underlying fundamental, which also leads to a higher expected project quality. This can be seen in Figure 3.

Lemma 2. ***(Improved Screening)*** *The posterior project quality upon receiving a high hard signal, $z\left(H\right)$, is increasing in the span of hard information η and in its precision α.*

Proof. See Appendix A.2. ☐

3.2 Learning upon Winning

While increases in the span and precision of hard information similarly improve overall screening efficiency regarding θ, they can have opposite effects on Bank B's residual uncertainty about the only-soft state θ_s^s. Since Bank B's beliefs about θ_s^s after receiving a positive hard signal determine the severity of the Winners Curse it faces, this residual uncertainty is key to understanding how changes in information technology shape credit market competition.

We first compute the expected quality of the only-soft state when both lenders get a high hard signal (HH, or the event of competition) and the specialized bank gets a soft signal s:

$$z_s^s\left(s\right) \equiv \mathbb{E}\left[\theta_s^s = 1 | HHs\right] = \frac{\mathbb{P}\left(\theta_s^s = 1, h^A = h^B = H, s\right)}{\mathbb{P}\left(HHs\right)}. \tag{30}$$

---

[14] Given the symmetry in the hard signals for the lenders, $z\left(H\right)$ also measures the posterior quality for Bank A after only observing a high hard signal.

Appendix A.3.1 gives the expression for (30), which depends on both the span η and the precision α of hard information.

In (30), $z_s^s(s)$ captures the expected quality of θ_s^s for Bank A when it observes signal s upon competition. Bank B, however, does not observe the soft signal realization s. Hence, Bank B is concerned about winning the competition for the borrower, only because the better-informed Bank A has received an unfavorable soft signal (Winner's Curse). More specifically, similar to the reasoning provided in Section 2.2.3 after (18), Bank B (conditional on $h^B = H$ and hence compete) cares about the left tail of the distribution of z_s^s, which, fixing any cutoff $\hat{z}$, is given by

$$\begin{aligned}\mathbb{P}\left(z_s^s\left(s\right) < \hat{z}|h^B = H\right) = \mathbb{P}\left(s < z_s^{s(-1)}\left(\hat{z}\right)|h^B = H\right) &= \int_0^{z_s^{s(-1)}(\hat{z})} \frac{p_{HH}\left(s\right)}{\mathbb{P}\left(h_B = H\right)} ds \\ &= \underbrace{\mathbb{P}\left(HH|\, h^B = H\right)}_{\text{prob. of facing competition}} \int_0^{\underbrace{z_s^{s(-1)}\left(\hat{z}\right)}_{\text{inferring } \theta_s^s}} \underbrace{\phi\left(s\,|HH\right)}_{s \text{ distribution in competition}} ds. \qquad (31)\end{aligned}$$

There are three channels through which information technology can affect the expression in (31). The first channel is by changing the probability of Bank B facing competition upon receiving a high hard signal. The second channel is through Bank B's inference about the only-soft fundamental θ_s^s upon winning. The third channel is by affecting the beliefs about the distribution of the soft signal upon competition. We analyze each of these effects below.

3.2.1 Probability of Facing Competition

The less informed Bank B cares about the realization of the soft signal—and the associated Winner's Curse—only when it expects to face competition for the borrower. That is, when Bank A receives a positive hard signal, given that Bank B receives one too. The first term in (31) captures the probability that Bank B assigns to this event upon observing $h^B = H$. Interestingly, the span and precision of hard information have opposite effects on this term, as Lemma 3 below show.

Lemma 3. ***(Beliefs about Competition)*** *The span and precision of hard information have opposite effects on* $\mathbb{P}\left(HH|\, h^B = H\right)$. *More specifically,*

$$\frac{d\mathbb{P}\left(HH|\, h^B = H\right)}{d\eta} < 0 \quad \textit{and} \quad \frac{d\mathbb{P}\left(HH|\, h^B = H\right)}{d\alpha} > 0.$$

Proof. See Appendix A.3.2. □

Intuitively, as the information span η increases, the hard signal reflects a broader range of fundamentals. Under a multiplicative structure, this makes it less likely for either bank to receive a positive hard signal since it requires more fundamental states to be favorable. As a result, the two hard signals become less correlated, reducing the probability that Bank B faces competition. Because $\mathbb{P}\left(HH|\, h^B = H\right)$ scales the left tail in (31), this reduction in competition mitigates the Winner's Curse on the soft signal faced by Bank B.

In contrast, as the precision of the hard signal increases, the two hard signals become more correlated, increasing the probability that Bank A also receives a positive signal given that Bank B does. In the extreme case where $\alpha = 1$, the hard signals are perfectly correlated and effectively public. Thus, a higher precision increases Bank B's perceived likelihood of facing competition upon receiving a positive signal, intensifying the Winner's Curse. Panel I in Figure 4 illustrates this comparison by plotting $\mathbb{P}\left(HH|\, h^B = H\right)$ against both η and α.

3.2.2 Inference from Winning

The second effect relates to the residual uncertainty about θ_s^s, captured by the integration limit $z_s^{s(-1)}\left(\hat{z};\eta,\alpha\right)$ in (31). This threshold represents the value of the soft signal received by Bank A that would induce a posterior belief $z_s^s = \hat{z}$ about θ_s^s. As shown in the lemma below, η and α have opposite effects on this threshold.

Lemma 4. ***(Inference about*** θ_s^s***)*** *The span and precision of hard information have opposite effects on* $z_s^{s(-1)}\left(\hat{z};\eta,\alpha\right)$*. More specifically,*

$$\frac{dz_s^{s(-1)}\left(\hat{z};\eta,\alpha\right)}{d\eta} < 0, \quad \textit{while} \quad \frac{dz_s^{s(-1)}\left(\hat{z};\eta,\alpha\right)}{d\alpha} \geq 0 \textit{ if } z_s^{s(-1)}(\hat{z}) < q_s.$$

Proof. See Appendix A.3.3. ☐

Panel II in Figure 4 illustrates this result. As the span of hard information η increases, the overlap between hard and soft fundamentals grows. Therefore fewer characteristics must be favorable for the "only-soft" fundamentals θ_s^s to be positive, and all else equal Bank B becomes more optimistic about θ_s^s. This implies that a lower soft signal suffices to induce the same level of expected quality $\hat{z}$, that is, $z_s^{s(-1)}\left(\hat{z};\eta,\alpha\right)$ decreases in η.

In contrast, as the precision α increases, the hard signal becomes more informative. Upon receiving a positive hard signal, Bank B is more certain that the "overlapping" characteristics covered by hard and soft information, that is, θ_s^h, are favorable. Since the soft signal s reflects $\theta_s \equiv \theta_s^h \theta_s^s$, Bank B updates its beliefs about θ_s^s downward. That is, a higher soft signal s is required to maintain the same expected quality of θ_s^s. Consequently, $z_s^{s(-1)}\left(\hat{z};\eta,\alpha\right)$ increases in α. This effect arises only when the hard and soft states are correlated; if $\eta = 0$, Bank B's beliefs about θ_s^s are independent of α.

3.2.3 Beliefs about Soft Signal upon Competition

Finally, the strength of the Winner's Curse faced by Bank B upon competition depends on the distribution of the soft signals received by Bank A. This is captured by $\phi\left(s|\, HH\right)$ in (31), which represents the density of s conditional on two positive hard signals:

$$\phi\left(s\,|HH\right)=\phi\left(s\right)+\left[\frac{\eta q}{\frac{(1-\alpha)^{2}}{2\alpha-1}+q_{h}^{h}\left(1-\eta\right)}\right]\cdot\left[\phi_{1}\left(s\right)-\phi_{0}\left(s\right)\right]. \tag{32}$$

Here, $\phi\left(s\right)$ is the unconditional distribution of the soft signal s, and $\phi_{\theta_s}\left(s\right)$, which we derive in (7) and (8), is the distribution of s conditional on the realization of soft fundamental θ_s.

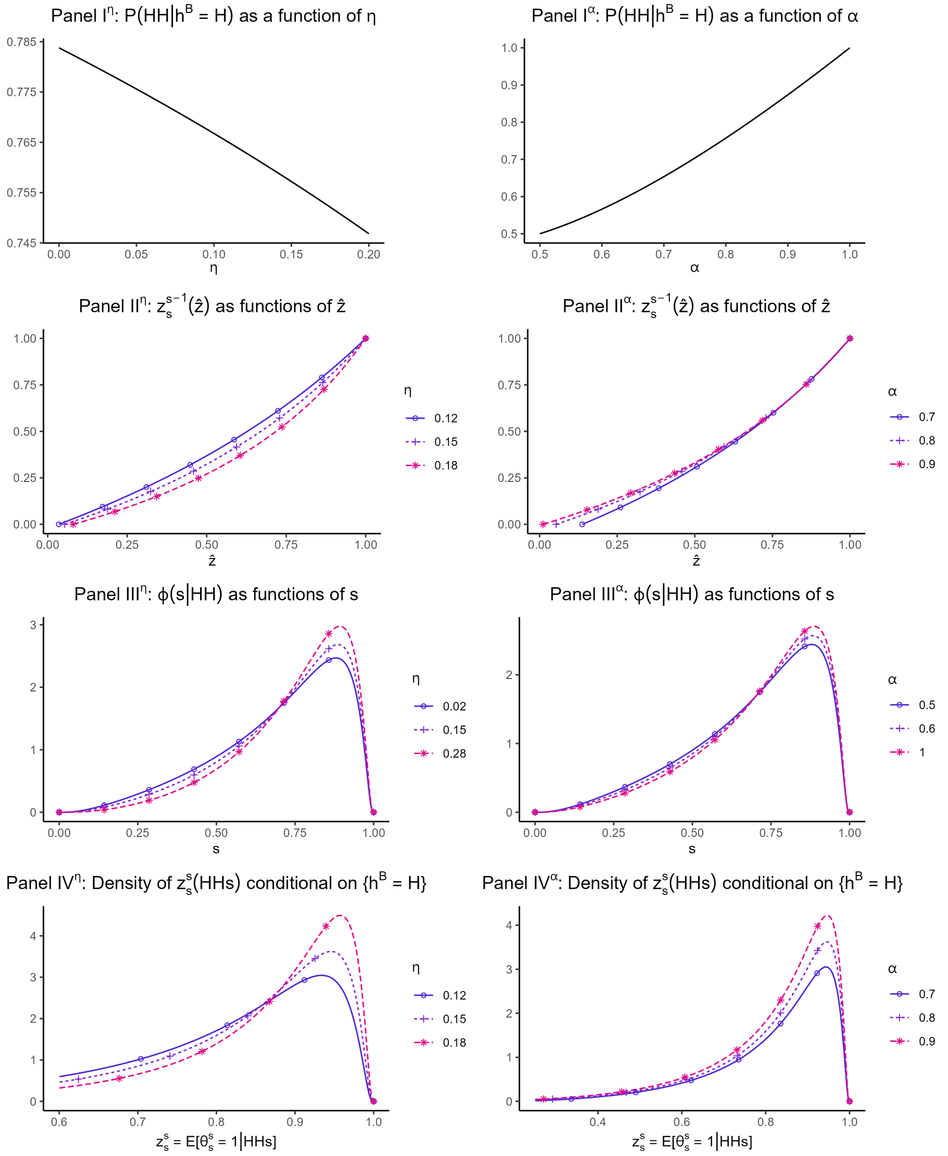
Lemma 5. ***(Conditional Distribution of the Soft Signal)*** *The distribution of the soft signal conditional on both lenders receiving a positive hard signal shifts to the right as the span and the precision of hard information increase. Formally, for soft signal below its prior mean $s < q_s$, we have*

$$\frac{d\phi\left(s|\, HH\right)}{d\eta}<0 \quad \text{and} \quad \frac{d\phi\left(s|\, HH\right)}{d\alpha}<0.$$

Proof. See Appendix A.3.4. □

The lemma above shows that when both banks receive H, the realization of s is more likely to be higher as the span and precision of hard information increase. Panel III in Figure 4 illustrates this result. When the hard information broadens (η increases), the hard signal becomes informative about more soft fundamentals. Thus, when both hard signals are positive, it is more likely that θ_s is also positive, and the conditional distribution $\phi\left(s|\, HH\right)$ puts more weight on $\phi_1\left(s\right)$. Given the monotone likelihood ratio property, we know $\phi_1\left(s\right)-\phi_0\left(s\right)<0$ for low values of s ($s < q_s$), implying that the soft signal is more likely to take higher values. Similarly, as the precision α increases, the hard signals provide a better assessment, including of the overlapping fundamentals in θ_s^h. This, in turn, makes it more likely for the soft fundamental to be positive upon HH.

It is worth emphasizing that the effects of η and α on $\phi\left(s|\, HH\right)$ do not explain the distinct impacts that span and precision have on the credit market (see Section 3.2.4). In fact, they resemble the effects of overall screening technology discussed in Section 3.1 and reflect the symmetric technological improvements for both lenders. Whether by increasing the correlation between the hard and soft signals (via greater span) or by making hard information "more public" (via higher precision), both dimensions raise the expected quality of the soft fundamentals upon competition.



Panel I$^{\eta}$: $P(HH|h^B = H)$ as a function of η

Panel I$^{\alpha}$: $P(HH|h^B = H)$ as a function of α

Panel II$^{\eta}$: $z_s^{s-1}(\hat{z})$ as functions of $\hat{z}$

Panel II$^{\alpha}$: $z_s^{s-1}(\hat{z})$ as functions of $\hat{z}$

Panel III$^{\eta}$: $\phi(s|HH)$ as functions of s

Panel III$^{\alpha}$: $\phi(s|HH)$ as functions of s

Panel IV$^{\eta}$: Density of $z_s^s(HHs)$ conditional on $\{h^B = H\}$

Panel IV$^{\alpha}$: Density of $z_s^s(HHs)$ conditional on $\{h^B = H\}$

Figure 4: **Conditional probability and density of posterior means of fundamentals.** Panel I$^{\eta}$ and I$^{\alpha}$ depict $\mathbb{P}\left(HH|h^B = H\right)$ as functions of η and α. Panel II$^{\eta}$ and II$^{\alpha}$ plots $s = z_s^{s-1}(\hat{z})$ as functions of $\hat{z}$, for three different levels of η and α respectively. Panel III$^{\eta}$ and III$^{\alpha}$ depict the density of signal s conditional on $h^A = h^B = H$, and Panel IV$_{\eta}^{B}$ and IV$_{\alpha}^{B}$ depict the density of posterior z_s^s. Baseline parameters: $\alpha = 0.8, \eta = 0.15, \tau = 1, q_h^h = 0.9$, and $q_s = 0.7$.

3.2.4 Overall Effect

Although span and precision shift the conditional distribution of the soft signal under competition (and the overall screening efficiency) in the same direction (Section 3.2.3), they operate through distinct economic mechanisms, and this distinction lies underneath the opposite effects of span and precision in the two previous sections (Section 3.2.1 and 3.2.2). Our discussion highlights that Bank B is mostly concerned with the signal received by its opponent because it reveals information about θ_s^s, for which Bank B lacks private information. The following theorem formally states this result under mild conditions, with illustration given by Panel IV of Figure 4.

Theorem 1. ***(Span and Precision on Winner's Curse on Only-Soft State)*** *The span and precision of hard information have opposite effects on Bank B's perceived left tail of the distribution of* z_s^s. *Formally, for all* z *such that* $z_s^{s(-1)}(z) < q_s$, *we have*

$$\frac{d\mathbb{P}\left(z_s^s \leq z |\, h^B = H\right)}{d\eta} < 0, \quad \textit{while} \quad \frac{d\mathbb{P}\left(z_s^s \leq z |\, h^B = H\right)}{d\alpha} > 0 \textit{ if } q_s^s < \frac{2q_h - 1}{q_h^s\left(4q_h^h - 2q_h - 1\right)}.$$

Proof. See Appendix A.4. ☐

We need condition $q_s^s < \frac{2q_h-1}{q_h^s\left(4q_h^h-2q_h-1\right)}$ in Theorem 1 to restrict the counterforce that a higher precision α associates competition with higher soft signal realizations; that is, shifts $\phi(s|HH)$ to the right. This shifting occurs through the correlated components of the hard and soft signal information, θ_s^h. When q_s^s is relatively small—specifically, below $\frac{2q_h-1}{q_h^s\left(4q_h^h-2q_h-1\right)}$—the only-soft state θ_s^s is more influential, as it spans a broader range of fundamentals. Since θ_s^s is unaffected by α, the impact of a greater α on the shift of the distribution of the soft signal (which reflects both θ_s^h and θ_s^s) tends to be muted when q_s^s is small.[15]

3.3 Bank Profits

Information technology, by affecting the severity of the information asymmetry between lenders, determines the competitiveness of the credit market. Following our findings in the previous section, we now show that the span and precision of information have opposite effects on the banks' equilibrium profits.

3.3.1 Information Span and Bank Profits

Despite an enlarged information span increasing the screening ability of both banks, it benefits Bank B relatively more than Bank A. In the proposition below, we show that an increase in the span η levels the playing field in the credit market.

Proposition 2. ***(Information Span on Equilibrium Profits)***

1. *The equilibrium profits of the non-specialized lender* π^B *are (weakly) increasing in* η.

---

[15] To see this result, the coefficient in (32), which is $\frac{\eta q}{\frac{(1-\alpha)^2}{2\alpha-1}+q_h^h(1-\eta)}$, captures the magnitude of the shift. One can rewrite this coefficient as $\frac{\eta q_h}{\frac{(1-\alpha)^2}{2\alpha-1}+q_h} \cdot q_s^s$, which is increasing in q_s^s.

Expected Profits

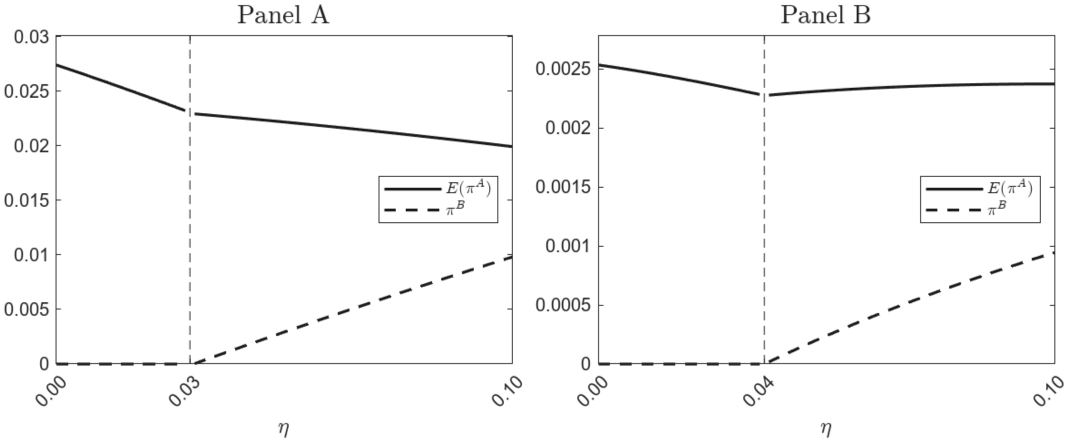


Figure 5: **Expected lender profits.** Panel A and Panel B plot expected lender profits against information span η under two sets of primitive parameters: Panel A, $\bar{r}$ = 0.36, q = 0.72, q_s = 0.9, $\alpha_u = \alpha_d = \alpha = 0.7$, τ = 1; Panel B, $\bar{r}$ = 0.33, q = 0.72, q_s = 0.9, $\alpha_u = \alpha_d = \alpha = 0.6$, $\tau = 0.1$. The solid lines correspond to Bank A while the dashed lines correspond to Bank B.

2. *In the region of positive-weak equilibrium, the impact of η on Bank B's profits dominates that on Bank A's profits:*
$$\frac{d\pi^B}{d\eta} > \frac{d}{d\eta}\mathbb{E}\left[\pi^A\right]. \tag{33}$$

Proof. See Appendix A.5. ☐

There are two forces that affect Bank B's profits following an increase in span η. First, as discussed above, an increase in span increases the overlap between the hard and soft states and reduces the informational asymmetry among the banks. Second, Bank A, endowed with a more accurate screening technology, competes more aggressively for the borrower.

As suggested in the first part of Proposition 2 and illustrated in Figure 5, there exists a threshold $\hat{\eta}$ that delimits the zero- and positive-weak regions. When $\eta < \hat{\eta}$ the two effects exactly offset each other in equilibrium, and π^B stays at zero in the zero-weak equilibrium region. For $\eta > \hat{\eta}$, Bank A's informational advantage (and Bank B's Winner's Curse associated with it) shrinks to the extent that Bank A—when receiving a sufficiently low signal $s = \hat{s}$—loses its local monopoly power and becomes the break-even lender. In this case, the technological advancement dominates the increase in competition from the perspective of Bank B, who starts making positive profits in this positive-weak equilibrium.

Moreover, as part 2) of Proposition 2 shows, in the region of positive-weak equilibria, the reduction in Bank A's informational advantage is evident in the behavior of the "profit gap" for the banks, which decreases with η. This result shows increasing the span levels the playing field, which we interpret as the credit market becoming more competitive.

Finally, while the information span η always helps Bank B (part 1 of Proposition 2), Bank A gains from improved screening too and hence its profits can also increase with η in the range of positive-weak equilibrium parameters ($\eta > \hat{\eta}$). Panel B in Figure 5 provides an example where Bank A's expected profits increase with η, whereas the opposite occurs in Panel A. Comparing the parameter configurations of the two panels in Figure 5, we find that $\mathbb{E}(\pi^A)$ is more likely to increase with η when the precision of signals—either τ for soft signal or α for hard signals—is low. For instance, when the precision of the soft signal τ is low, Bank A—initially holding a noisy signal about the soft state θ_s—gains more from the expanded span, as it reduces the uncertainty about θ_s considerably. In these settings, the benefits of improved screening outweigh the intensified competition from Bank B, leading to higher profits for Bank A. As we show in Section 3.4, this scenario may result in a Pareto improvement, with all agents in the economy enjoying greater surplus.

3.3.2 Information Precision and Bank Profits

The effect of an increase in the hard signal's precision α on equilibrium bank profits is quite involved, and in general, non-monotone. To understand the non-monotonicity, it is useful to consider two extreme cases. In an auction setting with asymmetric bidders, the uninformed bidder makes zero profit (Milgrom and Weber, 1982). When $\alpha = 0.5$ so that the hard signal is completely uninformative,[16] the model is identical to Milgrom and Weber (1982) where the uninformed lender B ignores the realization of h^B, randomizes its bids, and makes zero profits in equilibrium. On the other extreme, when $\alpha = 1$, hard information becomes public; and when $h^A = h^B = H$ we are back to Milgrom and Weber (1982) so that Bank B makes zero profits. In general, for values of $\alpha \in (0, 1)$, a positive-weak equilibrium (with $\pi^B > 0$) could arise, as shown in Panel D in Figure 6.

Hence, we show our formal results under restricted parameters. To focus on the contrast between span and precision, we set $\eta = 0$; this shuts down the improvement in the assessment of the overlapping characteristics θ_s^h when the precision increases. Providing a formal counterpart to Proposition 2, Proposition 3 shows that a higher precision benefits the specialized lender, Bank A.

Proposition 3. ***(Hard Signal Precision on Bank Profits.)*** *Suppose $\eta = 0$. In the range of zero-weak equilibrium, Bank A benefits more from a higher precision of hard signals; that is,*

$$\frac{d}{d\alpha}\mathbb{E}\left[\pi^A\right] > \frac{d\pi^B}{d\alpha} = 0. \tag{34}$$

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[16] Although this limiting case violates Assumption 1 which requires hard signals to be sufficiently strong, we have a well-defined equilibrium in this case a la Milgrom and Weber (1982) where both lenders ignore the hard signals.

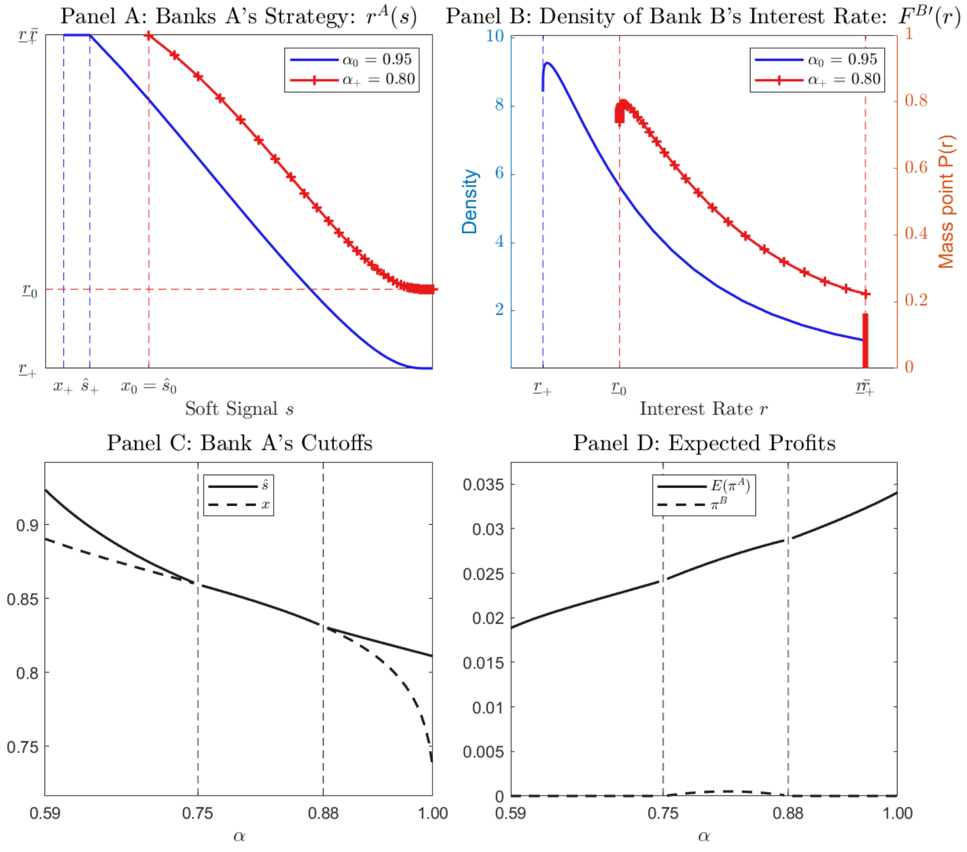

Figure 6: **Equilibrium strategies and profits for hard signal precision α.** Panel A depicts $r^A(s)$ as a function of s and Panel B plots $F^{B\prime}(r)$ as a function of r; strategies for $\alpha_+ = 0.8$ are depicted in red with markers while strategies with $\alpha_0 = 0.95$ are depicted in blue. Panel C depicts Bank A's thresholds $\hat{s} = \sup s^A(\overline{r})$ and $x = \sup s^A(\infty)$, and Panel D the expected profits for two lenders against α. Parameters: $\overline{r} = 0.36$, $q = 0.72$, $q_s = 0.9$, $\eta = 0$, and $\tau = 0.8$.

Proof. See Online Appendix B.4. □

Compared to Proposition 2, Proposition 3 concerns the region of zero-weak because we are interested in the scenario where α makes the specialized bank stronger. Under Assumption 2 (that is, $q_h > 0.5$), the more precise the hard signals, the more likely it is for lenders to compete ($h^A = h^B = H$) than to disagree and not compete ($h^A \neq h^B$). This tilt towards competition effectively increases the Winner's Curse that Bank B suffers from Bank A's soft signal. Hence, Bank A benefits more from increases in the hard signal precision, and its equilibrium profit improves.

Figure 6 plots the same equilibrium objects as Figure 2 (except $\eta = 0$ which is the case we focus on here), showing the comparative statics on α. First, Panels A and B illustrate the lenders' equilibrium pricing strategies, showing that lenders set more aggressive rates (lower rates) for $\alpha_+ < \alpha_0$. When α increases from $\alpha_+ = 0.8$ to $\alpha_0 = 0.9$, both lenders are competing more fiercely by quoting lower interest rates, so the equilibrium turns from positive-weak to zero-weak (hence α_0 for the larger α). In Panel C, the cutoff strategies of Bank A generally decrease as α increases; this reflects the standard learning effect—Bank A, receiving a more accurate positive hard signal, withdraws at a worse soft signal. Notably, $\hat{s}$ and x coincide for mid-values of α, which is consistent with the non-monotonicity of π^B. Finally, Panel D illustrates that Bank A's expected profits increase with α in the region of zero-weak equilibrium, and that the non-specialized lender B's profits π^B are non-monotone in α with $\pi^B = 0$ at the two limiting cases of $\alpha = \frac{1}{2}$ or 1.

3.4 Credit Allocation and Welfare

We now analyze how information span affects credit allocation and welfare. After presenting some comparative statics on aggregate markers of credit market health as a function of η , we formally show that greater information span on hard signals always improves welfare.

3.4.1 Information Span on Credit Market Outcomes

We focus on three aggregate markers of credit market health: loan approval rates, non-performance rates, and the probability of funding good/bad borrowers. We also investigate the expected NPV of funded projects as a measure of total welfare in the banking sector.

Figure 7 shows the comparative statics of equilibrium outcomes as a function of the span of hard information η. Two forces drive these results. First, a higher η assesses more characteristics and reduces Type II mistakes, lowering the probability of receiving a positive hard signal. Second, it alleviates the Winner's Curse faced by Bank B, leading to more aggressive bidding and participation in equilibrium.

Panel A shows the expected loan approval rates for the two lenders. As η increases, Bank A's approval rate rises due to better screening and more aggressive participation. For Bank B, the approval rate (dashed line) depends on whether it earns zero or positive profits. All the discontinuous jumps in Figure 7 around $\hat{\eta} \approx 0.03$ correspond to equilibrium regime switching, as Bank B moves from random participation (when $\pi^B = 0$) to full participation upon receiving $h^B = H$ (when $\pi^B > 0$). In a zero-weak equilibrium, the relaxation of the Winners Curse makes Bank B more likely to compete, raising its approval rate. In a positive-weak equilibrium, Bank B already always participates, and the decline in approval rate reflects the reduced likelihood of a positive signal.

Panel B shows the non-performing rates of loans made by Bank A (solid line) and Bank B (dashed line). Within the same equilibrium category (zero-weak or positive-weak), both decrease with the information span η as improved the screening reduces Type II errors and increases average loan quality.

Panel C plots the probability of funding good (solid line) and bad (dashed line) borrowers.

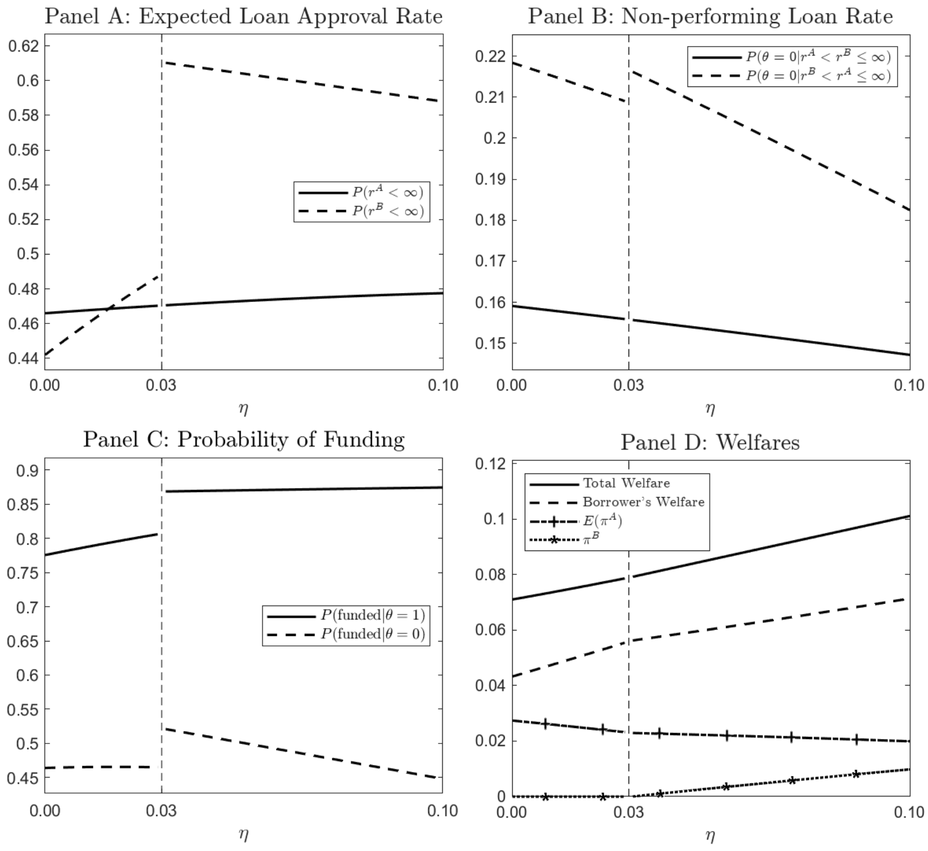

Figure 7: **Credit allocation and welfare.** Panel A and Panel B show the expected loan approval and non-performing rates, respectively. The solid lines correspond to Bank A while the dashed lines correspond to Bank B. Panel C depicts the probability of getting funded for a high-quality borrower (solid line) and a low-quality borrower (dashed line). Panel D illustrates aggregate welfare (solid line), borrower surplus (dashed line), and lenders' expected profits (dash-dotted line with cross markers for A while dotted line with star markers for B). All variables are depicted as a function of the span of hard information η. Priors q_h, q_s^s and information span η satisfy $q_h = q/q_s \cdot (1-\eta)$ and $q_s^s = q_s/(1-\eta)$. Parameters: $\bar{r} = 0.36$, $q = 0.72$, $q_s = 0.9$, $\tau = 1$ (top two panels) and $\alpha_u = \alpha_d = \alpha = 0.7$ (bottom two panels).

As a larger η represents a better screening technology, one would expect the probability of funding good loans to rise while that of bad loans to fall. This is indeed the case in Panel C when the equilibrium is in the positive-weak regime for $\eta > \hat{\eta} \approx 0.03$. However, in the zero-weak regime ($\eta < \hat{\eta} \approx 0.03$), a larger span attenuates the Winner's Curse and Bank B competes more aggressively, thereby extending more loans regardless of borrower types when the span increases.

3.4.2 Information Span and Welfare

The proposition below shows that total welfare increases with the span of hard information.

Proposition 4. *Total welfare, measured as expected net present value (NPV) of funded projects, strictly increases in information span η.*

Proof. See Online Appendix B.5. ☐

Panel D in Figure 7 shows how aggregate welfare and individual surpluses respond to increases in the information span η. Proposition 4 formally shows that overall aggregate welfare rises with η.[17] Note that welfare remains continuous at the zero-to-positive-weak transition as in Panel D: although loan quantity jumps, added loans yield zero NPV on average since both Bank B and borrowers break even at the cap rate $\bar{r}$.

In Panel D, for zero-weak equilibria under $\eta < \hat{\eta} \approx 0.03$, all welfare gains accrue to borrowers via a transfer from banks; while in a positive-weak equilibrium under $\eta > \hat{\eta}$, Bank B also gains from increased η. That is, all agents benefit from higher η, except Bank A in the positive-weak region. Yet, as demonstrated by Panel B in Figure 5, even Bank A could benefit when signal precision is low. In sum, broadening hard information—via modern data technology—can lead to a Pareto improvement.

4 Model Extensions

This section considers two extensions to our baseline model. First, we allow for correlated hard signals, motivated by open banking initiatives. Second, we consider an alternative modeling of hardening soft information by introducing a signal on θ_s^h, and show that both the equilibrium characterization and the key economic takeaways are robust to this alternative.

4.1 Correlated Hard Signals

A well-recognized consequence of advances in information technology is the increased correlation of hard information across lenders. For instance, open banking initiatives—by allowing customer-authorized data sharing—make lenders assessments more aligned (He, Huang, and Zhou, 2023; Babina, Bahaj, Buchak, De Marco, Foulis, Gornall, Mazzola, and Yu, 2025). We extend our model to capture this effect and show that increasing signal correlation, or making information more public, also features distinct implications for credit market equilibrium compared to the increase in information span.

We modify the hard information technology as follows. Suppose that, with probability $\rho_h \in [0, 1]$, lenders receive the same binary signal realization $h^c \in \{H, L\}$, while with probability $1 - \rho_h$ each lender receives an independent binary hard signal (just like our baseline). We solve this extension in Online Appendix B.6 and plot the comparative statics with respect to the correlation $\rho_h \in [0, 1]$ of hard signals across two lenders in Figure 8. The bottom two

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[17] As η rises, both screening and lender participation improve. While better screening always raises total welfare, the effect of increased participation is ambiguous and depends on the marginal borrowers efficiency. The main result of Proposition 4 is to show that the effects of screening dominate.

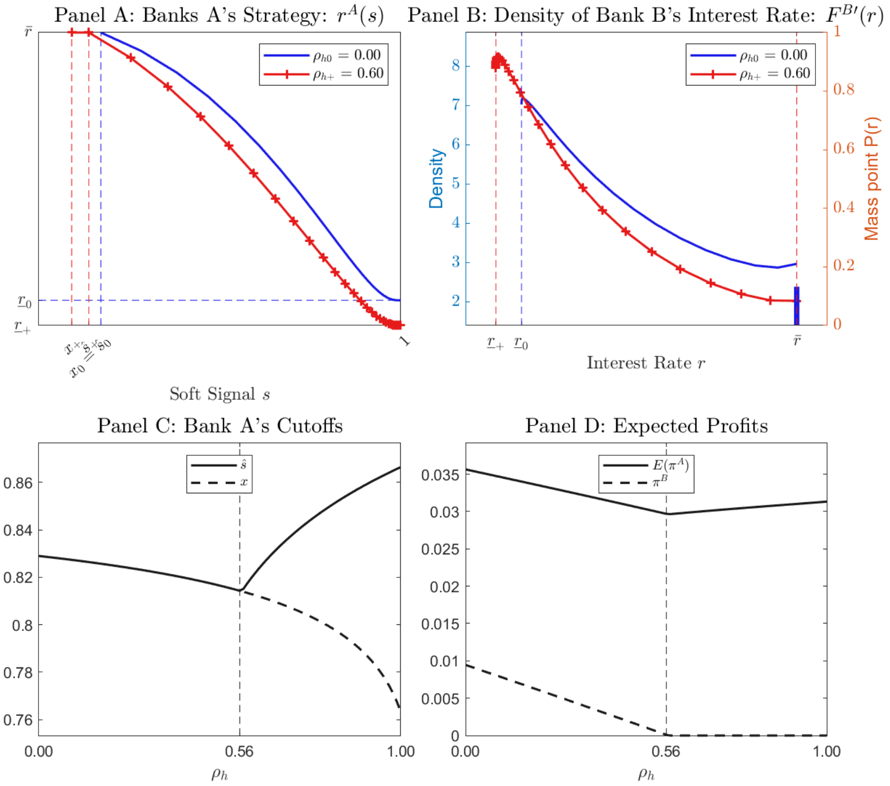

Figure 8: **Equilibrium strategies and profits for hard signal correlation ρ_h.** Panel A depicts $r^A(s)$ as a function of s and Panel B plots $F^{B\prime}(r)$ as a function of r; strategies for $\rho_{h+} = 0.6$ (a positive-weak equilibrium) are depicted in red with markers while strategies with $\rho_{h0} = 0$ (a zero-weak equilibrium) are depicted in blue. Panel C depicts Bank A's thresholds $\hat{s} = \sup s^A(\bar{r})$ and $x = \sup s^A(\infty)$, and Panel D depicts the expected profits for two lenders, both as a function of ρ_h. Parameters: $\overline{r} = 0.45$, $q_h = 0.8$, $q_s = 0.9$, $\eta = 0$, $\alpha = 0.7$, and $\tau = 1$.

panels show that a larger ρ_h makes a zero-weak equilibrium more likely. In the extreme case in which $\rho_h = 1$, the hard signal becomes a public signal; then Bank B becomes effectively uninformed, ending up with zero profits (Milgrom and Weber, 1982). It is therefore interesting to observe that the economic implications of ρ_h, which typically increases with data sharing, are qualitatively similar to those of greater signal precision but opposite to the effects of increasing information span (see the discussion in Section 3.3).

4.2 Additional Hard Signals on θ_s^h

In our analysis, we interpret the increase in the span of hard information as the outcome of "hardening" soft information, allowing hard signals to cover a broader range of fundamental states. Alternatively, one could introduce an additional signal about θ_s^h that is available to both banks, without altering the structure of the existing signals.

Consider an environment before soft information is hardened, so that $\eta = 0$. Suppose Big

Data technology now covers the overlapping fundamental state θ_h^s as in (3). Let $h_s^j \in \{H, L\}$ denote lender j's binary signal of θ_h^s, which we refer to as the hardened soft signal. For tractability, we assume that these signals are also decisive as in Section 1.2.3, so that each lender rejects the borrower as long as $h_s^j = L$.

Further, we assume that both banks' signal are perfectly correlated with $h_s^A = h_s^B = h_s^c$.[18] This "public" hardened soft signal captures the rising correlation correlation in signals generated by Big Data technology in a stark way,[19] leading to a simpler analysis. Finally, let α^s be the precision of h_s^c; that is, $h_s^c = H$ $(h_s^c = L)$ with probability $\alpha_s \in (\frac{1}{2}, 1)$ conditional on $\theta_s^h = 1$ $(\theta_s^h = 0)$.

This extension can be solved easily. Because competition occurs only when $h_s^c = H$, the relevant soft signal distribution becomes $\phi\left(s \mid h_s^c = H\right)$. Online Appendix B.7 shows that, once all distributions are replaced by those conditional on $h_s^c = H$, the model is isomorphic (up to a constant) to the $\eta = 0$ case with independent fundamentals and signals, allowing for a full characterization of the credit market equilibrium.

We draw two key insights from the resulting credit market equilibrium. First, adding a signal on θ_s^h—analogous to expanding the span of the hard signal—can level the playing field in credit markets with asymmetric lenders. This is most evident in the extreme case where $\theta_s^h = \theta_s$ (so $\theta_s^s = 1$ and there is no only-soft state) and $\alpha_s = 1$ (so a perfect precision for the additional signal), under which the hardened soft signal reveals the soft fundamental fully. In this scenario, Big Data technology eliminates Bank A's information advantage, which is equivalent to the symmetric-lender setting analyzed by Broecker (1990).

However, under more general parameters, the two modeling approaches can have different economic implications. This leads to our second, and arguably more important, point: just as the main insight of this paper, we show that in the alternative modeling the comparative statics with respect to the precision of the hardened soft signal (α_s) differ from those with respect to the information span η in our baseline model.

To see this, suppose that θ_s^h covers only a small subset of the soft fundamental states θ_s, so that the residual uncertainty about the only-soft states θ_s^s remains substantial. In Online Appendix B.7, we show that the comparative statics of α_s share the same sign as α (rather than η), particularly for the three effects analyzed in Section 3.2. For example, consider Bank B's inference about θ_s^s. As in Section 3.2.2, when Bank B receives a positive $h_s^B = H$ and competes against an opponent with soft signal s, a more precise h_s^B about θ_s^h leads it to update its belief about θ_s^s downward, thereby exacerbating the Winner's Curse due to

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[18] As in Section 4.1, any correlation ρ_s^h between the two hardened soft signals can be allowed; see details in Online Appendix B.7.

[19] In fact, $\rho_s^h = 1$ endogenously arises when the precision of the hardened soft signal becomes perfect $(\alpha_s \to 1)$: when h_s^j reveals θ_s^h perfectly, h_s^j's must be the same across two lenders.

residual uncertainty. Conceptually, this is because the precision of the hardened soft signal α_s functions in the same way as the precision of the hard signal α.

5 Concluding Remarks

One of the main roles of banks in the economy is to produce information for allocating credit. In this paper, we show that the nature of a bank's information technology—specifically, the distinction between the *span* and *precision* of hard information—shapes credit market equilibrium outcomes and the intensity of competition. We formalize the concept of information span as the breadth of fundamentals covered by signals and show that, while both span and precision improve screening (and thus allocative efficiency), they have opposing effects on residual uncertainty, strategic interaction, and market competitiveness.

At first glance, advances in information technology should benefit all lenders—including both specialized institutions and emerging fintechs. Indeed, large banks have led IT investments in recent years (He, Jiang, Xu, and Yin, 2025). Yet, the growing empirical literature on fintechs (e.g., Berg, Fuster, and Puri, 2022) suggests that new technologies have allowed less-established lenders to catch up, intensifying competition in the credit market. Motivated by these patterns, we develop a model with *asymmetric* lenders but *symmetric* technological improvements to study how expanding the span of hard information—particularly through the hardening of soft information—affects equilibrium outcomes.

Our analysis highlights that improvements in information span and precision operate through distinct channels. While greater precision tends to reinforce existing informational advantages, expansions in information span alter inference and competitive behavior in ways that can favor non-specialized lenders. More broadly, our results suggest that the type of technological progress in information processing—whether it improves span, precision, or public availability—matters for the structure and efficiency of credit markets. By making these distinctions explicit, we provide a framework for evaluating recent changes in financial data infrastructures and their implications for competition, access to credit, and welfare.

References

Abraham, Ittai, Susan Athey, Moshe Babaioff, and Michael Grubb, 2020, Peaches, lemons, and cookies: Designing auction markets with dispersed information, .

Arnosti, Nick, Marissa Beck, and Paul Milgrom, 2016, Adverse selection and auction design for internet display advertising, *American Economic Review* 106, 2852–2866.

Babina, Tania, Saleem Bahaj, Greg Buchak, Filippo De Marco, Angus Foulis, Will Gornall, Francesco Mazzola, and Tong Yu, 2025, Customer data access and fintech entry: Early evidence from open banking, *Journal of Financial Economics* 169, 103950.

Berg, Tobias, Andreas Fuster, and Manju Puri, 2022, Fintech lending, *Annual Review of Financial Economics* 14, 187–207.

Bertomeu, Jeremy, and Iván Marinovic, 2016, A theory of hard and soft information, *The Accounting Review* 91, 1–20.

Blickle, Kristian, Zhiguo He, Jing Huang, and Cecilia Parlatore, 2025, Information-based pricing in specialized lending, *Journal of Financial Economics* 172, 104–135.

Blickle, Kristian, Cecilia Parlatore, and Anthony Saunders, 2023, Specialization in banking, *FRB of New York Staff Report No. 967.*

Broecker, Thorsten, 1990, Credit-worthiness tests and interbank competition, *Econometrica* pp. 429–452.

Chen, Zhiguo, and Yajun Wang, 2023, Optimal sequential selling mechanisms and deal protections in mergers and acquisitions, *Journal of Finance.*

Corrao, Roberto, 2023, Mediation markets: The case of soft information, Discussion paper Working Paper.

Crawford, Gregory S, Nicola Pavanini, and Fabiano Schivardi, 2018, Asymmetric information and imperfect competition in lending markets, *American Economic Review* 108, 1659–1701.

Crawford, Vincent P, and Joel Sobel, 1982, Strategic information transmission, *Econometrica: Journal of the Econometric Society* pp. 1431–1451.

Ernst, Thomas, Chester Spatt, and Jian Sun, 2024, Would order-by-order auctions be competitive?, *Journal of Finance.*

Ghosh, Pulak, Boris Vallee, and Yao Zeng, 2021, Fintech lending and cashless payments, *The Journal of Finance.*

Goldstein, Itay, Chong Huang, and Liyan Yang, 2022, Open banking with depositor monitoring, *Working paper.*

Gorbenko, Alexander S., and Andrey Malenko, 2024, Competition and endogenous auction initiation, *Journal of Finance* Forthcoming version circulated as 2021 working paper.

Hardik, Nimbark, 2023, Digitalisation promotes adoption of soft information in sme credit evaluation: the case of indian banks, *Digital Finance* pp. 1–32.

Hauswald, Robert, and Robert Marquez, 2003, Information technology and financial services competition, *Review of Financial Studies* 16, 921–948.

He, Zhiguo, Jing Huang, and Jidong Zhou, 2023, Open banking: Credit market competition when borrowers own the data, *Journal of Financial Economics* 147, 449–474.

He, Zhiguo, Sheila Jiang, Douglas Xu, and Xiao Yin, 2025, Investing in lending technology: It spending in banking, *Management Science.*

Hendricks, Kenneth, and Robert H Porter, 1988, An empirical study of an auction with asymmetric information, *The American Economic Review* pp. 865–883.

Huang, Jing, 2023, Fintech expansion, *Available at SSRN 3957688.*

Huang, Yiping, Longmei Zhang, Zhenhua Li, Han Qiu, Tao Sun, and Xue Wang, 2020, Fintech credit risk assessment for smes: Evidence from china, .

Karapetyan, Artashes, and Bogdan Stacescu, 2014, Information sharing and information acquisition in credit markets, *Review of Finance* 18, 1583–1615.

Kim, Kyungmin, 2008, The value of an informed bidder in common value auctions, *Journal of Economic Theory* 143, 585–602.

Kremer, Michael, 1993, The o-ring theory of economic development, *The Quarterly Journal of Economics* 108, 551–575.

Liberti, Jose M., and Atif R. Mian, 2009, Estimating the effect of hierarchies on information use, *The Review of Financial Studies* 22, 4057–4090.

Liberti, José María, and Mitchell A Petersen, 2019, Information: Hard and soft, *Review of Corporate Finance Studies* 8, 1–41.

Marquez, Robert, 2002, Competition, adverse selection, and information dispersion in the banking industry, *Review of Financial Studies* 15, 901–926.

Milgrom, Paul, and Robert Weber, 1982, A theory of auctions and competitive bidding, *Econometrica* 50, 1089–1122.

Milgrom, Paul R, 1981, Good news and bad news: Representation theorems and applications, *The Bell Journal of Economics* pp. 380–391.

Paravisini, Daniel, and Antoinette Schoar, 2016, The incentive effect of scores: Randomized evidence from credit committees„ *NBER Working Paper, 19303.*

Povel, Paul, and Rajdeep Singh, 2006, Takeover contests with asymmetric bidders, *Review of Financial Studies* 19, 1399–1431.

Stein, Jeremy C., 2002, Information production and capital allocation: Decentralized versus hierarchical firms, *The Journal of Finance* 57, 1891–1921.

Vives, Xavier, 2019, Digital disruption in banking, *Annual Review of Financial Economics* 11, 243–272.

———, and Zhiqiang Ye, 2021, *Information technology and bank competition* (Centre for Economic Policy Research).

A Technical Appendices

A.1 Outline of Proof for Proposition 1

There are four parts of the proof: 1) showing monotonicity of $r^A(\cdot)$; 2) solving for equilibrium Bank B strategy $F^B(r)$; 3) solving for π^B and $(\hat{s}, x)$; and 4) global optimality of Bank A's strategy $r^A(\cdot)$. For details in see Online Appendix B.2, though we provide derivations for the FOCs in Eq. (23) and (24). Bank A's profits in (15) can be expressed as a function of $Q^A(r; s)$:

$$\pi^A(r, s) = Q^A(r; s) \cdot (1 + r) - \left[p_{HH}(s)\left(1 - F^B(r)\right) + p_{HL}(s)\right].$$

Taking derivative with respect to r and noticing $\frac{Q^{A\prime}(r;s)}{\mu_{HH}(s)} = -p_{HH}(s)F^{B\prime}(r)$, we arrive at the equation (23). Similarly, for Bank B, we can write its objective (16) as

$$\pi^B(r) = Q^B(r) \cdot (1 + r) - \left(\int_0^{s^A(r)} p_{HH}(t)dt + \overline{p}_{LH}r\right).$$

Taking derivative w.r.t. r and noticing that $\frac{Q^{B\prime}(r)}{\mu_{HH}(s^A(r))} = p_{HH}(s^A(r))s^{A\prime}(r)$, we arrive at the equation (24).

A.2 Proof of Lemma 2

Proof. Note that

$$z\left(H;\alpha,\eta\right)\equiv\mathbb{E}\left[\theta=1|\,h^{j}=H\right]=\frac{\mathbb{P}\left(\theta=1,h^{j}=H\right)}{\mathbb{P}\left(h^{j}=H\right)}=\frac{q\alpha}{1-\alpha+q_{h}^{h}\left(1-\eta\right)\left(2\alpha-1\right)},$$

which is strictly increasing in information span η since $\alpha > 0.5$. For signal precision α, we rewrite

$$z(H;\alpha)=\frac{q}{\underbrace{\left[1-q_{h}^{h}\left(1-\eta\right)\right]}_{+}\underbrace{\frac{1-\alpha}{\alpha}}_{\downarrow\text{ in }\alpha}+q_{h}^{h}\left(1-\eta\right)}.$$

The denominator strictly increases in $\frac{1-\alpha}{\alpha}$ and decreases in α. Therefore, $z(H;\alpha)$ strictly increases in α. $\square$

A.3 Bank B' Beliefs upon $h^B = H$

A.3.1 Derivation of z_s^s

We first calculate $p_{HH}(s)$ here which will be used below,

$$p_{HH}(s)=\underbrace{q\alpha^{2}\phi_{1}\left(s\right)}_{\theta=1}+\underbrace{\left(1-q_{h}^{h}\right)\left(1-\alpha\right)^{2}\phi\left(s\right)}_{\theta_{h}^{h}=0}+\underbrace{\left[\left(1-q_{s}\right)\alpha^{2}-\eta\left(2\alpha-1\right)\right]q_{h}^{h}\phi_{0}\left(s\right)}_{\theta_{h}^{h}=1,\theta_{s}=0}. \tag{35}$$

Eq. (35) calculates the probability of HHs depending on different realizations of θ_h^h, θ_s^h and θ_s^s. The third term for the joint probability for $\theta_h^h = 1$, $\theta_s = 0$ and HHs is $q_h^h\left[q_h^s\alpha^2(1-q_s^s)+(1-q_h^s)(1-\alpha)^2\right]\phi_0(s) = \left[\left(1-q_s\right)\alpha^2-\eta\left(2\alpha-1\right)\right]q_h^h\phi_0\left(s\right).$

The posterior mean of θ_s^s conditional on $\left\{h^A = h^B = H, s\right\}$, can be calculated as

$$\begin{aligned}z_{s}^{s}\left(s\right)\equiv\mathbb{E}\left[\theta_{s}^{s}|\,h^{A}=h^{B}=H,s\right]&=\frac{\mathbb{P}\left(\theta_{s}^{s}=1,h^{A}=h^{B}=H,s\right)}{p_{HH}\left(s\right)}\\&=\frac{\left[q_{h}^{h}\alpha^{2}+\left(1-q_{h}^{h}\right)\left(1-\alpha\right)^{2}\right]\cdot q_{s}^{h}q_{s}^{s}\phi_{1}\left(s\right)+q_{s}^{s}\left(1-q_{s}^{h}\right)\left(1-\alpha\right)^{2}\phi_{0}\left(s\right)}{p_{HH}(s)}.\end{aligned} \tag{36}$$

It is easy to check that $z_s^s\left(s\right)$ is strictly increasing in s as

$$z_{s}^{s}(s)=\frac{1}{1+\frac{\frac{1-q_{s}^{s}}{1-q_{s}}\cdot\overline{p}_{HH}}{\left[q_{h}^{h}\alpha^{2}+\left(1-q_{h}^{h}\right)(1-\alpha)^{2}\right]\cdot\underbrace{\frac{s}{1-s}}_{\uparrow\text{ in }\alpha}+\frac{q_{s}^{s}\left(1-q_{s}^{h}\right)}{1-q_{s}}(1-\alpha)^{2}}}.$$

Note that $\mathbb{P}\left(z_s^s\left(s\right)\in\left(z,z+dz\right)\right)=\mathbb{P}\left(HH,s\in\left(z_s^{s-1}\left(z\right),z_s^{s-1}\left(z\right)+dz_s^s\left(s\right)\right)\right).$ Then the density of $z_s^s(s)$ is

$$p_{HH}\left(z_{s}^{s-1}\left(z\right)\right)\frac{1}{z_{s}^{s\prime}\left(z_{s}^{s-1}\left(z\right)\right)}dz.$$

A.3.2 Proof of Lemma 3

Proof. The probability of competition upon $h^B = H$ is

$$\mathbb{P}\left(HH|\, h^B = H; \eta, \alpha\right) = \frac{q_h(\eta)\,\alpha^2 + (1 - q_h(\eta))\,(1-\alpha)^2}{q_h(\eta)\,\alpha + (1 - q_h(\eta))\,(1-\alpha)}, \tag{37}$$

where $q_h(\eta) = q_h^h(1-\eta)$ is a function of η. One can rewrite the expression as

$$\mathbb{P}\left(HH|\, h^B = H; \eta\right) = \frac{\frac{(1-\alpha)^2}{q_h^h(2\alpha-1)} + 1 - \eta}{\frac{1-\alpha}{q_h^h(2\alpha-1)} + 1 - \eta} = 1 - \underbrace{\frac{(1-\alpha)\,\alpha}{q_h^h\,(2\alpha-1)}}_{>0,\text{ as }\alpha\in(\frac{1}{2},1)} \frac{1}{\frac{1-\alpha}{q_h^h(2\alpha-1)} + 1 - \eta}.$$

Since $\alpha \in (\frac{1}{2}, 1)$, it is clear from this expression that $\mathbb{P}\left(HH|\, h^B = H; \eta\right)$ is strictly decreasing in η.

For the monotonicity in α, one can show that

$$\text{sgn}\left[\frac{\partial \mathbb{P}\left(HH|\, h^B = H; \alpha\right)}{\partial \alpha}\right] = \text{sgn}\left[q_h + 2q_h\left(\alpha^2 - \alpha\right) - (1-\alpha)^2\right].$$

The key term $M(\alpha) \equiv q_h + 2q_h\left(\alpha^2 - \alpha\right) - (1-\alpha)^2$ is strictly increasing in α for $\alpha \in (\frac{1}{2}, 1)$, since $M'(\alpha) = 2q_h(2\alpha - 1) + 2(1-\alpha) > 0$. Hence, for $\alpha \in (\frac{1}{2}, 1)$,

$$M(\alpha) \geq M(\frac{1}{2}) = \frac{q_h}{2} - \frac{1}{4} > 0,$$

where the last inequality follows from Assumption 2. Therefore, $\frac{\partial \mathbb{P}\left(HH|h^B = H;\alpha\right)}{\partial \alpha} > 0$. ☐

A.3.3 Proof of Lemma 4

Proof. Recall $s = z_s^{s(-1)}(z; \eta, \alpha)$ is the soft signal realization at which $z_s^s = z$. As discussed after Eq. (36), $z_s^s(s; \eta, \alpha)$ strictly increases in s. It remains to check the monotonicity of $z_s^s(s; \eta, \alpha)$ in η and α.

Using the definition of z_s^s in (36) and the expression of $p_{HH}(s)$ in (35), we have

$$z_s^s(s;\eta) = \frac{\left[q_h^h\alpha^2 + \left(1 - q_h^h\right)(1-\alpha)^2\right] \cdot q_s\phi_1(s) + \left(\frac{q_s}{1-\eta} - q_s\right)(1-\alpha)^2\,\phi_0(s)}{q\alpha^2\phi_1(s) + \left(1 - q_h^h\right)(1-\alpha)^2\,\phi(s) + \left[(1-q_s)\,\alpha^2 - \eta\,(2\alpha - 1)\right] q_h^h\phi_0(s)},$$

where we have used $q_s^h(\eta) = 1 - \eta$, $q_s^s(\eta) = \frac{q_s}{1-\eta}$. It is easy to check that the numerator increases in η be good) and the denominator $p_{HH}(s)$ decreases in η since $\alpha > \frac{1}{2}$. Therefore, when η increases, $z_s^s(s;\eta)$ becomes larger and we need a lower $s = z_s^{s(-1)}(z;\eta)$ to keep at the same threshold $z_s^s = z$.

For α, we rewrite $z_s^s(s;\alpha)$ in (36) as a function of $x = \frac{\alpha^2}{(1-\alpha)^2}$ (which increases in α):

$$z_s^s\left(s; x(\alpha) = \frac{\alpha^2}{(1-\alpha)^2}\right) = \frac{\overbrace{\left[q_h^h x + \left(1-q_h^h\right)\right] \cdot q_s^h q_s^s \phi_1(s) + q_s^s\left(1-q_s^h\right)\phi_0(s)}^{H(x)}}{\left[q_h^h x + \left(1-q_h^h\right)\right] q_s^h q_s^s \phi_1(s) + \underbrace{\left[q_h x + (1-q_h)\right](1-q_s^s)\phi_0(s)}_{G(x)} + q_s^s\left(1-q_s^h\right)\phi_0(s)}$$

$$= \frac{H(x)}{H(x) + G(x)},$$

where $H(x) \equiv \left[q_h^h x + \left(1-q_h^h\right)\right] q_s^h q_s^s \phi_1(s) + q_s^s\left(1-q_s^h\right)\phi_0(s)$ and $G(x) \equiv \left[q_h x + (1-q_h)\right](1-q_s^s)\phi_0(s)$. Then

$$\frac{\partial z_s^s(s;x)}{\partial x} = \frac{q\phi_1(s)\,G(x) - q_h(1-q_s^s)\phi_0(s)\,H(x)}{(H(x)+G(x))^2} = \frac{q\left(1-q_s^h\right)(1-q_s^s)\phi_0(s)\left[\phi_1(s)-\phi_0(s)\right]}{(H(x)+G(x))^2} < 0. \tag{38}$$

When $s < q_s$, $\phi_1(s) - \phi_0(s) = \left(\frac{s}{q_s} - \frac{1-s}{1-q_s}\right)\phi(s) < 0$, and the inequality follows. Hence, $z_s^s(s; x(\alpha))$ strictly decreases in $x(\alpha) = \frac{\alpha^2}{(1-\alpha)^2}$ which implies that $z_s^s(s;\alpha)$ strictly decreases in α. Since $z_s^s(s;\alpha)$ is strictly increasing in s, we need a higher s to keep $z_s^s = z$, i.e., $s = z_s^{s(-1)}(z;\alpha)$ strictly increases in α when $s < q_s$.

☐

A.3.4 Proof of Lemma 5

Proof. Using $p_{HH}(s)$ in Eq. (35) and $\overline{p}_{HH} = q_h(\eta)\alpha^2 + (1-q_h(\eta))(1-\alpha)^2$, one can calculate

$$\phi(s\,|HH;\eta,\alpha) = \frac{p_{HH}(s)}{\overline{p}_{HH}} = \phi(s) + \underbrace{\left[\frac{\eta q}{\frac{(1-\alpha)^2}{2\alpha-1} + q_h^h(1-\eta)}\right]}_{\uparrow \text{ in } \eta,\, \uparrow \text{ in } \alpha} \cdot \underbrace{\left[\phi_1(s) - \phi_0(s)\right]}_{<0 \text{ iff } s<q_s}$$

It is easy to check that the first bracketed term $\frac{\eta q}{\frac{(1-\alpha)^2}{2\alpha-1} + q_h^h(1-\eta)}$ strictly increases in η when $\alpha > \frac{1}{2}$. This term is also strictly increasing in α since $\frac{d\left[\frac{(1-\alpha)^2}{2\alpha-1}\right]}{d\alpha} < 0$. When $s < q_s$, the second bracketed term $\phi_1(s) - \phi_0(s) = \frac{s}{q_s}\phi(s) - \frac{1-s}{1-q_s}\phi(s) < 0$. Therefore, when $s < q_s$, $\phi(s\,|HH;\eta,\alpha)$ is strictly decreasing in both η and α. ☐

A.4 Proof of Theorem 1

Proof. The left tail event of interest is given in Eq. (31), which we replicate here:

$$\mathbb{P}\left(z_s^s \le z \middle| h^B = H\right) = \int_0^{s = z_s^{s(-1)}(z;\eta,\alpha)} \frac{p_{HH}(t;\eta,\alpha)}{\mathbb{P}(h^B = H;\eta,\alpha)} dt. \tag{39}$$

Part 1: the effect of η. From Lemma 4, the upper integration limit $s = z_s^{s(-1)}(z;\eta)$ of (39) is strictly decreasing in η. We decompose the integrand into

$$\frac{p_{HH}(s)}{\mathbb{P}(h^B = H)} = \mathbb{P}\left(HH \middle| h^B = H\right) \cdot \phi(s\,|HH).$$

Lemma 3 shows that $\mathbb{P}\left(HH|\, h^B = H; \eta\right)$ strictly decreases in η, and Lemma 5 shows that when $s < q_s$, the second term $\phi\left(s\,|HH; \eta\right)$ also strictly decreases in η. Taken together, (39) strictly decreases in η.

Part 2: The effect of α. From Lemma 4, the upper integration limit $s = z_s^{s(-1)}\left(z; \alpha\right)$ in (39) is strictly increasing in α when $s < q_s$. Now we show that the integrand in (39), $\frac{p_{HH}(t;\alpha)}{\mathbb{P}(h^B=H;\alpha)}$, is strictly increasing in α under condition $q_s \leq \frac{2q_h-1}{4q_h^h-2q_h-1}$. Let $N\left(\alpha\right) \equiv p_{HH}(t)$, which is given in Eq. (35), and $D\left(\alpha\right) \equiv \mathbb{P}(h^B = H) = q_h\alpha + (1-q_h)(1-\alpha)$ denote the numerator and denominator of $\frac{p_{HH}(t;\alpha)}{\mathbb{P}(h^B=H;\alpha)}$ respectively. Then

$$\operatorname{sgn}\left\{\frac{\partial \frac{p_{HH}(t;\alpha)}{\mathbb{P}(h^B=H;\alpha)}}{\partial\alpha}\right\} = \operatorname{sgn}\left\{N'D - D'N\right\} = \operatorname{sgn}\left\{\left[1 - \frac{\left(1-q_s^h\right)}{1-q_s}\left(1-t\right)\right]q_h^h - 1 + \left(2q_h-1\right)\alpha^2 + 2\left(1-q_h\right)\alpha\right\}. \tag{40}$$

Let $M\left(\alpha,t\right) \equiv \left[1 - \frac{\left(1-q_s^h\right)}{1-q_s}\left(1-t\right)\right]q_h^h - 1 + \left(2q_h-1\right)\alpha^2 + 2\left(1-q_h\right)\alpha$. Note that

$$\frac{\partial M\left(\alpha,t\right)}{\partial\alpha} > 0, \quad \frac{\partial M\left(\alpha,t\right)}{\partial t} > 0.$$

Then the minimum value of $M\left(\alpha,t\right)$ is when $\alpha = \frac{1}{2}$ and $s = 0$:

$$M(\alpha,t) \geq M\left(\alpha = \frac{1}{2}, t = 0\right) = -\frac{\left(1-q_s^h\right)q_h^h}{1-q_s} + q_h^h - \frac{2q_h+1}{4}.$$

Note that $q_h^h - \frac{2q_h+1}{4} \geq q_h - \frac{2q_h+1}{4} = \frac{2q_h-1}{4} > 0$ where the first inequality uses $q_h^h \geq q_h$ and the last inequality uses $q_h > \frac{1}{2}$. Then

$$q_s \leq \frac{2q_h-1}{4q_h^h-2q_h-1} \Leftrightarrow M\left(\frac{1}{2}, 0\right) \geq 0 \Rightarrow M(\alpha,t) \geq M\left(\alpha = \frac{1}{2}, s = 0\right) \geq 0.$$

Therefore, if $q_s \leq \frac{2q_h-1}{4q_h^h-2q_h-1}$, the integrand $\frac{p_{HH}(t)}{\mathbb{P}(h^B=H)}$ of (39) increases in α because the sign of derivative is determined by $M\left(\alpha,t\right) \geq 0$ (see Eq.(40)).

Taken together, the left tail event of interest in (39) strictly increases in α if $q_s \leq \frac{2q_h-1}{4q_h^h-2q_h-1}$.

□

A.5 Proof of Proposition 2

Lemma 6. *Define the function*

$$\Delta\pi(s) \equiv \pi^A(r^A(s), s) - \phi_1(s)\pi^B(r^A(s)). \tag{41}$$

Then the expected difference in lender profits can be expressed as

$$\mathbb{E}(\pi^A) - \pi^B = \begin{cases} \int_{\hat{s}}^{1} \Delta\pi(s)\, ds - \int_{0}^{x} \phi_1(s)\pi^B\, ds, & \textit{if } \pi^B > 0, \\ \int_{\hat{s}}^{1} \Delta\pi(s)\, ds + \int_{x}^{\hat{s}} \pi^A(s,\overline{r})\, ds, & \textit{if } \pi^B = 0. \end{cases} \tag{42}$$

The first term for $s \geq \hat{s}$ is driven by the difference in lending costs $C^j(s)$ where $j \in \{A, B\}$,

$$\Delta\pi\left(s\right) = -\underbrace{\left[\int_0^s \phi_1(t)\,dt \cdot p_{HH}(s) + p_{HL}(s)\right]}_{C^A(s)} + \underbrace{\phi_1(s)\left[\int_0^s p_{HH}(t)\,dt + \overline{p}_{LH}\right]}_{C^B(s)}. \tag{43}$$

Proof. See Online Appendix B.3.1. ☐

Lemma 7. *The break-even soft signals s_A^{be} and s_B^{be} defined in Eq. (55) and (53) satisfy*

$$\frac{\partial s_A^{be}}{\partial \eta} < 0, \quad \frac{\partial s_B^{be}}{\partial \eta} < 0.$$

Proof. See Online Appendix B.3.2. ☐

Proof of Proposition 2

Proof. From Lemma 6, the equilibrium profit gap between lenders is given by

$$\mathbb{E}(\pi^A) - \pi^B = \begin{cases} \int_{\hat{s}}^1 \Delta\pi(s)\,ds - \int_0^x \phi_1(s)\pi^B\,ds, & \text{if } \pi^B > 0, \\ \displaystyle\int_{\hat{s}}^1 \Delta\pi(s)\,ds + \int_x^{\hat{s}} \pi^A(s, \overline{r})\,ds, & \text{if } \pi^B = 0. \end{cases} \tag{44}$$

Step 1. We show that for $s \geq \hat{s}$, the profit gap $\int_{\hat{s}}^1 \Delta\pi\left(s\right) ds$ strictly decreases in η. In fact, we show the stronger claim that for any $s \geq \hat{s}$, we have $\frac{d\Delta\pi(s;\eta)}{d\eta} > 0$. From Lemma 6, when $s \geq \hat{s}$ and $r^A(s) \in [\underline{r}, \overline{r})$, lenders' profit gap is determined by the difference in lending costs

$$\Delta\pi\left(s;\eta\right) = -\underbrace{\left[\int_0^s \phi_1(t)\,dt \cdot p_{HH}(s;\eta) + p_{HL}(s)\right]}_{C^A(s)} + \underbrace{\phi_1(s)\left[\int_0^s p_{HH}(t;\eta)\,dt + \overline{p}_{LH}\right]}_{C^B(s)}. \tag{45}$$

Moreover, information span η does not affect lending costs when lenders disagree (HL or LH), which carries no information content as lenders share the same precision: $p_{HL}(s) = \overline{p}_{HL}\phi(s) = \alpha(1-\alpha)\phi(s)$, and $\phi_1(s)\overline{p}_{LH} = \phi_1(s)\alpha(1-\alpha)$. Hence, Eq. (45) is determined by lending costs in competition HH:

$$\frac{d\Delta\pi\left(s;\eta\right)}{d\eta} = \frac{d\left[\phi_1\left(s\right)\int_0^s p_{HH}\left(t;\eta\right)dt - p_{HH}\left(s;\eta\right)\int_0^s \phi_1\left(t\right)dt\right]}{d\eta}.$$

Using $p_{HH}(s;\eta)$ given in Eq. (35), we have

$$\frac{d\Delta\pi\left(s;\eta\right)}{d\eta} = q_h^h \underbrace{(2\alpha - 1)}_{+} \int_0^s \phi_0\left(t\right)\phi_0\left(s\right) \underbrace{\left[\frac{\phi_1\left(t\right)}{\phi_0\left(t\right)} - \frac{\phi_1\left(s\right)}{\phi_0\left(s\right)}\right]}_{-,MLRP} dt < 0.$$

The bracketed term $\frac{\phi_1(t)}{\phi_0(t)} - \frac{\phi_1(s)}{\phi_0(s)} = \frac{t}{1-t} - \frac{s}{1-s} < 0$ for $t < s$.

Step 2. We now show the first part of Proposition 2. In the zero weak regime, $\pi^B(\eta) = 0$ for all η. We aim to show that in the positive weak regime where $\pi^B(\eta) > 0$, Bank B's profit $\pi^B(\eta)$ is strictly increasing in η.

We intend to find a particular soft signal $s \geq \hat{s}(\eta)$ and show that Bank A's profit upon s is strictly increasing in η. Then Step 1 implies that Bank B' profit must be strictly increasing in η as well. Consider any η_1, η_2, where $\eta_1 < \eta_2$ and $\pi^B(\eta_1) > 0, \pi^B(\eta_2) > 0$ (positive weak.) From Proposition 1, the equilibrium threshold $\hat{s}(\eta) = s^{be}_A(\eta)$ in the positive weak regime. Then Lemma 7, which says $s^A_{be}(\eta)$ decreases in η, shows

$$\hat{s}(\eta_1) = s^A_{be}(\eta_1) > s^A_{be}(\eta_2) = \hat{s}(\eta_2). \tag{46}$$

Consider the equilibrium when $\eta = \eta_2$. Since the equilibrium is positive weak, Bank A breaks even upon soft signal $\hat{s}(\eta_2)$. In addition, Step 1 in the proof of Lemma 9 in Online Appendix B.2 shows that Bank A's profit conditional on the soft signal s, is strictly increasing in s. Hence,

$$\frac{\pi^A\left(r^A(\hat{s}(\eta_1)), \hat{s}(\eta_1); \eta_2\right)}{\phi(\hat{s}(\eta_1))} \underbrace{>}_{\text{profit} \uparrow \text{ in } s} \frac{\pi^A\left(r^A(\hat{s}(\eta_2)), \hat{s}(\eta_2); \eta_2\right)}{\phi(\hat{s}(\eta_2))} \underbrace{=}_{\text{def } \hat{s}(\eta_2)} 0. \tag{47}$$

The density adjustment $\frac{1}{\phi(s)}$ is included because the inequality holds for profit conditional on s (Lemma 9.) Now consider the equilibrium when $\eta = \eta_1$. From the definition of equilibrium threshold $\hat{s}(\eta_1)$,

$$\pi^A\left(r^A(\hat{s}(\eta_1)), \hat{s}(\eta_1); \eta_1\right) \underbrace{=}_{\text{def } \hat{s}(\eta_1)} 0. \tag{48}$$

We now focus on the particular soft signal realization $s = \hat{s}(\eta_1)$ that satisfies $\hat{s}(\eta_1) \geq \max\{\hat{s}(\eta_1), \hat{s}(\eta_2)\}$ (see (46)) so that Step 1 applies. From (47) and (48), we have:

$$\pi^A\left(r^A(\hat{s}(\eta_1)), \hat{s}(\eta_1); \eta_2\right) > 0 = \pi^A\left(r^A(\hat{s}(\eta_1)), \hat{s}(\eta_1); \eta_1\right).$$

From Step 1 which implies $\Delta\pi(\eta_2) < \Delta\pi(\eta_1)$, Bank B benefits more from the higher η than Bank A:

$$\phi_1(\hat{s}(\eta_1))\left[\pi^B(\hat{s}(\eta_1); \eta_2) - \pi^B(\hat{s}(\eta_1); \eta_1)\right] > \pi^A(r(\hat{s}(\eta_1)), \hat{s}(\eta_1); \eta_2) - \pi^A\left(r^A(\hat{s}(\eta_1)), \hat{s}(\eta_1); \eta_1\right) > 0.$$

Since Bank B makes a constant profit, the above inequality says $\pi^B(\eta_2) > \pi^B(\eta_1)$. This claim holds for any $\eta_1 < \eta_2$ in the positive weak regime.

Therefore, $d\pi^B(\eta)/d\eta > 0 (= 0)$ if $\pi^B(\eta) > 0 (= 0)$. Since $\frac{d\pi^B}{d\eta} > 0$ in the positive weak regime, there exists a threshold information span above which the equilibrium is positive weak.

Step 3. We show the second part of the proposition. From Lemma 6, in the positive weak regime,

$$\mathbb{E}(\pi^A; \eta) - \pi^B(\eta) = \int_{\hat{s}}^1 \Delta\pi(s; \eta)\, ds - \int_0^x \phi_1(s)\pi^B(\eta) ds.$$

Take derivative with respect to η,

$$\frac{d\left[\mathbb{E}(\pi^A; \eta) - \pi^B(\eta)\right]}{d\eta} = \int_{\hat{s}(\eta)}^1 \underbrace{\frac{d\Delta\pi(s; \eta)}{d\eta}}_{\text{Step 1:}<0} ds - \phi_1(s) \int_0^{x(\eta) = \hat{s}(\eta)} \underbrace{\frac{d\pi^B(\eta)}{d\eta}}_{\text{Step 2:}>0} ds - \underbrace{\left[\Delta\pi(\hat{s}; \eta) + \phi_1(s)\pi^B(\eta)\right]}_{= \pi^A(\overline{r}, \hat{s}) = 0,}.$$

We have shown $\frac{d\Delta\pi(s;\eta)}{d\eta} < 0$ for $s \geq \hat{s}$ in Step 1 and $\frac{d\pi^B(\eta)}{d\eta} > 0$ in Step 2, and the third bracketed term, which captures the effects on the integration limits, is zero. Therefore, $d\pi^B/d\eta > d\mathbb{E}\left[\tilde{\pi}^A\right]/d\eta$. □

B Online Appendix

B.1 Proof of Lemma 1

Proof. Note that the property of no gap implies common support $[\underline{r}, \overline{r}]$ (besides $\{\infty\}$.) If a bank's interest rate offering has a larger lower bound or a smaller upper bound interest rate than its competitor's, this constitutes a gap in the first bank's support.

To show that the distributions have no gap, suppose that the support of F^B has a gap $(r_1, r_2) \subset [\underline{r}, \overline{r}]$.[20] If follows that F^A should have no weight in this interval either, as any $r^A(s) \in (r_1, r_2)$ will lead to the same demand for Bank A and so a higher r will be more profitable. Moreover, at least one lender does not have a mass point at r_1 (it is impossible that both distributions have a mass point at $\overline{r}_1$). Then this lender's competitor has a profitable deviation by revising r_1 to $r \in (r_1, r_2)$ instead. Contradiction.

Regarding point mass, suppose that one lender's distribution, say F^B has a mass point at some $\tilde{r} \in [\underline{r}, \overline{r})$. Then Bank A would strictly prefer to quote $r^A = \tilde{r} - \epsilon$ over any $r^A(s) \in [\tilde{r}, \tilde{r} + \epsilon]$. In other words, the support of F^A must have a gap over $[\tilde{r}, \tilde{r} + \epsilon]$, contradicting the property of no gaps which we have shown. Finally, it is possible that one lender has a mass at $\overline{r}$, but impossible for both lenders to have a mass point at $\overline{r}$ (or both lender has a profitable deviation to undercut its competitor.)

□

B.2 Proof of Proposition 1

Since the derivations of equilibrium strategies are largely included in the main text, this proof aims to fill in several important steps. More specifically, in Part 1 we first prove that Bank A's equilibrium interest rate quoting strategy as a function of specialized signal $r^A(s)$ is always decreasing. In Part 2, we fill in the details in deriving equilibrium $F^B(r)$, and in Part 3 we derive the equilibrium profit π^B and cutoff threshold $\hat{s}$. Finally, in Part 4, we formally prove the monotonicity of r^A together with Bank A's FOC imply the global optimality of Bank A's strategy.

Part 1: The monotonicity of $r^A(\cdot)$. We have the next lemma which shows that $r^A(\cdot)$ is single-peaked over the space of $[0, 1]$.

Lemma 8. *Given any exogenous $\pi^B \geq 0$, $r^A(\cdot)$ single-peaked over $\mathcal{S} = [0, 1]$ with a maximum point.*

Proof. When $r \in [\underline{r}, \overline{r})$, the derivative of $r^A(s)$ with respect to s is

$$\frac{dr^A(s)}{ds} = \frac{p_{HH}(s)\left(\overbrace{\int_0^s p_{HH}(t)\left[\mu_{HH}(t) - \mu_{HH}(s)\right] dt}^{M_1(s)<0,\ \text{and}M_1'(s)<0} + \overbrace{\overline{p}_{LH}\overline{\mu}_{LH} - \left(\pi^B + \overline{p}_{LH}\right)\mu_{HH}(s)}^{M_2(s)?0,\ \text{but}M_2'(s)<0}\right)}{\left(\int_0^s p_{HH}(t) \cdot \mu_{HH}(t)\, dt + \overline{p}_{LH}\overline{\mu}_{LH}\right)^2}.$$

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[20] The same argument follows if the support of F^A has a gap in the conjectured equilibrium. In this case, for Bank B, any quotes within the gap lead to the same demand of the same posterior quality of customers (as inferred from Bank A's strategy.)

As $\mu_{HH}(t) < \mu_{HH}(s)$ for $t \in [0, s)$, the first term in the bracket $M_1(s) < 0$, and

$$M_1'(s) = -\frac{\partial \mu_{HH}(s)}{\partial s} \int_0^s p_{HH}(t)\, dt < 0.$$

For $M_2(s) = \overline{p}_{LH}\overline{\mu}_{LH} - \left(\pi^B + \overline{p}_{LH}\right)\mu_{HH}(s)$, the sign is ambiguous, but $M_2(s)$ is decreasing in s. This implies that $M_1(s) + M_2(s)$ decreases in s. It is easy to verify that $M_1(0) + M_2(0) > 0$ and $M_1(1) + M_2(1) < 0$. Therefore, $r^A(s)$ initially increases and then decreases, implying that it is single-peaked. ☐

Suppose the peak of $r^A(s)$ occurs at $\tilde{s}$. Then, for all $s < \tilde{s}$, Bank A can earn higher profits by charging $r(s) = \tilde{r}$—this is the standard "ironing" technique. Accordingly, we define the following ironed strategy formally (here, we also account for the capping $r \leq \overline{r}$):

$$r^A_{ironed}(s) \equiv \sup_{t \in [s,1]} \min\left(r^A(t), \overline{r}\right).$$

By definition $r^A_{ironed}(s)$ is monotone decreasing.

We now argue that in equilibrium, π^B and $\underline{r}$ adjust so that $r^A(s)$ is monotone decreasing over $[x, 1]$. Since we define $r^A(s) = \infty$ for $s < x$, monotonicity over the entire signal space $[0, 1]$ follows immediately. There are two subcases to consider.

1. Case 1: $\tilde{r} = \overline{r}$. In this case, the expression $r^A(s) = \frac{\pi^B + \int_0^s p_{HH}(t)dt + \overline{p}_{LH}}{\int_0^s p_{HH}(t) \cdot \mu_{HH}(t)dt + \overline{p}_{LH}\overline{\mu}_{LH}} - 1$ used in Lemma 10 and 8 does not apply to $s < \tilde{s}$, as it is defined only over the left-closed-right-open interval $[\underline{r}, \overline{r})$. In the region of $s < \tilde{s}$, $r^A(s)$ is determined by Bank A's optimality condition: since r^A does not affect the competition from Bank B (winning probability at $1 - F^B\left(\overline{r}^-\right)$), Bank A simply sets the maximum possible rate $r^A(r) = \overline{r}$ unless doing so becomes unprofitable (i.e., for $s < x$). (This corresponds to a zero-weak equilibrium with $\pi^B = 0$, and no "ironing" occurs in this case.)

2. Case 2: $\tilde{r} < \overline{r}$. Bank A quotes $\tilde{r}$ for all $s < \hat{s}$ after ironing. However, this cannot constitute an equilibrium—Bank A leaves a gap in the interval $[\tilde{r}, \overline{r}]$, contradicting with point 3) in Lemma 1 which rules out gaps in equilibrium. Intuitively, B would have a profitable deviation to raise its quotes within $[\tilde{r}, \overline{r}]$ to $\overline{r}$. Hence, this case leads to a contradiction. There is no "ironing" either in this case; intuitively, in equilibrium, π^B and $\underline{r}$ adjust upward, so that the peak point $\tilde{s}$ coincides with $\overline{r}$.

We hence have shown that Bank A's strategy $r^A(s)$ that decreases in s.

Part 2: Equilibrium strategy $F^B(r)$. This part provides the supplementary analysis for the key steps provided after Proposition 1.

First, we directly derive lenders' first-order conditions to show that they correspond to (23) and (24) that are expressed in $Q^A(r; s)$ and $Q^B(r)$. Bank A's profits in (15) can be expressed as a function of $Q^A(r; s)$:

$$\pi^A(r, s) = Q^A(r; s) \cdot (1 + r) - \left[p_{HH}(s)\left(1 - F^B(r)\right) + p_{HL}(s)\right].$$

Taking derivative with respect to r and noticing $\frac{Q^{A\prime}(r;s)}{\mu_{HH}(s)} = -p_{HH}(s)F^{B\prime}(r)$, we arrive at the equation (23).

Similarly, for Bank B, we can write its objective (16) as

$$\pi^B(r) = Q^B(r) \cdot (1 + r) - \left(\int_0^{s^A(r)} p_{HH}(t)dt + \overline{p}_{LH}r\right).$$

Taking derivative with respect to r and noticing that $\frac{Q^{B\prime}(r)}{\mu_{HH}(s^A(r))} = p_{HH}(s^A(r))s^{A\prime}(r)$, we arrive at the equation (24).

Then evaluating (23) at the equilibrium borrower type $s = s^A(r)$ and combining it with (24), we have the key ODE (25) which we reproduce here:

$$\frac{d}{dr}\left[\frac{Q^A(r;s)}{Q^B(r)}\right]\Bigg|_{s=s^A(r)} = 0.$$

Now derive Bank B's strategy from this key ODE. Factoring out s in $Q^A(r;s)$, we obtain the following ordinary differential equation:

$$\frac{d}{dr}\left\{\frac{p_{HH}(s)\mu_{HH}(s)\left[1-F^B(r)\right]+p_{HL}(s)\mu_{HL}(s)}{\int_0^{s^A(r)} p_{HH}(t)\mu_{HH}(t)dt+\overline{p}_{LH}\overline{\mu}_{LH}}\right\} = 0, \tag{49}$$

which implies that the function inside the curly brackets is a constant independent of r. Since signals are conditionally independent given $\theta = 1$, we have $p_{h^Ah^B}(s)\mu_{h^Ah^B}(s) = \mathbb{P}(\theta = 1, h^A, h^B, s) = \mathbb{P}(\theta = 1, h^A, h^B)\phi_1(s) = \overline{p}_{h^Ah^B}\overline{\mu}_{h^Ah^B}\phi_1(s)$, (or (14)) where $\phi_1(s) = \frac{s\phi(s)}{q_s}$ is the conditional density of s given $\theta_s = 1$. Then (49) can be rewritten as

$$\frac{d}{dr}\left\{\frac{\overline{p}_{HH}\overline{\mu}_{HH}\left[1-F^B(r)\right]+\overline{p}_{HL}\overline{\mu}_{HL}}{\overline{p}_{HH}\overline{\mu}_{HH}\int_0^{s^A(r)}\phi_1(t)dt+\overline{p}_{LH}\overline{\mu}_{LH}}\cdot\phi_1(s)\right\} = 0. \tag{50}$$

What is more, given that hard signals are symmetric across lenders, i.e., $\overline{p}_{HL}\overline{\mu}_{HL} = \overline{p}_{LH}\overline{\mu}_{LH}$, $1 - F^B(r)$ is proportional to $\int_0^{s^A(r)}\phi_1(t)dt = \frac{\int_0^{s^A(r)} t\phi(t)dt}{q_s}$. Using the boundary condition $F^B(\underline{r}) = 0$ where $s^A(\underline{r}) = 1$, we solve for $F^b(r)$ in the interior strategy space,

$$1 - F^B(r) = \frac{\int_0^{s^A(r)} t\phi(t)\,dt}{q_s}, \text{ for } r \in (\underline{r}, \overline{r}). \tag{51}$$

Bank B's strategy on $\overline{r}$ depends on whether it is profitable in equilibrium: it either places a mass of $1 - F^B(\overline{r}^-) = \frac{1}{q_s}\int_0^{\hat{s}} t\phi(t)\,dt > 0$ there if $\pi^B > 0$, or withdraws ($r = \infty$) if $\pi^B = 0$.

Part 3: Equilibrium Bank B's profit π^B and Bank A's threshold cutoff $\hat{s}$. Define the following auxiliary functions

$$\hat{\pi}^B\left(\overline{r}; s^A(\overline{r}) = s\right) = \int_0^s p_{HH}(t)\left[\mu_{HH}(t)(\overline{r}+1)-1\right]dt + \overline{p}_{LH}\left[\overline{\mu}_{LH}(\overline{r}+1)-1\right], \tag{52}$$

which is Bank B's profits when assuming that quoting $r^B = \overline{r}$ wins Bank A if its soft signal is below s in competition (HH). We define s_B^{be} as the threshold where Bank B's auxiliary profits break even,

$$\hat{\pi}^B\left(\overline{r}; s^A(\overline{r}) = s_B^{be}\right) = 0. \tag{53}$$

In addition, we define the following auxiliary profit function for Bank A,

$$\begin{aligned}&\hat{\pi}^A\left(\overline{r}\,\middle|\, s; F^B\left(\overline{r}\right)=\int_s^1\frac{t\phi\left(t\right)}{q_s}dt\right)\\ =&\frac{1}{\phi(s)}\left\{p_{HH}\left(s\right)\underbrace{\int_0^s\frac{t\phi\left(t\right)}{q_s}dt}_{=1-F^B\left(\overline{r}\right)}\left[\mu_{HH}\left(s\right)\left(1+\overline{r}\right)-1\right]+p_{HL}\left(s\right)\left[\mu_{HL}\left(s\right)\left(1+\overline{r}\right)-1\right]\right\}.\end{aligned}\tag{54}$$

This function is conditional on the soft signal realization s such that the profit level does not depend on signal distribution $\phi(s)$. The equation assumes that Bank A receiving soft signal s wins with probability $\int_0^s\frac{s\phi(s)}{q_s}ds$ when quoting $\overline{r}$. We define s_A^{be} as the threshold where Bank A's auxiliary profits break even,

$$\hat{\pi}^A\left(\overline{r}\left|s_A^{be}; F^B\left(\overline{r}\right)=\int_{s_A^{be}}^1\frac{s\phi\left(s\right)}{q_s}ds\right.\right)=0.\tag{55}$$

Lemma 9. *Bank A's auxiliary profit* $\hat{\pi}^A(\overline{r}|s)$ *in* (54) *strictly increases in* s.

Proof. We calculate the key terms in (54) as a function of s in order to show monotonicity. First,

$$p_{HH}\left(s\right)=\left[\underbrace{\left(1-q_h^h\right)}_{\theta_h^h=0}\phi\left(s\right)+\underbrace{q_h^h\left(1-q_s^h\right)}_{\theta_h^h=1,\theta_s^h=0}\phi_0\left(s\right)\right]\left(1-\alpha\right)^2+\left[\underbrace{q_h^hq_s^h\left(1-q_s^s\right)}_{\theta_h^h=\theta_s^h=1,\theta_s^s=0}\phi_0\left(s\right)+\underbrace{q}_{\theta=1}\phi_1\left(s\right)\right]\alpha^2.\tag{56}$$

Using the expressions of conditional density $\phi_1(s)=\frac{s\phi(s)}{q_s}$ and $\phi_0(s)=\frac{(1-s)\phi(s)}{1-q_s}$, we have

$$\begin{aligned}\frac{p_{HH}\left(s\right)}{\phi\left(s\right)}=&\left(1-q_h^h\right)\left(1-\alpha\right)^2+\left[q_h^h\left(1-q_s\right)\alpha^2-q_h^h\left(1-q_s^h\right)\left(2\alpha-1\right)\right]\frac{1-s}{1-q_s}+q_h^hs\alpha^2\\ =&q_h^h\alpha^2+\left(1-q_h^h\right)\left(1-\alpha\right)^2-q_h^h\left(1-q_s^h\right)\underbrace{\left(2\alpha-1\right)}_{+}\frac{1-s}{1-q_s},\end{aligned}$$

which shows that $\frac{p_{HH}(s)}{\phi(s)}$ is strictly increasing in s as $\alpha>\frac{1}{2}$. In addition,

$$\frac{p_{HL}(s)}{\phi(s)}=\frac{\overline{p}_{HL}\phi(s)}{\phi(s)}=\overline{p}_{HL}$$

is a constant in s. Then Bank A's auxiliary profit $\hat{\pi}^A(\overline{r}|s)$ in (54) can be rewritten as

$$\hat{\pi}^A\left(\overline{r}\,\middle|\, s; F^B\left(\overline{r}\right)=\int_s^1\phi_1(t)dt\right)=\underbrace{\frac{p_{HH}\left(s\right)}{\phi(s)}}_{\uparrow\text{ in }s}\underbrace{\int_0^s\phi_1\left(t\right)dt}_{\uparrow\text{ in }s}\left[\underbrace{\mu_{HH}\left(s\right)}_{\uparrow\text{ in }s}\left(1+\overline{r}\right)-1\right]+\overline{p}_{HL}\left[\underbrace{\mu_{HL}\left(s\right)}_{\uparrow\text{ in }s}\left(1+\overline{r}\right)-1\right].$$

Since the posterior beliefs $\mu_{HH}(s)$ and $\mu_{HL}(s)$, and the winning probability $\int_s^1\phi_1(t)dt$ are strictly increasing in s as well, we have the desired claim that $\hat{\pi}^A(\overline{r}|s)$ is strictly increasing in s. ☐

Now we characterize the equilibrium $\hat{s}$ and π^B. First, the equilibrium $\hat{s}\equiv\arg\sup_s\left\{s:r^A\left(s\right)\geq\overline{r}\right\}$ either equals s_A^{be} or s_B^{be}. To see this, if $\pi^B = 0$, we have $\hat{s} = s_B^{be}$ by construction. If $\pi^B > 0$, then Bank B always makes an offer upon H, i.e., $F^B(\overline{r}) = 1$. We also know that $F^B\left(\overline{r}^-\right) = 1 - \frac{\int_0^{s^A(r)=\hat{s}} t\phi(t)dt}{q_s} < 1$, because Bank A must reject the borrower when s realizes as close to 0 and $\hat{s} > 0$. Hence, $F^B(r)$ has a point mass at $\overline{r}$, and it follows that $F^A(r)$ is open at $\overline{r}$: $\hat{s} = x$ and Bank A's equilibrium profit $\pi^A\left(r^A(\hat{s})|\hat{s}\right) = 0$, which is exactly the definition of s_A^{be}. In addition, Eq. (55) gives a unique solution of s_A^{be} inside $(0, 1)$, because $\hat{\pi}^A\left(\overline{r}|s_A^{be}\right)$ is strictly increasing in s_A^{be} (Lemma 9), with $\hat{\pi}^A\left(\overline{r}|s_A^{be} = 0\right) < 0$ and $\hat{\pi}^A\left(\overline{r}|s_A^{be} = 1\right) = \frac{1}{\phi(1)}\left\{p_{HH}(1)\left[\mu_{HH}(1)(1+\overline{r}) - 1\right] + p_{HL}(1)\left[\mu_{HL}(1)(1+\overline{r}) - 1\right]\right\} > 0$—the latter is implied by Bank A's willingness to make an offer given $h^A = H$. Therefore, $\hat{s} = s_A^{be}$ when $\pi^B > 0$.

We check the property of Eq. (53) and its solution s_B^{be}. We have

$$\frac{\partial \hat{\pi}^B\left(\overline{r}|s = s_B^{be}\right)}{\partial s_B^{be}} = p_{HH}(s_B^{be})\left[\mu_{HH}(s_B^{be})(1+\overline{r}) - 1\right],$$

so $\hat{\pi}^B\left(\overline{r}\,\middle|\,s = s_B^{be}\right)$ is strictly decreasing in s_B^{be} when $\mu_{HH}(s_B^{be}) \in [0, \frac{1}{1+\overline{r}})$ and strictly increasing in s_B^{be} when $\mu_{HH}(s_B^{be}) \geq \frac{1}{1+\overline{r}}$. At the endpoints $s_B^{be} = 0$ and 1, $\hat{\pi}^B\left(\overline{r}\,\middle|\,s_B^{be} = 1\right) = \overline{p}_{\cdot H}\left[\overline{\mu}_{\cdot H}(1+\overline{r}) - 1\right] > 0$ from Assumption 1 regarding Bank B's participation, but the sign of $\hat{\pi}^B\left(\overline{r}\,\middle|\,s_B^{be} = 0\right) = \overline{p}_{LH}[\overline{\mu}_{LH}(1+\overline{r}) - 1]$ is ambiguous. When $\hat{\pi}^B\left(\overline{r}\,\middle|\,s_B^{be} = 0\right) < 0$, there is at most one solution s_B^{be} inside $(0, 1)$. On the other hand, when $\hat{\pi}^B\left(\overline{r}\,\middle|\,s_B^{be} = 0\right) > 0$, there are at most two solutions of s_B^{be} inside $(0, 1)$, s_{B1}^{be} and s_{B2}^{be}, with $\mu_{HH}(s_{B1}^{be})(1+\overline{r}) - 1 < 0$ and $\mu_{HH}(s_{B2}^{be})(1+\overline{r}) - 1 > 0$. We argue that only the larger solution s_{B2}^{be} is a candidate for equilibrium $\hat{s}$. To see this, we first show that $s_A^{be} > s_{B1}^{be}$: otherwise, if $s_A^{be} < s_{B1}^{be}$, then

$$\mu_{HL}(s_A^{be})(1+\overline{r}) - 1 \underbrace{<}_{\mu_{HL}(s)<\mu_{HH}(s)} \mu_{HH}(s_A^{be})(1+\overline{r}) - 1 \underbrace{<}_{s_A^{be}<s_{B1}^{be}} \mu_{HH}(s_{B1}^{be})(1+\overline{r}) - 1 < 0,$$

which leads to the contradictory implication that $\hat{\pi}^A\left(\overline{r}, s_A^{be}\right) < 0$ (by construction $\hat{\pi}^A\left(\overline{r}, s_A^{be}\right) = 0$.) Hence, $s_A^{be} > s_{B1}^{be}$. Then if equilibrium $\hat{s} = s_{B1}^{be}$, Bank A makes negative profits upon $\hat{s} = s_{B1}^{be}$, because $\hat{\pi}^A(\overline{r}, s)$ decreases in s and $\hat{\pi}^A\left(r^A(\hat{s}) = \overline{r}, \hat{s} = s_{B1}^{be}\right) < \hat{\pi}^A\left(\overline{r}, s = s_A^{be}\right) = 0$. Therefore, in the case of $\hat{\pi}^B\left(\overline{r}\,\middle|\,s_B^{be} = 0\right) > 0$ and there are at most two solutions of s_B^{be} inside $(0, 1)$, only the larger one is relevant. For the following analysis, we restrict s_B^{be} to be this largest solution $s_B^{be} \equiv \sup\{s_B^{be} \in (0,1) | \hat{\pi}^B\left(\overline{r}\,\middle|\,s_B^{be}\right) = 0\}$. If there is no solution of s_B^{be}, it is implied that equilibrium $\hat{s} = s_A^{be}$ and we define $s_B^{be} = 0$.

Now we show that equilibrium $\hat{s} = \max\{s_A^{be}, s_B^{be}\}$ and the comparison between s_A^{be} and s_B^{be} determines whether the equilibrium is positive-weak or zero-weak. To see this, in the first case of $s_B^{be} < s_A^{be}$, suppose $\hat{s} = s_B^{be}$. Then Bank A's equilibrium profit upon $\hat{s}$, $\pi^A(r^A(\hat{s})|\hat{s}) = \hat{\pi}^A\left(\overline{r}|s_B^{be}\right)$, is negative because $\hat{\pi}^A(\overline{r}|s)$ increases in s (Lemma 9) and $\hat{\pi}^A\left(\overline{r}|s = s_B^{be}\right) < \hat{\pi}^A\left(\overline{r}|s = s_A^{be}\right) = 0$; this is a contradiction. Hence, when $s_B^{be} < s_A^{be}$, $\hat{s} = s_A^{be}$. Because $\hat{\pi}^B(r = \overline{r}; s)$ is strictly increasing in $s \in (s_B^{be}, 1)$ as discussed above, Bank B's equilibrium profit $\hat{\pi}^B\left(r = \overline{r}; s = \hat{s} = s_A^{be}\right) > \hat{\pi}^B\left(r = \overline{r}; s = s_B^{be}\right) = 0$, i.e., the equilibrium is positive weak.

In the other case of $s_A^{be} < s_B^{be}$, suppose $\hat{s} = s_A^{be}$ and then Bank B's equilibrium profit (at $\overline{r}$) is $\hat{\pi}^B\left(r = \overline{r}; s = \hat{s} = s_A^{be}\right)$. From the discussion about Eq. (53) above, when Eq. (53) has one solution in $(0, 1)$, $\pi^B(r = \overline{r}; s)$ is negative for $s \in (0, s_B^{be})$, which applies to $s_A^{be} < s_B^{be}$; when Eq. (53) has two solutions in $(0, 1)$, $\hat{\pi}^B(r = \overline{r}; s)$ is negative for $s \in (s_{B1}^{be}, s_{B2}^{be})$, which applies to $s_{B1}^{be} < s_A^{be} < s_{B2}^{be}$. (Recall we took $s_B^{be} = s_{B2}^{be}$.) Hence, Bank B's equilibrium profit $\hat{\pi}^B\left(r = \overline{r}; s = \hat{s} = s_A^{be}\right) < 0$, which is a contradiction. Therefore, when $s_A^{be} < s_B^{be}$, equilibrium $\hat{s} = s_B^{be}$

and the equilibrium is zero-weak by construction. In addition,

$$\begin{aligned}0 =& \frac{\int_0^{s_A^{be}} p_{HH}(s)\,ds}{q_s}\left[\mu_{HH}\left(s_A^{be}\right)(1+\overline{r})-1\right]+p_{HL}\left[\mu_{HL}\left(s_A^{be}\right)(1+\overline{r})-1\right]\\ =& \frac{\int_0^{s_B^{be}} p_{HH}(s)\,ds}{q_s}\left[\mu_{HH}(x)(1+\overline{r})-1\right]+p_{HL}\left[\mu_{HL}(x)(1+\overline{r})-1\right]\\ \geq& \frac{p_{HH}\int_0^{s_A^{be}} t\phi(t)\,dt}{q_s}\left[\mu_{HH}(x)(1+\overline{r})-1\right]+p_{HL}\left[\mu_{HL}(x)(1+\overline{r})-1\right].\end{aligned}$$

The first equality is the definition of s_A^{be}, $\hat{\pi}^A\left(\overline{r}\left|s_A^{be}\right.\right)=0$, the second equality is Bank A's equilibrium break-even condition $\pi^A(\overline{r}|x)=0$ where winning probability in competition is $1-F^B(\overline{r}^-)=\frac{\int_0^{\hat{s}} t\phi(t)dt}{q_s}=\frac{\int_0^{s_B^{be}} t\phi(t)dt}{q_s}$, and the last inequality uses $s_B^{be}>s_A^{be}$ in this case. This means $x<s_A^{be}<s_B^{be}=\hat{s}$, and the distribution of Bank A's quote has a point mass at $\overline{r}$.

Part 4: Global optimality of Bank A's strategy . We now prove the global optimality of Bank A's strategy, which also rules out mixed strategy for Bank A in equilibrium.

Write Bank A's value $\Pi^A(r,s)$ as a function of its interest rate quote and specialized signal, in the event of $g^A=H$ and s. (We use π to denote the equilibrium profit but Π for the profit under any strategy.) Recall that Bank A solves the following problem:

$$\max_r \Pi^A(r,s)=\underbrace{p_{HH}(s)}_{h^A=H,h^B=H,s}\underbrace{\left[1-F^B(r)\right]}_{A\text{ wins}}\left[\mu_{HH}(s)(1+r)-1\right]+\underbrace{p_{HL}(s)}_{h^A=H,h^B=L,s}\left[\mu_{HL}(s)(1+r)-1\right]. \tag{57}$$

The first order condition (FOC) with respect to r is

$$\begin{aligned}0=&\Pi_r^A(r(s),s)\\ =&\underbrace{p_{HH}(s)\left[-\frac{dF^B(r)}{dr}\right]}_{\text{marginal customer}}\left[\underbrace{\left[\mu_{HH}(s)(1+r)-1\right]}_{\text{customer payoff}}\right]+\underbrace{p_{HH}(s)\left[1-F^B(r)\right]\mu_{HH}(s)}_{\text{existing customers who repay}}+p_{HL}(s)\mu_{HL}(s).\end{aligned} \tag{58}$$

One useful observation is that, on the support, $\mu_{HH}(s)(1+r)-1>0$ must hold. Otherwise, we have $\mu_{HL}(s)(1+r)-1<\mu_{HH}(s)(1+r)-1\leq 0$, implying that Bank A's profit would be negative and it would exit.

Lemma 10. *Consider s_1,s_2 in the interior domain with corresponding interest rate r_1, $r_2\in[\underline{r},\overline{r})$. The marginal value of quoting r_2 for type $s=s_1$, i.e. $\Pi_r^A(r_2,s_1)$, has the same sign as s_2-s_1.*

Proof. Consider the FOC in (58), evaluated at type s_1 but quoting r_2:

$$\begin{aligned}\Pi_r^A(r_2,s_1)=&p_{HH}(s_1)\left[-\frac{dF^B(r_2)}{dr}\right]\left[\mu_{HH}(s_1)(1+r_2)-1\right]\\ &+p_{HH}(s_1)\left[1-F^B(r_2)\right]\mu_{HH}(s_1)+p_{HL}(s_1)\mu_{HL}(s_1).\end{aligned} \tag{59}$$

The FOC at type s_2 yields

$$\begin{aligned}0=\Pi_r^A\left(r_2,s_2\right)=&p_{HH}\left(s_2\right)\left[-\frac{dF^B\left(r_2\right)}{dr}\right]\left[\mu_{HH}\left(s_2\right)\left(1+r_2\right)-1\right]\\&+p_{HH}\left(s_2\right)\left[1-F^B\left(r_2\right)\right]\mu_{HH}\left(s_2\right)+p_{HL}\left(s_2\right)\mu_{HL}\left(s_2\right),\end{aligned}$$

or

$$\frac{dF^B\left(r_2\right)}{dr}=\frac{p_{HH}\left(s_2\right)\left[1-F^B\left(r_2\right)\right]\mu_{HH}\left(s_2\right)+p_{HL}\left(s_2\right)\mu_{HL}\left(s_2\right)}{p_{HH}\left(s_2\right)\left[\mu_{HH}\left(s_2\right)\left(1+r_2\right)-1\right]}. \tag{60}$$

Plugging in this term to (59), $\Pi_r^A\left(r_2,s_1\right)$ becomes

$$\begin{aligned}\Pi_r^A\left(r_2,s_1\right)&=\left[\phi_1\left(s_1\right)-\frac{p_{HH}\left(s_1\right)}{p_{HH}\left(s_2\right)}\cdot\frac{\mu_{HH}\left(s_1\right)\left(1+r_2\right)-1}{\mu_{HH}\left(s_2\right)\left(1+r_2\right)-1}\cdot\phi_1\left(s_2\right)\right]\left\{\overline{p}_{HH}\overline{\mu}_{HH}\left[1-F^B\left(r_2\right)\right]+\overline{p}_{HL}\overline{\mu}_{HL}\right\}\\&=\frac{p_{HH}\left(s_1\right)\phi_1\left(s_2\right)-\phi_1\left(s_1\right)p_{HH}\left(s_2\right)}{p_{HH}\left(s_2\right)\left[\mu_{HH}\left(s_2\right)\left(1+r_2\right)-1\right]}\left\{\overline{p}_{HH}\overline{\mu}_{HH}\left[1-F^B\left(r_2\right)\right]+\overline{p}_{HL}\overline{\mu}_{HL}\right\},\end{aligned} \tag{61}$$

where $\overline{p}_{h^Ah^B}\equiv\mathbb{P}\left(h^A,h^B\right)$, $\overline{\mu}_{h^Ah^B}\equiv\mathbb{P}\left(\theta=1\left|h^A,h^B\right.\right)$ are defined in Section 2.2.1 and $\phi_1\left(s\right)\equiv\phi\left(s\left|\theta_s=1\right.\right)=\frac{s\phi(s)}{q_s}$ is the conditional density of soft signal given $\theta_s=1$. Because $\mu_{HH}\left(s\right)\left(1+r\right)-1>0$ as we noted right after Eq. (58), the sign of $\Pi_r^A\left(r_2,s_1\right)$ depends on $p_{HH}\left(s_1\right)\phi_1\left(s_2\right)-p_{HH}\left(s_2\right)\phi_1\left(s_1\right)$.

Last, we argue that the sign of $p_{HH}\left(s_1\right)\phi_1\left(s_2\right)-p_{HH}\left(s_2\right)\phi_1\left(s_1\right)$ depends on s_2-s_1. Note that the event HH correlates with the soft signal s only through θ_s^h, which in turn affects θ_s. We have

$$p_{HH}\left(s\right)=\mathbb{P}\left(\theta_s=1,HH\right)\phi_1\left(s\right)+\mathbb{P}\left(\theta_s=0,HH\right)\phi_0\left(s\right),$$

i.e., the event of two positive hard signals HH is uninformative about the soft signal s once we condition on the soft state θ_s. Using this observation,

$$p_{HH}\left(s_1\right)\phi_1\left(s_2\right)-p_{HH}\left(s_2\right)\phi_1\left(s_1\right)=\mathbb{P}\left(\theta_s=0,HH\right)\phi_0\left(s_1\right)\phi_0\left(s_2\right)\left[\frac{\phi_1\left(s_2\right)}{\phi_0\left(s_2\right)}-\frac{\phi_1\left(s_1\right)}{\phi_0\left(s_1\right)}\right]. \tag{62}$$

$\phi_1\left(s\right)\equiv\phi\left(s\left|\theta_s=1\right.\right)=\frac{s\phi(s)}{q_s}$ and $\phi_0\left(s\right)\equiv\phi\left(s\left|\theta_s=0\right.\right)=\frac{(1-s)\phi(s)}{q_s}$ are the conditional densities of s, and $\frac{\phi_1(s_2)}{\phi_0(s_2)}-\frac{\phi_1(s_1)}{\phi_0(s_1)}=\frac{s_2}{1-s_2}-\frac{s_1}{1-s_1}$ shares the same sign as s_2-s_1. ☐

Fist, the monotonicity of $r^A\left(\cdot\right)$ implied by Lemma 10 helps us show that Bank A uses a pure strategy. To see this, consider any $s_1>s_2$ that induce interior quotes $r_1,r_2\in[\underline{r},\overline{r})$, however close. Monotonicity requires that $\sup r^A(s_1)<\inf r^A(s_2)$ in equilibrium. Combining this with Part 3 of Lemma 1—which states that the induced distribution $F^A(\cdot)$ has no gaps—this implies that Bank A must adopt a pure strategy in the interior of $[\underline{r},\overline{r})$, i.e., for all $s\geq\hat{s}$. Finally, the following argument shows pure strategy for $s<\hat{s}$: i) Randomization at $s=0$ is a zero-measure set and is thus irrelevant; ii) For $0<s<\hat{s}$, Bank A chooses between quoting $\overline{r}$ and ∞, which generically results in different values, ruling out mixing in this region.

Second, if $r^A\left(\cdot\right)$ is decreasing globally over $[0,1]$ (which we have shown in Part 1), then the first-order condition is sufficient to ensure global optimality. Consider a type s_1 who would like to deviate to $\check{r}>r_1$. Then

$$\Pi^A\left(\check{r},s_1\right)-\Pi^A\left(r_1,s_1\right)=\int_{r_1}^{\check{r}}\Pi_r^A\left(r,s_1\right)dr.$$

Given the monotonicity of $r^A\left(s\right)$, we can find the corresponding type $s\left(r\right)$ for $r\in\left[r_1,\check{r}\right]$. From Lemma 10, we know that

$$\operatorname{sgn}\left(\Pi_r^A\left(r, s_1\right)\right) = \operatorname{sgn}\left(s(r) - s_1\right),$$

which is negative given $s\left(r\right) < s_1$, suggesting a loss for deviation. An analogous argument applies for any $\check{r} < r_1$, establishing that there is no profitable global deviation for Bank A.

B.3 Additional Proofs for Proposition 2

B.3.1 Proof of Lemma 6

Proof. Since Bank B uses mixed strategy and earns a constant profit π^B for any $r \in [\underline{r}, \overline{r}]$, we have

$$\pi^B = \pi^B \underbrace{\int_0^1 \phi_1(s)ds}_{=1} = \int_{\hat{s}}^1 \phi_1(s)\pi^B(r^A(s))ds + \int_0^{\hat{s}} \phi_1(s)\pi^B ds.$$

Here, recall $\phi_1(s) \equiv \frac{s\phi(s)}{q_s}$ is the conditional density of s given good soft fundamental $\theta_s = 1$, so that $\int_0^1 \phi_1(s)ds = 1$. In the second equality, we specify $r = r^A(s)$ in $\pi^B(\cdot)$. Then,

$$\mathbb{E}(\pi^A) - \pi^B = \int_0^1 \pi^A(r^A(s), s)ds - \pi^B = \int_{\hat{s}}^1 \Delta\pi(s)ds + \int_x^{\hat{s}} \left[\pi^A(\overline{r}, s) - \phi_1(s)\pi^B\right] ds - \int_0^x \phi_1(s)\pi^B ds, \quad (63)$$

where $\Delta\pi(s)$ is given in Eq. (41). For the first equality, note that $\pi^A(r^A(s), s)$, as defined in Eq. (15), already includes the density of s, so $\mathbb{E}(\pi^A) = \int_0^1 \pi^A(r^A(s), s)ds$. The second equality uses Bank A's equilibrium strategy in Eq. (26) that it quotes $\overline{r}$ when receiving $s \in [x, \hat{s})$ and exits and makes zero profits when receiving $s < x$. Specifying zero-weak or positive-weak equilibrium in (63), we have the desired claim in (44): if the equilibrium is positive weak ($\pi^B > 0$), $\hat{s} = x$ and the second term in (63) is zero; if the equilibrium is zero weak, $\pi^B = 0$ in the second and third term of (63).

We analyze the first term in (44) and show it is driven by the difference in lending costs. Introduce $Y^j(s)$ to denote lender $j \in \{A, B\}$'s revenue from good borrowers and $C^j(s)$ to denote lending costs. For $s \geq \hat{s}$, we rewrite a lender's equilibrium profit as $Y^j(s) - C^j(s)$, which are

$$\begin{aligned}\pi^A\left(r^A\left(s\right), s\right) = & \underbrace{\left\{\left[1 - F^B\left(r^A(s)\right)\right] p_{HH}\left(s\right)\mu_{HH}\left(s\right) + p_{HL}\left(s\right)\mu_{HL}\left(s\right)\right\}\left(1 + r^A\left(s\right)\right)}_{Y^A(s)} \\ & - \underbrace{\left\{\left[1 - F^B\left(r^A(s)\right)\right] p_{HH}\left(s\right) + p_{HL}\left(s\right)\right\}}_{C^A(s)}, \end{aligned} \quad (64)$$

and

$$\begin{aligned}\phi_1\left(s\right)\pi^B\left(r^A\left(s\right)\right) = & \underbrace{\phi_1\left(s\right)\left[\int_0^s p_{HH}\left(t\right)\mu_{HH}\left(t\right)dt + \overline{p}_{LH}\overline{\mu}_{LH}\right]\left(1 + r^A\left(s\right)\right)}_{Y^B(s)} \\ & - \underbrace{\phi_1\left(s\right)\left[\int_0^s p_{HH}\left(t\right)dt + \overline{p}_{LH}\right]}_{C^B(s)}, \end{aligned} \quad (65)$$

where density adjustment $\phi_1(s)$ is added for Bank B.

Using Bank B's equilibrium strategy in (27), we have

$$1 - F^B\left(r^A(s)\right) = \int_0^s \phi_1(t)\,dt.$$

Additionally, since signals are independent conditional on $\theta = 1$, it follows that

$$p_{h^Ah^B}(s)\mu_{h^Ah^B}(s) = \mathbb{P}(\theta = 1, h^A, h^B, s) = \mathbb{P}(\theta = 1, h^A, h^B)\phi_1(s) = \overline{p}_{h^Ah^B}\overline{\mu}_{h^Ah^B}\phi_1(s). \tag{66}$$

The second equality separates $\mathbb{P}(s|\theta = 1)$, which equals the conditional density $\phi_1(s) \equiv \phi(s|\theta_s = 1)$ ($\theta_s = 1$ is implied by $\theta = 1$. Hence, both lenders make the same revenue in competition (when HH realizes)

$$\underbrace{\int_0^s \phi_1(t)dt}_{\text{A wins}} \cdot \underbrace{p_{HH}(s)\,\mu_{HH}(s)}_{\text{A's good borrower}} = \underbrace{\phi_1(s)}_{\text{measure adjustment}} \underbrace{\int_0^s}_{\text{B wins}} \underbrace{p_{HH}(t)\,\mu_{HH}(t)}_{\text{B's good borrower}} dt = \overline{p}_{HH}\overline{\mu}_{HH}\phi_1(s)\int_0^s \phi_1(t)dt.$$

Moreover, because lenders share the same precision in hard signals, they also make the same revenue when the other bank rejects the borrower upon L,

$$\underbrace{p_{HL}(s)\,\mu_{HL}(s)}_{\text{A's good borrower}} = \overline{p}_{HL}\overline{\mu}_{HL}\phi_1(s) = \underbrace{\phi_1(s)\overline{p}_{LH}\overline{\mu}_{LH}}_{\text{B's good borrower}}.$$

The first equality follows from conditional independence of signals given $\theta = 1$ (see Eq. 66). The second one is from the symmetric hard information technology between lenders (specifically, $\overline{p}_{HL} = \overline{p}_{LH} = \alpha(1-\alpha)$ and $\overline{\mu}_{HL} = \overline{\mu}_{LH} = q$.)

Therefore, when $s \geq \hat{s}$, lenders make the same revenue from good type borrowers, $Y^A(s) = Y^B(s)$, so

$$\Delta\pi(s) = [Y^A(s) - Y^B(s)] - [C^A(s) - C^B(s)] = [C^A(s) - C^B(s)], \tag{67}$$

which is the desired claim (43).

□

B.3.2 Proof of Lemma 7.

Proof. The break-even soft signal s_A^{be} satisfies Eq. (55), which we reproduce below:

$$\hat{\pi}^A(\overline{r}|s;\eta) \equiv \frac{1}{\phi(s)}\left\{p_{HH}(s;\eta)\frac{\int_0^s t\phi(t)\,dt}{q_s}\left[\mu_{HH}(s;\eta)(1+\overline{r}) - 1\right] + p_{HL}(s)\left[\mu_{HL}(s)(1+\overline{r}) - 1\right]\right\}. \tag{68}$$

Here, s_A^{be} is implicitly defined by $\hat{\pi}^A\left(\overline{r}|s_A^{be};\eta\right) = 0$, which defines an implicit function linking $s = s_A^{be}$ and η. From the Implicit Function Theorem,

$$\frac{ds(\eta)}{d\eta} = -\frac{\frac{\partial\hat{\pi}^A}{\partial\eta}}{\frac{\partial\hat{\pi}^A}{\partial s}}.$$

From Lemma 9, $\hat{\pi}^A$ is strictly increasing in s_A^{be}. Now we analyze $\frac{\partial\hat{\pi}^A}{\partial\eta}$ by examining the monotonicity of the key terms in Eq. (68) with respect to η. Note that the joint events of signal realizations and good project is independent of η,

$$p_{HH}\left(s\right)\mu_{HH}\left(s\right)=q\alpha^{2}\cdot\frac{s\phi\left(s\right)}{q_{s}},\quad p_{HL}\left(s\right)\mu_{HL}\left(s\right)=q\alpha\left(1-\alpha\right)\cdot\frac{s\phi\left(s\right)}{q_{s}}.$$

In addition,

$$p_{HH}(s)=q\alpha^{2}\phi_{1}\left(s\right)+\left(1-q_{h}^{h}\right)\left(1-\alpha\right)^{2}\phi\left(s\right)+\left[\left(1-q_{s}\right)\alpha^{2}-\eta\left(2\alpha-1\right)\right]q_{h}^{h}\phi_{0}\left(s\right) \tag{69}$$

decreases in η since $\alpha>\frac{1}{2}$. Because lenders share the same hard information technology, the non-competition event HL is independent of η.

$$p_{HL}\left(s\right)=\overline{p}_{HL}\cdot\phi(s). \tag{70}$$

Taken together, we have

$$\hat{\pi}^{A}\left(\overline{r}\right|s;\eta)=\frac{1}{\phi\left(s\right)}\left\{\underbrace{\frac{\int_{0}^{s}t\phi\left(t\right)dt}{q_{s}}}_{\text{indept of }\eta}\left[\underbrace{p_{HH}\left(s\right)\mu_{HH}\left(s\right)}_{\text{indept of }\eta}\left(1+\overline{r}\right)-\underbrace{p_{HH}\left(s;\eta\right)}_{\text{decrease in }\eta}\right]+\underbrace{p_{HL}\left(s\right)\mu_{HL}\left(s\right)}_{\text{indept of }\eta}\left(1+\overline{r}\right)-p_{HL}\left(s\right)\right\},$$

and it is easy to see that $\frac{\partial\hat{\pi}^{A}}{\partial\eta}>0$. Therefore, the implicit function theorem implies

$$\frac{ds_{A}^{be}\left(\eta\right)}{d\eta}<0.$$

Part 2. Similarly, s_{B}^{be} is implicitly defined by $\hat{\pi}^{B}(\overline{r};s^{A}(\overline{r})=s_{B}^{be},\eta)=0$, and we reproduce (52):

$$\hat{\pi}^{B}(\overline{r};s,\eta)=\int_{0}^{s}p_{HH}\left(t;\eta\right)\left[\mu_{HH}\left(t;\eta\right)\left(\overline{r}+1\right)-1\right]dt+\overline{p}_{LH}\left[\overline{\mu}_{LH}\left(\overline{r}+1\right)-1\right].$$

It is straightforward to see that $\hat{\pi}^{B}(\overline{r};s,\eta)$ strictly increases in s, and it remains to discuss the monotonicity with respect to η. Similar as in the previous argument, the quantity of Bank B's good borrowers is independent of the span η under the multiplicative-characteristic setting:

$$\int_{0}^{s}p_{HH}\left(t\right)\mu_{HH}\left(t\right)dt=q\alpha^{2}\Phi\left(s\right|\theta_{s}=1)=q\alpha^{2}\cdot\frac{\int_{0}^{s}t\phi\left(t\right)dt}{q_{s}},$$
$$\overline{p}_{LH}\overline{\mu}_{LH}=q\alpha\left(1-\alpha\right).$$

In addition, the probability of disagreement in hard signals is also independent of η as $\alpha^{A}=\alpha^{B}$,

$$\overline{p}_{LH}=\alpha\left(1-\alpha\right).$$

The remaining key term—the probability that Bank B wins in competition $\int_{0}^{s}p_{HH}\left(t;\eta\right)dt$—decreases with η. Taken together, $\hat{\pi}^{B}(\overline{r};s,\eta)$ strictly increases in s. The Implicit Function Theorem then implies that

$$\frac{\partial s_{B}^{be}\left(\eta\right)}{\partial\eta}<0.$$

□

B.4 Proof of Proposition 3

Proof. From Lemma 6, under the parameter range where the equilibrium is zero weak,

$$\mathbb{E}\left(\pi^A;\alpha\right) - \pi^B\left(\alpha\right) = \int_{\hat{s}(\alpha)}^{1} \Delta\pi\left(s;\alpha\right) ds + \int_{x}^{\hat{s}(\alpha)} \pi^A\left(\overline{r}, s;\alpha\right) ds, \tag{71}$$

where $\Delta\pi\left(s\right) = \pi^A\left(r^A\left(s\right), s\right) - \phi_1\left(s\right)\pi^B\left(r^A\left(s\right)\right)$. We separate our analysis into two cases depending on whether $s \geq \hat{s}\left(\alpha\right)$.

Case 1: $s \geq \hat{s}$. In this case, lenders quote interior rates $r^A(s) \in [\underline{r}, \overline{r})$. Then Lemma 6 shows that for any $s \geq \hat{s}$, lenders earn the same revenue from good borrowers when quoting $r^A(s)$, and the profit gap is determined by their lending costs, denoted as $C^A\left(s;\alpha\right)$ and $C^B\left(s;\alpha\right)$:

$$\Delta\pi\left(s;\alpha\right) = -\underbrace{\left[\int_0^s \phi_1\left(t\right) dt \cdot p_{HH}\left(s;\alpha\right) + p_{HL}\left(s;\alpha\right)\right]}_{C^A(s;\alpha)} + \underbrace{\phi_1\left(s\right)\left[\int_0^s p_{HH}\left(t;\alpha\right) dt + \overline{p}_{LH}\left(\alpha\right)\right]}_{C^B(s;\alpha)}. \tag{72}$$

Therefore we need to study

$$\frac{d\Delta\pi\left(s;\alpha\right)}{d\alpha} = \int_0^s \left[\phi_1\left(s\right)\frac{\partial p_{HH}\left(t;\alpha\right)}{\partial\alpha} - \frac{\partial p_{HH}\left(s;\alpha\right)}{\partial\alpha}\phi_1\left(t\right)\right] dt + \left[\frac{\partial\overline{p}_{LH}\left(\alpha\right)}{\partial\alpha}\phi_1\left(s\right) - \frac{\partial p_{HL}\left(s;\alpha\right)}{\partial\alpha}\right]. \tag{73}$$

To calculate the first term, one can show that under $\eta = 0$ (independent hard and soft signals),[21]

$$p_{HH}\left(s;\alpha\right) = \underbrace{\left[q_h\alpha^2 + \left(1-q_h\right)\left(1-\alpha\right)^2\right]}_{\overline{p}_{HH}(\alpha)}\phi\left(s\right)$$

which gives us

$$\frac{\partial p_{HH}\left(s;\alpha\right)}{\partial\alpha} = \frac{\partial\overline{p}_{HH}\left(\alpha\right)}{\partial\alpha}\cdot\phi\left(s\right) = \left[2\alpha q_h - 2\left(1-\alpha\right)\left(1-q_h\right)\right]\phi\left(s\right). \tag{74}$$

Then the first term in (73) is

$$\int_0^s \left[\phi_1\left(s\right)\frac{\partial p_{HH}\left(t;\alpha\right)}{\partial\alpha} - \frac{\partial p_{HH}\left(s;\alpha\right)}{\partial\alpha}\phi_1\left(t\right)\right] dt = 2\left(\alpha + q_h - 1\right)\phi\left(s\right)\int_0^s \frac{\phi\left(t\right)}{q_s}\left(s-t\right) dt$$

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[21] For general $\eta \in [0,1]$, we show

$$p_{HH}\left(s;\alpha\right) = \left[\underbrace{\left(1-q_h^h\right)}_{\theta_h^h=0}\phi\left(s\right) + \underbrace{q_h^h\left(1-q_s^h\right)}_{\theta_h^h=1,\theta_s^h=0}\phi_0\left(s\right)\right]\left(1-\alpha\right)^2 + \left[\underbrace{q_h^h q_s^h\left(1-q_s^s\right)}_{\theta_h^h=\theta_s^h=1,\theta_s^s=0}\phi_0\left(s\right) + \underbrace{q}_{\theta=1}\phi_1\left(s\right)\right]\alpha^2$$

Following the steps in this proof, one can prove our desired result of $\frac{d\Delta\pi(s;\alpha)}{d\alpha} > 0$ for Case 1 if $q_s < \frac{2q_h-1}{2q_h^h-1}$. However, Case 2 is much more difficult for general η.

where we used $\phi_1(s) \equiv \frac{s\phi(s)}{q_s}$ and $\phi_0(s) \equiv \frac{(1-s)\phi(s)}{1-q_s}$. For the second term in (73),

$$\frac{\partial \overline{p}_{LH}(\alpha)}{\partial \alpha}\phi_1(s) - \frac{\partial p_{HL}(s;\alpha)}{\partial \alpha} = (1-2\alpha)\,\phi(s)\left(\frac{s}{q_s}-1\right). \tag{75}$$

Therefore, for any $s \geq \hat{s}$, the effect of precision on $\Delta\pi(s;\alpha)$ in (73) is (recall we assume sufficiently good hard state $q_h > 0.5$):

$$\begin{aligned}\frac{d\Delta\pi(s;\alpha)}{d\alpha} &= \underbrace{2(q_h+\alpha-1)}_{\geq 2\alpha-1\text{if } q_h \geq 0.5}\phi(s)\int_0^s \frac{\phi(t)(s-t)}{q_s}dt - (2\alpha-1)\,\phi(s)\left(\frac{s}{q_s}-1\right)\\ &\geq (2\alpha-1)\,\phi(s)\left\{\int_0^s \frac{\phi(t)(s-t)}{q_s}dt - \left(\frac{s}{q_s}-1\right)\right\}.\end{aligned}$$

So it remains to show that the curly bracketed term $\int_0^s \frac{\phi(t)(s-t)}{q_s}dt - \left(\frac{s}{q_s}-1\right) > 0$. To this end, note when $s=1$, this term equals $\frac{1}{q_s}(1-q_s) - \left(\frac{1}{q_s}-1\right) = 0$. In addition, this term decreases in s as

$$\frac{d\left[\frac{1}{q_s}\int_0^s (s-t)\,\phi(t)\,dt - \left(\frac{s}{q_s}-1\right)\right]}{ds} = \frac{1}{q_s}\left[\Phi(s)-1\right] < 0.$$

This proves that the bracketed term is positive for $s \in [\hat{s}, 1)$.

This concludes the proof of Case 1 that $\frac{d\Delta\pi(s;\alpha)}{d\alpha} > 0$ for any $s \geq \hat{s}$.

Case 2: $s \in [x, \hat{s}]$. We aim to show that under the parameter range where zero weak equilibrium arises, $\frac{d\Delta\pi(s;\alpha)}{d\alpha} = \frac{d\pi^A\left(s, r^A(s)=\overline{r}\right)}{d\alpha} > 0$ for any $x \leq s \leq \hat{s}$.

Bank A's equilibrium profit upon $s \in [x, \hat{s}]$ is

$$\pi^A\left(r^A(s)=\overline{r}, s;\alpha\right) = \underbrace{\int_0^{\hat{s}(\alpha)} \phi_1(t)\,dt}_{=1-F^B\left(\overline{r}^-;\alpha\right)} \cdot p_{HH}(s;\alpha)\left[\mu_{HH}(s;\alpha)(1+\overline{r})-1\right] + p_{HL}(s;\alpha)\left[\mu_{HL}(s;\alpha)(1+\overline{r})-1\right].$$

Note the probability that Bank A in competition, $1-F^B\left(\overline{r}^-;\alpha\right) = \int_0^{\hat{s}(\alpha)}\phi_1(t)\,dt$, is affected by the signal precision α because the equilibrium soft signal threshold $\hat{s}(\alpha)$ is a function of α. Hence, the effects of α on Bank A's profits consist of the direct effect of improved screening technology, $\frac{\partial\pi^A(\overline{r},s)}{\partial\alpha}$, and the indirect effect of competition through $\hat{s}(\alpha)$:

$$\frac{d\pi^A(\overline{r},s)}{d\alpha} = \underbrace{\frac{\partial\pi^A(\overline{r},s)}{\partial\alpha}}_{\text{screening}} + \underbrace{\phi_1(\hat{s})\,\frac{d\hat{s}(\alpha)}{d\alpha}}_{=\frac{\partial\left[1-F^B\left(\overline{r}^-;\alpha\right)\right]}{\partial\alpha}\text{, winning prob}} \cdot \underbrace{p_{HH}(s;\alpha)\left[\mu_{HH}(s;\alpha)(1+\overline{r})-1\right]}_{\text{marginal borrower}}. \tag{76}$$

Now let us characterize the indirect effect. In the zero weak equilibrium, $\hat{s}$ is determined by Bank B's break-even condition when it quotes $\overline{r}$,

$$0 = \pi^B(\overline{r};\hat{s},\alpha) = \int_0^{\hat{s}} p_{HH}(t;\alpha)\left[\mu_{HH}(t;\alpha)(\overline{r}+1)-1\right]dt + \overline{p}_{LH}(\alpha)\left[\overline{\mu}_{LH}(\alpha)(\overline{r}+1)-1\right].$$

Then from implicit function theorem,

$$\frac{d\hat{s}\left(\alpha\right)}{d\alpha}=-\frac{\frac{\partial\pi^{B}\left(\overline{r};\hat{s},\alpha\right)}{\partial\alpha}}{\frac{\partial\pi^{B}\left(\overline{r};\hat{s},\alpha\right)}{\partial\hat{s}}}=-\frac{\frac{\partial\pi^{B}\left(\overline{r};\hat{s},\alpha\right)}{\partial\alpha}}{p_{HH}\left(\hat{s}\right)\left[\mu_{HH}\left(\hat{s}\right)\left(\overline{r}+1\right)-1\right]}.$$

Use this in Eq. (76), we have

$$\frac{d\pi^{A}\left(\overline{r},s\right)}{d\alpha}=\frac{\partial\pi^{A}\left(\overline{r},s;\hat{s},\alpha\right)}{\partial\alpha}-\phi_{1}\left(\hat{s}\right)\frac{\partial\pi^{B}\left(\overline{r};\hat{s},\alpha\right)}{\partial\alpha}\cdot\frac{p_{HH}\left(s;\alpha\right)\left[\mu_{HH}\left(s;\alpha\right)\left(1+\overline{r}\right)-1\right]}{p_{HH}\left(\hat{s};\alpha\right)\left[\mu_{HH}\left(\hat{s};\alpha\right)\left(1+\overline{r}\right)-1\right]}. \tag{77}$$

It is useful to compare Eq. (77) with the expression in Case 1, $\frac{d\pi^{A}\left(r^{A}(s),s;\alpha\right)}{d\alpha}=\frac{d\Delta\pi(s;\alpha)}{d\alpha}=\frac{\partial\pi^{A}\left(r^{A}(s),s;\alpha\right)}{\partial\alpha}-\phi_{1}\left(s\right)\frac{\partial\pi^{B}\left(r^{A}(s);s,\alpha\right)}{\partial\alpha}$. While both expressions share a similar structure, in Case 2, Bank A's soft signal $s\in[x,\hat{s})$ is no longer aligned with the competition from Bank B, as the winning probability becomes fixed at $\int_{0}^{\hat{s}}\phi_{1}\left(t\right)dt$. Moreover, there is an additional adjustment term $\frac{p_{HH}(s;\alpha)[\mu_{HH}(s;\alpha)(1+\overline{r})-1]}{p_{HH}(\hat{s};\alpha)[\mu_{HH}(\hat{s};\alpha)(\overline{r}+1)-1]}$ capturing α's effect on $\hat{s}(\alpha)$.

To examine the sign of Eq. (77), we first decompose lenders' equilibrium profits into revenue from good borrowers and lending costs. Let Y^{j} denote lender revenue from good borrowers and C^{j} as lending costs for lender $j\in\{A,B\}$:

$$\begin{aligned}
Y^{A}\left(s,\overline{r};\hat{s},\alpha\right)\equiv&\left[\int_{0}^{\hat{s}}\phi_{1}\left(t\right)dt\cdot p_{HH}\left(s;\alpha\right)\mu_{HH}\left(s;\alpha\right)+p_{HL}\left(s;\alpha\right)\mu_{HL}\left(s;\alpha\right)\right]\left(1+\overline{r}\right),\\
C^{A}\left(s,\overline{r};\hat{s},\alpha\right)\equiv&\int_{0}^{\hat{s}}\phi_{1}\left(t\right)dtp_{HH}\left(s;\alpha\right)+p_{HL}\left(s;\alpha\right),\\
Y^{B}\left(\overline{r};\hat{s},\alpha\right)\equiv&\phi_{1}\left(\hat{s}\right)\left[\int_{0}^{\hat{s}}p_{HH}\left(t;\alpha\right)\mu_{HH}\left(t;\alpha\right)dt+\overline{p}_{LH}\left(\alpha\right)\overline{\mu}_{LH}\left(\alpha\right)\right]\left(\overline{r}+1\right),\\
C^{B}\left(\overline{r};\hat{s},\alpha\right)\equiv&\phi_{1}\left(\hat{s}\right)\left[\int_{0}^{\hat{s}}p_{HH}\left(t;\alpha\right)dt+\overline{p}_{LH}\left(\alpha\right)\right].
\end{aligned}$$

Note that density adjustment $\phi_{1}\left(\hat{s}\right)$ is added for Bank B, and the definition treats $\hat{s}$ as independent of α as we aim to characterize the direct effect of screening ($\frac{\partial\pi^{j}}{\partial\alpha}$), and the indirect effect has already been characterized in (77). Then Eq. (77) could be rewritten as

$$\begin{aligned}
\frac{d\pi^{A}\left(s,\overline{r}\right)}{d\alpha}=&\frac{1}{p_{HH}\left(\hat{s}\right)\left[\mu_{HH}\left(\hat{s}\right)\left(\overline{r}+1\right)-1\right]}\cdot\\
&\left\{\overbrace{\frac{\partial Y^{A}\left(s,\overline{r};\alpha\right)}{\partial\alpha}p_{HH}\left(\hat{s}\right)\left[\mu_{HH}\left(\hat{s}\right)\left(\overline{r}+1\right)-1\right]-\frac{\partial Y^{B}\left(\overline{r};\hat{s},\alpha\right)}{\partial\alpha}p_{HH}\left(s\right)\left[\mu_{HH}\left(s\right)\left(1+\overline{r}\right)-1\right]}^{\equiv I}\right.\\
&\left.+\overbrace{\frac{\partial C^{B}\left(\overline{r};\hat{s},\alpha\right)}{\partial\alpha}p_{HH}\left(s\right)\left[\mu_{HH}\left(s\right)\left(1+\overline{r}\right)-1\right]-\frac{\partial C^{A}\left(s,\overline{r};\alpha\right)}{\partial\alpha}p_{HH}\left(\hat{s}\right)\left[\mu_{HH}\left(\hat{s}\right)\left(\overline{r}+1\right)-1\right]}^{\equiv II}\right\}.
\end{aligned} \tag{78}$$

In this equation, $\frac{1}{p_{HH}(\hat{s})[\mu_{HH}(\hat{s})(\overline{r}+1)-1]}>0$ from Bank A's participation condition upon $\hat{s}$: since the marginal borrower's payoff from competition case $\mu_{HH}(\hat{s})(\overline{r}+1)-1$ is higher than that in the non-competition case $\mu_{LH}(\hat{s})(\overline{r}+1)-1$, the former must be positive for Bank A to participate. The curly bracketed term is decomposed into I—the effect of α on the gap of lender revenue after the adjustment of $\frac{p_{HH}(s)[\mu_{HH}(s)(1+\overline{r})-1]}{p_{HH}(\hat{s})[\mu_{HH}(\hat{s})(\overline{r}+1)-1]}$, and II—the effect on the gap of lending costs after adjustment.

Now we calculate both I and II when $\eta = 0$. Because of the multiplicative structure, $p_{HH}(s)\,\mu_{HH}(s) = q\alpha^2\phi_1(s)$ and $p_{HL}(s)\,\mu_{HL}(s) = q\alpha(1-\alpha)\,\phi_1(s)$. Then

$$\begin{aligned}\frac{\partial Y^A(s,\overline{r};\hat{s},\alpha)}{\partial\alpha} &= \left[2\alpha\int_0^{\hat{s}}\phi_1(t)\,dt + (1-2\alpha)\right]q(1+\overline{r})\cdot\phi_1(s),\\ \frac{\partial Y^B(\overline{r};\hat{s},\alpha)}{\partial\alpha} &= \left[2\alpha\int_0^{\hat{s}}\phi_1(t)\,dt + (1-2\alpha)\right]q(1+\overline{r})\cdot\phi_1(\hat{s}).\end{aligned}$$

and I can be simplified as

$$I = \left[2\alpha\int_0^{\hat{s}}\phi_1(t)\,dt + (1-2\alpha)\right]q(1+\overline{r})\cdot\overline{p}_{HH}\left[\phi_1(\hat{s})\,\phi(s) - \phi_1(s)\,\phi(\hat{s})\right].$$

To calculate II, from Case 1 we know that for $s = \hat{s}$,

$$\frac{d\Delta\pi\left(\hat{s}, r^A(\hat{s}) = \overline{r};\alpha\right)}{d\alpha} = \frac{\partial C^B(\overline{r};\hat{s},\alpha)}{\partial\alpha} - \frac{\partial C^A(\hat{s},\overline{r};\alpha)}{\partial\alpha} > 0. \tag{79}$$

Since $\mu_{HH}(s)(1+\overline{r})-1>0$ from Bank A's participation condition,

$$\begin{aligned}II &\equiv -\frac{\partial C^A(s,\overline{r};\hat{s},\alpha)}{\partial\alpha}p_{HH}(\hat{s})\left[\mu_{HH}(\hat{s})(\overline{r}+1)-1\right] + \frac{\partial C^B(\overline{r};\hat{s},\alpha)}{\partial\alpha}p_{HH}(s)\underbrace{\left[\mu_{HH}(s)(1+\overline{r})-1\right]}_{+}\\ &\underbrace{>}_{(79)} -\frac{\partial C^A(s,\overline{r};\hat{s},\alpha)}{\partial\alpha}p_{HH}(\hat{s})\left[\mu_{HH}(\hat{s})(\overline{r}+1)-1\right] + \frac{\partial C^A(\hat{s},\overline{r};\hat{s},\alpha)}{\partial\alpha}p_{HH}(s)\left[\mu_{HH}(s)(1+\overline{r})-1\right]\\ &= -\left[\int_0^{\hat{s}}\phi_1(t)\,dt\cdot\frac{\partial\overline{p}_{HH}}{\partial\alpha} + \frac{\partial\overline{p}_{HL}}{\partial\alpha}\right]q\alpha^2(\overline{r}+1)\left[\phi_1(\hat{s})\,\phi(s) - \phi_1(s)\,\phi(\hat{s})\right]\end{aligned}$$

We now combine I and II to arrive at

$$I + II = q(1+\overline{r})\frac{\phi(\hat{s})\,\phi(s)}{q_s}\cdot\underbrace{(\hat{s}-s)}_{+,\text{ as }s\in[x,\hat{s})}\left\{\int_0^{\hat{s}}\phi_1(t)\,dt\left(2\alpha\overline{p}_{HH} - \frac{\partial\overline{p}_{HH}}{\partial\alpha}\alpha^2\right) + \underbrace{(1-2\alpha)}_{-,\text{ as }\alpha>\frac{1}{2}}\left(\overline{p}_{HH}-\alpha^2\right)\right\} > 0,$$

because of the following two results: (note $\overline{p}_{HH} = q_h\alpha^2 + (1-q_h)(1-\alpha)^2$)

$$\begin{aligned}2\alpha\overline{p}_{HH} - \frac{\partial\overline{p}_{HH}}{\partial\alpha}\alpha^2 &= 2\alpha(1-q_h)(1-\alpha)^2 + 2(1-q_h)(1-\alpha)\,\alpha^2 > 0,\text{and}\\ \overline{p}_{HH}-\alpha^2 = q_h\alpha^2 + (1-q_h)(1-\alpha)^2 - \alpha^2 &= (1-q_h)\left[(1-\alpha)^2-\alpha^2\right]\underbrace{<}_{\alpha>\frac{1}{2}}0.\end{aligned}$$

Thanks to (78), this concludes the proof of $\frac{d\pi^A(s,\overline{r})}{d\alpha} > 0$ for Case 2 when $s\in[x,\hat{s})$.

In sum, we have shown that in the range of zero weak equilibrium, for any $s \geq x$, $\pi^A\left(r^A(s), s\right)$ strictly increases in α when $\eta = 0$. This means that $\frac{d\mathbb{E}\left(\pi^A\right)}{d\alpha} > 0 = \frac{d\pi^B}{d\alpha}$ which is the desired claim.

$\square$

B.5 Proof of Proposition 4

Proof. Recall that $r^j = \infty$ means lender $j \in \{A, B\}$ does not make an offer, and then $\min\left\{r^A, r^B\right\} < \infty$ means that the borrower is funded. Let W denote the total welfare,

$$W \equiv \mathbb{P}\left(\theta = 1, \min\left\{r^A, r^B\right\} < \infty\right)\overline{r} - \mathbb{P}\left(\theta = 0, \min\left\{r^A, r^B\right\} < \infty\right), \tag{80}$$

where funding a good borrower generates $\overline{r}$ in net social gain while funding a bad borrower costs 1 dollar. A borrower is funded when at least one lender is making an offer ($r^B < \infty$ or $s \geq x$) in the event of HH and when Bank A (B) makes an offer in the event of HL (LH). Recall that $\overline{p}_{h^Ah^B} \equiv \mathbb{P}\left(h^A, h^B\right)$, $p_{h^Ah^B}(s)\,ds \equiv \mathbb{P}\left(h^A, h^B, s \in ds\right)$, and $\overline{\mu}_{h^Ah^B} \equiv \mathbb{P}\left(\theta = 1 \left| h^A, h^B\right.\right)$, $\mu_{h^Ah^B} \equiv \mathbb{P}\left(\theta = 1 \left| h^A, h^B, s\right.\right)$, and then

$$\begin{aligned}
\mathbb{P}\left(\theta = 1, \min\left\{r^A, r^B\right\} < \infty\right) &= \underbrace{\overline{p}_{HH}\overline{\mu}_{HH} - \int_0^x p_{HH}(t)\,\mu_{HH}(t)\,dt \cdot \left(1 - F^B(\overline{r})\right)}_{HH} \\
&\quad + \underbrace{\int_x^1 p_{HL}(t)\,\mu_{HL}(t)\,dt}_{HL} + \underbrace{\overline{p}_{LH}\overline{\mu}_{LH}F^B(\overline{r})}_{LH}, \\
\mathbb{P}\left(\theta = 0, \min\left\{r^A, r^B\right\} < \infty\right) &= \underbrace{\overline{p}_{HH}\left[1 - \overline{\mu}_{HH}\right] - \int_0^x p_{HH}(t)\left[1 - \mu_{HH}(t)\right]dt \cdot \left(1 - F^B(\overline{r})\right)}_{HH} \\
&\quad + \underbrace{\int_x^1 p_{HL}(t)\left[1 - \mu_{HL}(t)\right]dt}_{HL} + \underbrace{\overline{p}_{LH}\left[1 - \overline{\mu}_{LH}\right]F^B(\overline{r})}_{LH}.
\end{aligned}$$

When the information span η changes, there are two types of effects. The first is improvement in screening as η affects the conditional distribution of hard signals. The second is the indirect effects as η affects lender participation upon H—$x(\eta)$ for Bank A and $F^B(\overline{r};\eta) = 1 - \mathbf{1}_{\pi^B=0} \cdot \frac{\int_0^{\hat{s}(\eta)} t\phi(t)dt}{q_s}$ for Bank B are both functions of η.

We separate our analysis into the regime of zero weak equilibrium and the regime of positive weak equilibrium.

Zero weak equilibrium. In the zero weak equilibrium, we aim to show that span increases total welfare because the indirect effects of lender participation are zero. Take derivative of welfare in (80) with respecet to Bank A's participation threshold x,

$$\begin{aligned}
\frac{\partial W}{\partial x} &= \left\{-p_{HH}\left(x\right)\mu_{HH}\left(x\right)\left(1-F^{B}\left(\overline{r}\right)\right)-p_{HL}\left(x\right)\mu_{HL}\left(x\right)\right\}\overline{r} \\
&\quad +p_{HH}\left(x\right)\left[1-\mu_{HH}\left(x\right)\right]\left(1-F^{B}\left(\overline{r}\right)\right)+p_{HL}\left(x\right)\left[1-\mu_{HL}\left(x\right)\right] \\
&= -\left\{p_{HH}\left(x\right)\left(1-F^{B}\left(\overline{r}\right)\right)\left[\mu_{HH}\left(x\right)\left(1+\overline{r}\right)-1\right]+p_{HL}\left(x\right)\left[\mu_{HL}\left(x\right)\left(1+\overline{r}\right)-1\right]\right\} \\
&= -\pi^{A}\left(\overline{r},x\right)=0.
\end{aligned}$$

When x increases, the social marginal borrower who lost credit access is exactly Bank A's marginal borrower for whom it breaks even at $\overline{r}$ upon signal x, so x's effects on welfare is zero. Similarly, regarding Bank B's participation upon H, which is $F^B(\overline{r})$, we have

$$\begin{aligned}
\frac{\partial W}{\partial F^{B}\left(\overline{r}\right)} &= \left\{\int_{0}^{x}\left[p_{HH}\left(t\right)\mu_{HH}\left(t\right)\right]dt+\overline{p}_{LH}\overline{\mu}_{LH}\right\}\overline{r}-\int_{0}^{x}\left\{p_{HH}\left(t\right)\left[1-\mu_{HH}\left(t\right)\right]\right\}dt-\overline{p}_{LH}\left(1-\overline{\mu}_{LH}\right) \\
&= \int_{0}^{x}p_{HH}\left(t\right)\left[\mu_{HH}\left(t\right)\left(\overline{r}+1\right)-1\right]dt+\overline{p}_{LH}\left[\overline{\mu}_{LH}\left(\overline{r}+1\right)-1\right]=\pi^{B}\left(\overline{r}\right)=0.
\end{aligned}$$

That is to say, the marginal borrower who gains credit access is Bank B's marginal borrower for whom it quotes $\overline{r}$ and breaks even, so $F^B\left(\overline{r}\right)$'s effects on total welfare is also zero.

Therefore, as information span increases, total welfare is affected through the screening technology only:

$$\begin{aligned}
\frac{dW}{d\eta} &= \underbrace{\frac{\partial}{\partial\eta}\left\{-\int_{0}^{x}p_{HH}\left(t;\eta\right)dt\cdot\left[1-F^{B}\left(\overline{r}\right)\right]\right\}}_{\text{screening, }\downarrow\text{Type II errors}}+\underbrace{\frac{\partial W}{\partial x}\frac{\partial x}{\partial\eta}+\frac{\partial W}{\partial F^{B}\left(\overline{r}\right)}\frac{\partial F^{B}\left(\overline{r}\right)}{\partial\eta}}_{\text{participation effect=0}} \\
&= \left(2\alpha-1\right)q_{h}^{h}\int_{0}^{\hat{s}}\phi_{1}\left(t\right)dt\int_{0}^{x}\phi_{0}\left(s\right)dt>0,
\end{aligned}$$

where $\phi_1\left(s\right)\equiv\frac{s\phi(s)}{q_s}$ and $\phi_0\left(s\right)\equiv\frac{(1-s)\phi(s)}{1-q_s}$ are the conditional densities of s. The second equality uses Eq. (35) which shows $p_{HH}(s)$ is linearly decreasing in η, and $1-F^B(\overline{r})=\int_0^{\hat{s}}\phi_1(t)dt$ from Proposition 1. The equation shows that span improves screening via reducing Type II errors in the event of HH. To see this, Type I errors remains fixed under the multiplicative structure (conditional on $\theta=1$, the distributions of hard signals are irrelevant of η). In addition, because of the symmetric hard signal precision, the disagreement events HL and LH do not have information content and are irrelevant of η either.

Positive weak equilibrium. In the positive weak equilibrium, Bank B always makes an offer upon H. The effect from Bank A's participation is no longer zero because there is a gap between the marginal borrower x's value to Bank A and the social gain. Specifically, in competition (HH), as x increases, there is transfer between the lenders as Bank B wins this marginal borrower but total welfare does not change.

First, we calculate the total welfare in (80). In this case,

$$\mathbb{P}\left(\theta=1,\min\left\{r^A,r^B\right\}<\infty\right)=\underbrace{q\alpha}_{\theta=1,h^B=H}+q\underbrace{\alpha\left(1-\alpha\right)}_{HL}\underbrace{\int_{x(\eta)}^{1}\phi_1\left(t\right)dt}_{s\geq x|\theta=1},$$

$$\mathbb{P}\left(\theta=0,\min\left\{r^A,r^B\right\}<\infty\right)=\underbrace{\left(1-q_h(\eta)\right)\left(1-\alpha\right)^2+q_h(\eta)\cdot\left(1-q_s^s(\eta)\right)\alpha^2}_{\theta=0,HH}+\underbrace{\left(1-q\right)\alpha\left(1-\alpha\right)}_{\theta=0,LH}$$
$$+\underbrace{\left(1-\alpha\right)\alpha\left[\left(1-q_h^h\right)q_s\cdot\int_{x(\eta)}^{1}\phi_1\left(t\right)dt+\left(1-q_s\right)\int_{x(\eta)}^{1}\phi_0\left(t\right)dt\right]}_{\theta=0,HL,s\geq x}.$$

Because $W\equiv\mathbb{P}\left(\theta=1,\min\left\{r^A,r^B\right\}<\infty\right)\overline{r}-\mathbb{P}\left(\theta=0,\min\left\{r^A,r^B\right\}<\infty\right)$, we have

$$\begin{aligned}\frac{dW}{d\eta}&=-q_h'\left(\eta\right)\left(2\alpha-1\right)-\left(1-\alpha\right)\alpha\cdot\frac{\partial x}{\partial\eta}\cdot\left\{\left[q\overline{r}-\left(1-q_h^h\right)q_s\right]\phi_1\left(x\right)-\left(1-q_s\right)\phi_0\left(x\right)\right\}\\&=q_h^h\cdot\left[\alpha^2-\left(1-\alpha\right)^2\right]-\left(1-\alpha\right)\alpha\cdot\frac{\partial x}{\partial\eta}\cdot\left\{\left[q_h^h\left(\overline{r}+1\right)-1\right]x\phi\left(x\right)-\left(1-x\right)\phi\left(x\right)\right\}\\&=\underbrace{q_h^h(2\alpha-1)}_{\text{screening}}-\underbrace{\frac{\partial x}{\partial\eta}\cdot\left(1-\alpha\right)\alpha\phi\left(x\right)\cdot\left[q_h^h\cdot x\cdot\left(\overline{r}+1\right)-1\right]}_{A\text{'s participation}}.\end{aligned}\tag{81}$$

We argue that $\frac{\partial x}{\partial\eta}<0$ and $q_h^hx\left(1+\overline{r}\right)-1<0$, so the effect of A's participation is negative. First, we have shown $\frac{\partial s_A^{be}}{\partial\eta}<0$ in Lemma 7; as $x=\hat{s}=s_A^{be}$ in the positive weak equilibrium, we have $\frac{\partial x}{\partial\eta}<0$. Second, Bank A's break even condition upon x is

$$\begin{aligned}0&=\pi^A\left(\overline{r},x;\eta\right)\\&=p_{HH}\left(x;\eta\right)\left[1-F^B\left(\overline{r}^-;\eta\right)\right]\left[\mu_{HH}\left(x;\eta\right)\left(1+\overline{r}\right)-1\right]+p_{HL}\left(x\right)\left[\mu_{HL}\left(x\right)\left(1+\overline{r}\right)-1\right]\\&=p_{HH}\left(x;\eta\right)\int_0^{x(\eta)}\phi_1\left(t\right)dt\left[\mu_{HH}\left(x;\eta\right)\left(1+\overline{r}\right)-1\right]+\alpha\left(1-\alpha\right)\phi\left(x\right)\left[q_h^hx\left(1+\overline{r}\right)-1\right],\end{aligned}\tag{82}$$

where the last equation uses $p_{HL}(s)=\alpha(1-\alpha)\phi(s)$ and $\mu_{HL}(s)=\frac{q\alpha(1-\alpha)\phi_1(s)}{p_{HL}(s)}=q_h^hs$. As $\mu_{HH}\left(x\right)>\mu_{HL}(x)=q_h^hx$ from belief updating, $q_h^hx\left(1+\overline{r}\right)-1<0<\mu_{HH}\left(x\right)\left(1+\overline{r}\right)-1$ must hold for Bank A to break even.

Now we aim to characterize the indirect effect on Bank A's participation and show that it is dominated. We apply the Implicit Function Theorem on Bank A's break-even condition, $0=\pi^A\left(\overline{r}|\,x;\eta\right)\equiv\frac{\pi^A(\overline{r},x;\eta)}{\phi(x)}$, to solve for $\frac{\partial x}{\partial\eta}$, where $\pi^A\left(\overline{r},x;\eta\right)$ is provided in (82). We condition on the realization of soft signal x to shut down the effects of signal density $\phi(\cdot)$ on Bank A's profits. From Eq. (82), we have

$$\begin{aligned}
\frac{\partial \pi^A\left(\overline{r}\middle|\, x;\eta\right)}{\partial \eta} &= -\frac{\partial}{\partial \eta}\left[\frac{p_{HH}\left(x;\eta\right)}{\phi(x)}\right]\cdot\int_0^x \phi_1\left(t\right)dt \underbrace{=}_{(35)} q_h^h(2\alpha-1)\frac{\phi_0\left(x\right)}{\phi(x)}\cdot\int_0^x \phi_1\left(t\right)dt>0,\\
\frac{\partial \pi^A\left(\overline{r}\middle|\, x;\eta\right)}{\partial x} &= \underbrace{\frac{p_{HH}\left(x\right)}{\phi\left(x\right)}\phi_1\left(x\right)\left[\mu_{HH}\left(x\right)\left(1+\overline{r}\right)-1\right]}_{+}\\
&\quad \underbrace{+\int_0^x \phi_1\left(t\right)dt\cdot\frac{\partial\left\{\frac{p_{HH}(x;\eta)}{\phi(x)}\left[\mu_{HH}\left(x\right)\left(1+\overline{r}\right)-1\right]\right\}}{\partial x}+\overline{p}_{HL}\cdot\frac{\partial\left[q_h^h x\left(1+\overline{r}\right)-1\right]}{\partial x}}_{+}>0. \qquad (83)
\end{aligned}$$

For $\frac{\partial \pi^A(\overline{r}|x;\eta)}{\partial \eta}$, the second inequality uses Eq. (35) which implies

$$\frac{p_{HH}\left(x\right)}{\phi\left(x\right)}=q_h^h\alpha^2+\left(1-q_h^h\right)\left(1-\alpha\right)^2-\frac{\eta\left(2\alpha-1\right)}{1-q_s}q_h^h\left(1-x\right). \tag{84}$$

For $\frac{\partial \pi^A(\overline{r}|x;\eta)}{\partial x}$, the second and third terms are positive because $\frac{p_{HH}(x)}{\phi(x)}$ in (84), and posterior beliefs $\mu_{HH}(x)$, $q_h^h x$ are all increasing in x.

Use the Implicit Function Theorem and the fact that the last two terms in Eq. (83) are positive, we have a lower bound of $\frac{\partial x}{\partial \eta}$:

$$0>\frac{\partial x}{\partial \eta}=-\frac{\frac{\partial \pi^A(\eta,x)}{\partial \eta}}{\frac{\partial \pi^A(\eta,x)}{\partial x}}>-\frac{q_h^h(2\alpha-1)\phi_0\left(x\right)\cdot\int_0^x\phi_1\left(t\right)dt}{p_{HH}\left(x\right)\phi_1\left(x\right)\left[\mu_{HH}\left(x\right)\left(1+\overline{r}\right)-1\right]}.$$

Use this inequality in Eq. (81), we have

$$\begin{aligned}
\frac{dW}{d\eta} &= q_h^h(2\alpha-1)-\frac{\partial x}{\partial \eta}\left(1-\alpha\right)\alpha\phi\left(x\right)\overbrace{\left[q_h^h x\left(\overline{r}+1\right)-1\right]}^{-}\\
&> q_h^h(2\alpha-1)+\frac{q_h^h(2\alpha-1)\phi_0\left(x\right)\int_0^x\phi_1\left(t\right)dt}{p_{HH}\left(x\right)\phi_1\left(x\right)\left[\mu_{HH}\left(x\right)\left(1+\overline{r}\right)-1\right]}\left(1-\alpha\right)\alpha\phi\left(x\right)\left[q_h^h x\left(\overline{r}+1\right)-1\right]\\
&= q_h^h(2\alpha-1)+\frac{q_h^h(2\alpha-1)\phi_0\left(x\right)\int_0^x\phi_1\left(t\right)dt}{p_{HH}\left(x\right)\phi_1\left(x\right)\left[\mu_{HH}\left(x\right)\left(1+\overline{r}\right)-1\right]}\left\{-p_{HH}\left(x\right)\int_0^x\phi_1\left(t\right)dt\left[\mu_{HH}\left(x\right)\left(1+\overline{r}\right)-1\right]\right\}\\
&= q_h^h(2\alpha-1)\left[1-\frac{\phi_0\left(x\right)\left(\int_0^x\phi_1\left(t\right)dt\right)^2}{\phi_1\left(x\right)}\right],
\end{aligned}$$

where the second last equation holds from Bank A's break even condition upon x in Eq. (82). The second bracketed term in the last equation equals

$$1-\frac{\phi_0\left(x\right)\cdot\left(\int_0^x\phi_1\left(t\right)dt\right)^2}{\phi_1\left(x\right)}=1-\frac{\left(1-x\right)\left(\int_0^x t\phi\left(t\right)dt\right)^2}{q_s\left(1-q_s\right)x}.$$

From the Cauchy-Schwartz inequality,

$$q_s\left(1-q_s\right)=\int_0^1 t\phi\left(t\right)dt\int_0^1\left(1-t\right)\phi\left(t\right)dt\geq\left(\int_0^1\sqrt{t\left(1-t\right)}\phi\left(t\right)dt\right)^2,$$

therefore we have the following inequality

$$\begin{aligned}1-\frac{\phi_0\left(x\right)\cdot\left(\int_0^x\phi_1\left(t\right)dt\right)^2}{\phi_1\left(x\right)}&=1-\frac{\left(1-x\right)\left(\int_0^x t\phi\left(t\right)dt\right)^2}{q_s(1-q_s)x}\\&\geq1-\frac{1-x}{x}\cdot\left(\frac{\int_0^x t\phi\left(t\right)dt}{\int_0^1\sqrt{t\left(1-t\right)}\phi\left(t\right)dt}\right)^2\\&=1-\left(\sqrt{\frac{1-x}{x}}\frac{\int_0^x t\phi\left(t\right)dt}{\int_0^1\sqrt{t\left(1-t\right)}\phi\left(t\right)dt}\right)^2\\&=1-\left(\frac{\int_0^x\sqrt{t}\cdot\sqrt{t-tx}\phi\left(t\right)dt}{\int_0^1\sqrt{t}\cdot\sqrt{x-tx}\phi\left(t\right)dt}\right)^2\geq0.\end{aligned}$$

Here, the last inequality follows because $x\leq1$ and $t\leq x$. Therefore, total welfare strictly increases in the case of positive weak equilibrium as well.

□

B.6 Derivation of Correlated Hard Signals

Another aspect of information technology advancement is that the lenders' hard information signals become more correlated. Formally, with probability ρ_h, lenders receive the same signal realization $h^c\in\{H,L\}$ and

$$\mathbb{P}\left(h^c=H\left|\theta_h=1\right.\right)=\mathbb{P}\left(h^c=L\left|\theta_h=0\right.\right)=\alpha;$$

with probability $1-\rho_h$, each receives an independent hard signal according to Eq. (4).

With more correlated hard signals or a higher ρ_h, lenders are more likely to agree on the customer quality and so more likely to compete (the event of HH). In terms of inference, the information content of competition (HH) declines due to correlation: $\mu_{HH}\left(\rho_h\right)$ decreases in ρ_h; in contrast, the posterior upon disagreement remains unchanged at the prior q_h, as in the baseline model. Taken together, correlated hard signals intensifies competition—lenders are more likely to compete, but not more concerned about the winner's curse.

B.7 Appendix for Additional Hard Signals on θ_s^h

Lender problem under $\rho_s^h=1$. We show that Bank A and Bank B's problem can be rewritten as in our baseline setup. Recall that the hard signals h^j for $j\in\{A,B\}$ are informative about θ_h^h, and the hardened soft signal h_s^c is a public signal ($\rho_s^h=1$) about θ_h^s.

For notation, since h^j is independent from h_s^c and the soft signal s, we separately define the joint probability and the posterior belief of hard signal realizations as $p_{h^Ah^B}\equiv\mathbb{P}\left(h^A,h^B\right)$ and $\mu_{h^Ah^B}\equiv\mathbb{P}(\theta=1|h^A,h^B)$ respectively. For correlated h_s^c and s, we introduce $p_{h_s^c}^s(s)\equiv\mathbb{P}\left(h_s^c,s\in ds\right)$ as the joint probability of h_s^c and the soft signal s falling in the interval $(s,s+ds)$ and $\mu_{h_s^c}^s(s)\equiv\mathbb{P}(\theta=1|h_s^c,s)$ as the posterior belief; we also define $\overline{p}^s_{h^c_s} = \mathbb{P}\left(h^c_s\right)$ and $\overline{\mu}^s_{h^c_s}(s) \equiv \mathbb{P}(\theta = 1 | h^c_s)$ for the hardened soft signal h^c_s alone. Then, Bank B's lending profits when quoting r and $h^B = h^B_s = H$, are similar to (16):

$$\pi^B\left(r\right) = \int_0^{s^A(r)} \underbrace{p_{HH} p^s_H(t)}_{h^A=h^B=h^c_s=H,t} \left[\mu_{HH}\mu^s_H(t)\left(r+1\right) - 1\right] dt + \underbrace{p_{LH}\overline{p}^s_H}_{h^A=L,h^B=h^c_s=H} \left[\mu_{LH}\overline{\mu}^s_H\left(r+1\right) - 1\right]. \tag{85}$$

And, Bank A's profit when quoting r and $\left\{h^A = H, h^c_s = H, s\right\}$ is similar to (15):

$$\pi^A\left(r,s\right) = \underbrace{p_{HH} p^s_H(s)}_{h^A=h^B=h^c_s=H,s} \underbrace{\left[1 - F^B\left(r\right)\right]}_{A \text{ wins}} \left[\mu_{HH}\mu^s_H(s)\left(1+r\right) - 1\right] + \underbrace{p_{HL} p^s_H(s)}_{h^A=h^c_s=H,h^B=L,s} \left[\mu_{HL}\mu^s_H(s)\left(1+r\right) - 1\right]. \tag{86}$$

Now let us redefine the soft signal to be $\tilde{s} \equiv \mu^s_H(s) \equiv \mathbb{P}(\theta = 1 | h^c_s = H, s)$, and let $\tilde{\phi}(s) \equiv \frac{p^s_H(s)}{\overline{p}^s_H}$ to be its density. Then banks' objective functions could be rewritten as

$$\begin{aligned}
\pi^A\left(r,s\right) &= \overline{p}^s_H \left\{p_{HH}\tilde{\phi}(s)\left[1 - F^B\left(r\right)\right]\left[\mu_{HH}\tilde{s}\left(1+r\right) - 1\right] + p_{HL}\left[\mu_{HL}\tilde{s}\left(1+r\right) - 1\right]\right\}, \\
\pi^B\left(r\right) &= \overline{p}^s_H \left\{\int_0^{s^A(r)} p_{HH}\tilde{\phi}(t)\left[\mu_{HH}\tilde{t}\left(r+1\right) - 1\right] dt + p_{LH}\left[\mu_{LH}\tilde{s}\left(r+1\right) - 1\right]\right\}.
\end{aligned}$$

They are isomorphic to our baseline model with $\eta = 0$, with a gap being the constant $\overline{p}^s_H$.

Lender problem under $\rho^h_s < 1$. In this part, we assume that the hardened soft signal (which is about θ^s_h) is no longer public across lenders. We introduce h^j_s to denote lender j's hardened soft signal for $j \in \{A, B\}$. For simplicity, we continue to assume that both the hard signal h^j (which is about θ^h_h) and the hardened soft signal h^j_s are decisive, such that a lender rejects the borrower when either h^j or h^j_s is negative, and is willing to participate only when $h^j = h^j_s = H$.

Notation is largely the same from that in the previous discussion before Eq. (85). One modification here is for the hardened soft signal: we use $p^s_{h^A_s h^B_s}(s) \equiv \mathbb{P}(h^A_s, h^B_s, s \in ds)$ to represent the joint probability of signals and $\mu^s_{h^A_s h^B_s}(s) \equiv \mathbb{P}\left(\theta = 1 \left| h^A_s, h^B_s, s \in ds\right.\right)$ for the posterior belief; we define $\overline{p}^s_{h^A_s h^B_s} \equiv \mathbb{P}(h^A_s, h^B_s)$ and $\mu^s_{h^A_s h^B_s} \equiv \mathbb{P}\left(\theta = 1 \left| h^A_s, h^B_s\right.\right)$ accordingly, where the overline means integrating over possible soft signal realizations.

We use Bank A as an example. Bank A's profits when quoting r, which requires $h^A = h^A_s = H$, are:

$$\begin{aligned}
\pi^A\left(r,s\right) =& \underbrace{p_{HH} p^s_{HH}(s)}_{h^B=h^B_s=H} \underbrace{\left[1 - F^B\left(r\right)\right]}_{A \text{ wins}} \left[\mu_{HH}\mu^s_{HH}(s)\left(1+r\right) - 1\right] + \underbrace{p_{HL} p^s_{HH}(s)}_{h^B=L,h^B_s=H} \left[\mu_{HL}\mu^s_{HH}(s)\left(1+r\right) - 1\right] \\
&+ \underbrace{p_{HH} p^s_{HL}(s)}_{h^B=H,h^B_s=L} \left[\mu_{HH}\mu^s_{HL}(s)\left(1+r\right) - 1\right] + \underbrace{p_{HL} p^s_{HL}(s)}_{h^B=h^B_s=L} \left[\mu_{HL}\mu^s_{HL}(s)\left(1+r\right) - 1\right] \\
=& p_{HH} p^s_{HH}(s)\left[1 - F^B\left(r\right)\right]\left[\mu_{HH}\mu^s_{HH}(s)\left(1+r\right) - 1\right] \\
&+ \left[p_{HL} p^s_{HH}(s)\mu_{HL}\mu^s_{HH}(s) + p_{HH} p^s_{HL}(s)\mu_{HH}\mu^s_{HL}(s) + p_{HL} p^s_{HL}(s)\mu_{HL}\mu^s_{HL}(s)\right]\left(1+r\right) \\
&- \left[p_{HL} p^s_{HH}(s) + p_{HH} p^s_{HL}(s) + p_{HL} p^s_{HL}(s)\right].
\end{aligned} \tag{87}$$

As seen from the first equation, Bank A participates only when $h^A = h^A_s = H$ and expects competition from

Bank B only when $h^B = h_s^B = H$—otherwise, when either h^B or h_s^B is negative, Bank A is the only lender making the offer (non-competition case.) The second equality rewrites Bank A's profits by regrouping Bank A's revenue and lending costs in the non-competition case.

Now, we combine a lender's hard signal and hardened soft signal into *combined hard signal* $\tilde{h}^j$: this combined signal is positive only when both signals are positive, and is negative when either signal is negative,

$$\{\tilde{h}^j = H\} \equiv \{h^j = h_s^j = H\}, \quad \{\tilde{h}^j = L\} \equiv \{h^j = L, \text{ or } h_s^j = L.\}$$

Note that this combined signal is binary, and decisive for lending—lender j rejects the borrower upon $\tilde{h}^j = L$. Accordingly, we introduce $\tilde{p}_{\tilde{h}^A \tilde{h}^B}(s) \equiv \mathbb{P}(\tilde{h}_s^A, \tilde{h}_s^B, s \in ds)$ to represent the joint probability of signals and $\tilde{\mu}_{\tilde{h}^A \tilde{h}^B}(s) \equiv \mathbb{P}\left(\theta = 1 \middle| \tilde{h}_s^A, \tilde{h}_s^B, s \in ds\right)$ for the posterior belief; we define $\overline{\tilde{p}}_{h_s^A h_s^B} \equiv \mathbb{P}(\tilde{h}_s^A, \tilde{h}_s^B)$ and $\overline{\tilde{\mu}}_{\tilde{h}_s^A \tilde{h}_s^B} \equiv \mathbb{P}\left(\theta = 1 \middle| h_s^A, h_s^B\right)$ accordingly. Using the definition of $\tilde{h}^j$, we have

$$\tilde{p}_{HL}(s) \equiv p_{HL} p_{HH}^s(s) + p_{HH} p_{HL}^s(s) + p_{HL} p_{HL}^s(s),$$

and

$$\tilde{p}_{HH}(s) = p_{HH} p_{HH}^s(s), \ \tilde{\mu}_{HH}(s) = \mu_{HH} \mu_{HH}^s(s),$$

where we used the fact that hard signals are independent from hardened soft signals. In addition, for any event X where X refers to some signal realizations, $p_X \mu_X = \mathbb{P}(\theta = 1, X)$, so

$$\tilde{p}_{HL}(s)\,\tilde{\mu}_{HL}(s) = p_{HL} p_{HH}^s(s) \mu_{HL} \mu_{HH}^s(s) + p_{HH} p_{HL}^s(s) \mu_{HH} \mu_{HL}^s(s) + p_{HL} p_{HL}^s(s) \mu_{HL} \mu_{HL}^s(s).$$

Therefore, Bank A's profits in Eq. (87) could be rewritten as:

$$\pi^A(r, s) = \tilde{p}_{HH}(s)\left[1 - F^B(r)\right]\left[\tilde{\mu}_{HH}(s)(1 + r) - 1\right] - \tilde{p}_{HL}(s)\left[\tilde{\mu}_{HL}(s)(1 + r) - 1\right]. \tag{88}$$

Similarly, the objective function Bank B could be rewritten as

$$\pi^B(r) = \int_0^{s^A(r)} \tilde{p}_{HH}(t)\left[\tilde{\mu}_{HH}(t)(r + 1) - 1\right] dt + \overline{\tilde{p}}_{LH}\left[\overline{\tilde{\mu}}_{LH}(r + 1) - 1\right].$$

There are two points worth noting. First, the combined hard signals and the soft signal are conditionally independent given $\theta = 1$, which is the key condition for tractability in our baseline model. Hence, credit market equilibrium under this alternative setting could be characterized in a similar way as in our baseline model. Second, this alternative setting is not isomorphic to our baseline model, because the binary combined hard signal $\tilde{h}^j$ is no longer symmetric in Type I and Type II mistakes. While this does not affect equilibrium characterization (symmetric signals are not required), the model implications could be quantitatively different (Type II mistakes are more prevalent).

The effect of α_s under $\rho_s^h = 1$. We discuss the effect of the precision of the hardened soft signal α_s on the three key terms capturing the Winner's Curse for Bank B in competition (introduced in Section 3.2.) First, it is easy to see the probability of competition conditional on Bank B receiving positive signals, that is $\mathbb{P}(h^A = h^B = h_s^c = H | h^B = h_s^c = H)$ in this alternative model, is a constant in α_s (therefore, weakly increasing in α_s). This is because the hardened soft signal h_s^c is public under $\rho_s^h = 1$ and it is independent of the hard signals h^j for $j \in \{A, B\}$:

$$\mathbb{P}(h^A = h^B = h_s^c = H | h^B = h_s^c = H) = \frac{\mathbb{P}\left(h^A = h^B = h_s^c = H\right)}{\mathbb{P}\left(h^B = h_s^c = H\right)} = \frac{\mathbb{P}\left(h^A = h^B = H\right)\mathbb{P}\left(h_s^c = H\right)}{\mathbb{P}\left(h^B = H\right)\mathbb{P}\left(h_s^c = H\right)}.$$

Next, we show that when α_s increases, the distribution of the soft signal conditional on competition (the event of $h^A = h^B = h_s^c = H$) shifts to the right

$$\begin{aligned}
\phi\left(s | h^A = h^B = h_s^c = H\right) = \phi\left(s | h_s^c = H\right) &= \frac{q_s^h \alpha_s \left[q_s^s \phi_1(s) + (1 - q_s^s)\phi_0(s)\right] + \left(1 - q_s^h\right)(1 - \alpha_s)\phi_0(s)}{q_s^h \alpha_s + (1 - q_s^h)(1 - \alpha_s)} \\
&= \frac{q_s^h \alpha_s \left[q_s^s \phi_1(s) + \phi_0(s) - q_s^s \phi_0(s)\right] + \left(1 - q_s^h\right)(1 - \alpha_s)\phi_0(s)}{q_s^h \alpha_s + (1 - q_s^h)(1 - \alpha_s)} \\
&= \phi_0(s) + \underbrace{\frac{q_s^h q_s^s}{q_s^h + (1 - q_s^h)\left(\frac{1}{\alpha_s} - 1\right)}}_{>0, \uparrow \text{ in } \alpha} \underbrace{\left[\phi_1(s) - \phi_0(s)\right]}_{<(>)0 \text{ if } s<(>)q_s}.
\end{aligned}$$

The first equality uses the fact that hard signals, which are only informative about θ_h^h, are independent of s under this alternative setting. The key coefficient in the last equation, $\frac{q_s^h q_s^s}{q_s^h + (1 - q_s^h)\left(\frac{1}{\alpha_s} - 1\right)} > 0$, means that upon competition, lenders update their beliefs about s and put more weight towards $\phi_1(s)$—that is, the soft fundamental is more likely to be good ($\theta_s = 1$).

Lastly, we examine the effect of α_s on $z_s^{s(-1)}(z)$. We apply the Implicit Function Theorem on

$$z_s^s(s; \alpha_s) = \frac{q_s^h \alpha_s q_s^s \phi_1(s) + \left(1 - q_s^h\right)(1 - \alpha_s) q_s^s \phi_0(s)}{q_s^h \alpha_s \left[q_s^s \phi_1(s) + (1 - q_s^s)\phi_0(s)\right] + (1 - q_s^h)(1 - \alpha_s)\phi_0(s)}.$$

It is easy to check that $z_s^s(s; \alpha_s)$ increases in s.

$$z_s^s(s; \alpha_s) = \frac{1}{1 + \overbrace{\frac{\left[(1 - q_s^h)(1 - \alpha_s) + q_s^h \alpha_s\right](1 - q_s^s)}{q_s^h \alpha_s q_s^s \frac{\phi_1(s)}{\phi_0(s)} + \left(1 - q_s^h\right)(1 - \alpha_s) q_s^s}}^{\uparrow \text{ in } s}}.$$

Now we discuss how $z_s^s(s; \alpha_s)$ changes with α_s. Define $M(\alpha_s) \equiv q_s^h \alpha_s q_s^s \phi_1(s) + \left(1 - q_s^h\right)(1 - \alpha_s) q_s^s \phi_0(s)$ and $N(\alpha_s) \equiv q_s^h \alpha_s (1 - q_s^s)\phi_0(s) + \left(1 - q_s^h\right)(1 - q_s^s)(1 - \alpha_s)\phi_0(s)$ and then $z_s^s(s; \alpha_s) = \frac{M(\alpha_s)}{M(\alpha_s) + N(\alpha_s)}$. Take derivative with respect to α_s:

$$\begin{aligned}
\operatorname{sgn}\left[\frac{\partial z_s^s(s; \alpha_s)}{\partial \alpha_s}\right] &= \operatorname{sgn}\left\{M'(\alpha_s)\left[M(\alpha_s) + N(\alpha_s)\right] - M(\alpha_s)\left[M'(\alpha_s) + N'(\alpha_s)\right]\right\} \\
&= \operatorname{sgn}\left[M'(\alpha_s) N(\alpha_s) - M(\alpha_s) N'(\alpha_s)\right].
\end{aligned}$$

Note that $M(\alpha_s), N(\alpha_s)$ are linear in α_s so

$$\left[M'(\alpha_s) N(\alpha_s) - M(\alpha_s) N'(\alpha_s)\right]' = M'(\alpha_s) N'(\alpha_s) - M'(\alpha_s) N'(\alpha_s) = 0,$$

which means $M'(\alpha_s)\,N(\alpha_s) - M(\alpha_s)\,N'(\alpha_s)$ is a constant inα_s. Therefore,

$$\begin{aligned}\operatorname{sgn}\left[\frac{\partial z_s^s(s;1)}{\partial \alpha_s}\right] &= \operatorname{sgn}\left[M'(1)\,N(1) - M(1)\,N'(1)\right] \\ &= \operatorname{sgn}\left\{\left(1-q_s^h\right)\left(1-q_s^s\right) q_s^h q_s^s \phi_0(s)\underbrace{\left[\phi_1(s)-\phi_0(s)\right]}_{<0 \text{ when } s<q_s}\right\}.\end{aligned}$$

This means that $z_s^s(s;\alpha_s)$ is strictly decreasing in α_s when $s < q_s$. Taken together, since $z_s^s(s;\alpha_s)$ increases in s and decreases in α_s when $s < q_s$, the threshold signal that induces belief z, $s = z^{s(-1)_s}(z)$ increases in α_s.

In sum, in this alternative model, the effect of α_s on the Winner's Curse faced by Bank B resembles the effect of signal precision instead of information span in our baseline model.