Demand Elasticity in Dynamic Asset Pricing*

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Abstract

Standard demand elasticity estimation treats investors' demand slopes as stable objects that can be traced out by exogenous residual supply shifts. We show this identification strategy fails in dynamic settings: supply shocks cause demand curves to tilt and shift through general equilibrium effects. The mechanism is intuitive — investors' demand depends on the entire distribution of current and future returns, including volatility, covariances, and correlations with investment opportunities. Supply shocks that change today's prices inevitably reshape future return distributions, moving the demand curve itself. We develop and calibrate a dynamic model to quantify this mismeasurement. The measured slope is approximately 40% of its conceptual counterpart, implying that demand curves are substantially steeper than estimated. This distortion operates through two channels: endogenous risk (altered volatility and covariances) and amplified intertemporal hedging (changed correlation with investment opportunities). The distortion remains sizable even for infinitesimal or purely transitory shocks.

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1 Introduction

Measuring the sensitivity of investors' asset demand to changes in returns is essential for understanding a wide range of questions faced by academics, practitioners, and policymakers: How much should GameStop's price react to a persistent surge in retail demand? How much should the yield curve drop in response to a central bank's QE program? How much should green industry stocks appreciate if ESG funds are expected to double in size over the next decade?

The standard approach of demand elasticity estimation treats the slope of investor's demand as a stable object that can be traced out by exogenous shifts in residual supply. In this paper, we show that in a dynamic setting, standard general equilibrium forces cause the demand curve to both tilt and shift in response to residual supply shocks invalidating the identification assumption. In general, investors' demand for an asset with a given expected return depends on the entire distribution of current and future returns, often captured by volatility, return comovement and correlation with future investment opportunities (Merton, 1971). If a residual supply shock changes the equilibrium price, it is likely to reshape the distribution of future returns as well, moving the demand curve itself.

To understand the economic mechanism and to sign and quantify the resulting mismeasurement, we develop and calibrate a dynamic model with shifts in residual supply. We find that the measured slope is approximately 40% of its conceptual counterpart—demand curves are substantially steeper than standard estimates suggest. This distortion arises from two distinct channels: an endogenous risk effect, whereby supply shocks alter return volatility and covariances, and an amplified intertemporal hedging effect, whereby these shocks change how returns comove with future investment opportunities. While the effect is larger for shocks with greater persistence, we show that distortion does not diminish as the identifying shock becomes fully transitory.

Our multi-asset-multi-agent dynamic asset pricing model is in continuous time. Assets are claims on risky cash flows driven by a multi-dimensional Brownian shock. The economy is populated by two types of agents: hedgers and investors. Investors are long-lived and solve a canonical consumption-saving problem, with optimal portfolio decisions similar to that in Merton (1971). For simplicity, hedgers are assumed to be short-lived. Hedgers receive a stochastic endowment, which is correlated with the assets' cash flows. This correlation is changing over time driven by a Poisson process. This leads to periodic shifts in hedgers' demand for the assets, that is, shifts in investors' residual supply. The intensity of the Poisson process parametrizes the persistence of the supply shock. In equilibrium, asset prices adjust in response.

Our main question is that whether observing the price and quantity response due to an exogenous shift in residual supply can help recover the slope of the representative investor's demand curve.

Consider a single risky asset. In a static version of our economy, the answer is affirmative. As in the Markowitz model, the slope of investors' static demand is the ratio of risk-tolerance and return variance. Both are primitives in a static economy. When hedgers demand less of an asset, institutions have to hold more, leading to an increase in its expected equilibrium return. Comparing the changes in investors' demand for the assets and the induced changes in expected returns reveals the slope of their demand curve.

This is in contrast even with the simplest case of our dynamic model where investors have log-utility. In line with Merton (1971), the slope of log-investor's demand curve is simply the inverse of return variance. However, in a dynamic model return volatility is an endogenous object. In fact, when hedgers partially liquidate their positions following the shock, investors have to increase their risk exposure in equilibrium, which gives rise to higher volatility of the asset return. This is a standard amplification effect: Investors' larger risk exposure implies that adverse future shocks reduce their risk-bearing capacity, increasing risk-premium, resulting in a larger price drop, amplifying the initial effect.[1] Because the slope of the investors' demand curve is the inverse of the return volatility, the shift in residual supply decreases the slope we wish to recover, leading to a flatter measured slope relative to its conceptual counterpart. Interestingly, we show that this effect is bounded away from zero even in the limit when the residual supply shock is fully transitory. Figure 1 illustrates this point with a calibrated example.

Allowing for multiple assets adds a covariance channel to this mechanism. As the endogenous volatility of all assets is due to investors' fluctuation of wealth, when investors hold more of a given asset, both the asset's own volatility and its comovement with every other asset increase. Intuitively, assets with idiosyncratic dividend risk become more correlated when the pricing kernel—a function of investors' aggregate wealth—becomes more volatile. In our multi-asset economy, the slope of investors' demand curve is proportional to the inverse of the covariance matrix. As a result, the variance and covariance channels work in the same direction. Both tilt the demand curve, making the measured slope flatter than its conceptual counterpart. We call this distortion through the endogenous covariance matrix the *endogenous risk effect*.

When investors' relative risk-aversion is different from one, there is an additional effect connected to a general equilibrium version of Merton's intertemporal hedging demand. In our economy, each investor's relative demand for different assets depends on the portfolio of other investors. It is so, because each understands that if others overweight a given asset, whenever that asset performs well, the groups' risk-bearing capacity improves, risk premia decrease,

[1] Larger risk-exposure leading to larger endogenous volatility is a standard effect in typical dynamic asset pricing models with heterogeneous agents, including Longstaff and Wang (2012), He and Krishnamurthy (2013), Brunnermeier and Sannikov (2014), and Kondor and Vayanos (2019), among others.

making investment opportunities worse. As Merton (1971) explains, when the investor's risk-aversion is larger than one, she wants to hedge against that, increasing her exposure to the overweighted assets. Therefore, when hedgers are shocked and sell some assets to investors, each investor's demand slope shifts upward due to this force. Again, the residual supply shock affects the demand curve. A smaller-than-one-risk-aversion implies a downward shift. This is the *amplified intertemporal hedging effect.*

Finally, investors will also be concerned whether the shift in residual supply is persistent. Each investor understands that if and when households' demand shifts back and they buy back the shocked assets, prices will adjust affecting the wealth of investors who previously overweighted the shocked assets. Again, foreseeing this effect alters the required change in expected returns today for financial institutions to be willing to hold the increased supply of shocked assets. That is, again the slope of their demand curve is affected by the shift in residual supply. We refer to this effect as *hedging against the reversal.*

We formalize our observations by comparing two objects: the *measured slope* and the *conceptual slope.* The former is simply the equilibrium change in investors' demand per unit change in expected returns due to the supply shock. Instead, as the conceptual counterpart of the former, the latter captures the standard economic concept of the slope of a demand curve as derived from the first-order condition—that is, the optimal quantity response of a representative investor per unit change in instantaneous expected returns.

We consider two variants of the conceptual slope. The *myopic slope* measures the investor's response to a fully transitory change in expected returns. The advantage of this concept is its simplicity. As we show, analogously to the static slope, the myopic slope is derived from the product of risk-tolerance and the inverse of the (now endogenous) covariance matrix.

Instead, the *dynamic slope* is parametrized to allow for a wide variety of changes in future price path. Formally, it gives the answer to the question of how the long-term investor would change her portfolio shares under a counterfactual dividend process which would increase the expected returns until a Poisson shock hits. It is parameterized by the size and direction of change in expected returns, the persistence of the shock, and the fraction of returns which comes as a flow as opposed to a lump sum at reversal. While this concept is more complex, it gives a fairer comparison between measured and conceptual slope.[2] In particular, by choosing its parameters carefully, we show that the measured slope does not match its conceptual counterpart even if they correspond to the same change in the instantaneous expected return vector and to similar changes in the follow-up price path.

In the second part of the paper we quantify our effects by building two calibrated applications. In our first application, we are interested in our effects in the factor-elasticity

[2]This is in line with the emphasis in Gabaix and Koijen (2022) that the persistence of the shock should matter for the response in quantities.

context. We calibrate our parameters to Ben-David, Li, Rossi and Song (2021) who measure the price effect of a persistent change in the demand from mutual funds due to a change in Morningstar's rating methodology. In our second application, we are interested in the likely magnitude of "index inclusion" (Shleifer, 1986; Greenwood and Sammon, 2024); hence, this application is in a micro-elasticity context.

Although, in line with Gabaix and Koijen (2022), the magnitudes of the measured slope differ across the factor-elasticity and micro-elasticity exercises, we find similar quantitative patterns of mismeasurement across our two calibrations. First, we note the conceptual benchmarks of myopic slope and dynamic slope are quantitatively very close. Therefore, the mismeasurement is virtually the same regardless of which one we compare the measured slope to. Second, in each of our calibrations the measured slope is approximately 40% of its conceptual counterparts. This suggests that investors' demand curves might be significantly steeper than their typical estimates. Third, our decomposition shows that the endogenous risk effect tends to be the bulk of the deviation. The amplified intertemporal hedging effect goes against the endogenous risk effect whenever investors are more risk-averse than the log investor and amplifies it otherwise. Quantitatively, this second component can be responsible for up to 30% of the mismeasurement when the implied shock in returns is large and persistent. Finally, we find that the hedging for reversal effect tends to be small.

We conclude with a discussion on the implications of our work to the potential way forward. Our results imply that focusing on transitory shocks reduces the mismeasurement, but cannot eliminate it. We also point out that controlling for risk measures aiming to address the endogenous risk-channel would introduce a bad-control problem (Angrist and Pischke, 2009). Hence, we recommend a richer characterization of the structure of asset demand, taking into account the two-way interaction between changes in the distribution of future return paths and the allocation of risk across agents.

Literature Review. To our knowledge, this paper is the first to highlight that exogenous shocks to residual supply—arising from dynamic general equilibrium effects through changing risk and investment opportunities—can alter the very demand slope that econometricians seek to estimate. In the empirical literature, the closest evidence to our mechanism comes from Haddad, Moreira and Muir (2024), who show that the Federal Reserve's quantitative easing program—interpretable as a series of residual supply shocks from investors' perspective—changed market perceptions of the future return distribution of government bonds, thereby shifting investors' demand for them. This finding is consistent with our central insight that residual supply shocks can endogenously reshape demand curves. Building on this intuition, our paper connects and extends several strands of the existing literature.

The seminal papers of demand system estimation, (Koijen and Yogo, 2019; Koijen, Richmond and Yogo, 2023; Gabaix and Koijen, 2022) set up dynamic models as the conceptual background for their estimates. However, in these frameworks the possibility of our effects is ruled out by assumption. For instance, in Koijen and Yogo (2019) the form of the covariance matrix is determined by an assumption under which it is independent of the residual supply shocks. This rules out the endogenous risk effect. Similarly, the main model in Gabaix and Koijen (2022) (as its generalized version in Gabaix, Koijen, Mainardi, Oh and Yogo (2025)) assumes that investors' response to changes in expected returns is driven by an exogenous, constant coefficient. In contrast, our model allows for endogenous risk deriving the equilibrium covariance matrix from first principles.

The recent papers of An (2024), Fuchs, Fukuda and Neuhann (2025) and Haddad, He, Huebner, Kondor and Loualiche (2025) (HHHKL, from now on) focus on problems stemming from the fact that investors' demand curve for a given asset potentially depends on the prices of all assets in the investment opportunity set. HHHKL work out a flexible method to estimate each coefficient of the resulting demand curve, using the cross-section for relative elasticity between assets with same observables and the time-series for substitution across assets with different observables.[3] As these papers focus on a static economy where demand curve coefficients are exogenously given by primitives, while at the same time we allow the econometrician to observe the effect of the shock to all prices, our work is orthogonal to this literature. That is, all our effects are above and beyond the ones highlighted in these papers.

Perhaps closest to our paper is the concurrent work by van Binsbergen, David and Opp (2025), who also highlight that supply shocks as instruments might violate the exclusion restriction in a dynamic economy. van Binsbergen et al. (2025) focus on bias in price-elasticity estimation (i.e. the partial derivative of demand to the current price) arising from the disconnect between price-shifts and the corresponding shifts in expected returns. Since expected returns drive investors' demand for financial assets, price-elasticity estimates are unbiased only if the price-level shift is fully offset by a corresponding resolution return in the subsequent period.[4] Price-shift build-ups violate this condition. In contrast, our work directly focuses on the mismeasurement of investors' response to expected return shifts, side-stepping this disconnect. As shown in Section 6.2, our mechanism applies whether the econometrician estimates return elasticity or price elasticity.

As we mentioned, the endogenous risk effect is present in a wide range of models with heterogeneous agents with and without explicit financial frictions (e.g. Longstaff and Wang, 2012; He and Krishnamurthy, 2013; Brunnermeier and Sannikov, 2014; Kondor and Vayanos, 2019). Thanks to its simple structure, we build our model on Kondor and Vayanos (2019).

[3] In Section 6.2, we show the investors' endogenous slope matrix derived in our fully dynamic model satisfies the restriction imposed in HHHKL.

[4] As a connected empirical point, Davis, Kargar and Li (2023) show that focusing on price elasticity instead of return elasticity might be problematic, because "price pass-through"—a concept closely related to the pace of resolution of the price shock —strongly influences this mapping. See also An and Zheng (2025) for a dynamic asset pricing model with a role for the predictability of flow shocks.

None of these papers focus on the problem of recovering demand sensitivities using exogenous residual supply shocks. We are also unaware of any other models that emphasize the amplified intertemporal hedging effect.

Finally, in the companion paper He, Kondor and Li (2025), we take a step back from the view that asset markets can be seen as a demand system with well-defined supply and demand shocks. Instead, we focus on the quantitative effect of an exogenous supply change on equilibrium returns and prices.[5] As we explain in this paper, this shock affects the demand curves of each group, hence the multiplier does not correspond to the inverse of the demand elasticity for any group. Still, we provide sophisticated benchmarks for the literature on what "canonical" models imply on this aggregate multiplier and how sensitive it is to changes of parameters in the economy.

2 The Tilt and the Shift: A Minimal Demand System

To set the stage, we start with an illustrative example. We present a demand system with the minimal reduced-form properties required for our main effects to be present. In the dynamic model, these properties arise endogenously.

Minimal demand system. Consider the following minimal demand system with a single asset, which is given as follows:

$$\begin{aligned} \text{hedger's demand:} \quad & \hat{x} = c\,[\mathbb{E}(R) - r] + \varepsilon, \\ \text{investor's demand:} \quad & \hat{y} = b^{\xi}\,[\mathbb{E}(R) - r] + a^{\xi} + \eta, \\ \text{marker clearing:} \quad & S = x + y. \end{aligned} \tag{1}$$

Here, r denotes the exogenous risk-free rate while $\mathbb{E}(R)$ denotes the asset's *endogenous* expected return. The individual demand of any representative hedger and investor, denoted by $\hat{x}$ and $\hat{y}$ respectively, depends on the expected excess return $\mathbb{E}(R) - r$ with $\{\varepsilon, \eta\}$ being orthogonal error terms. We will consider below the effect of two realizations of the demand shock, $\varepsilon = \{\varepsilon^{\xi}\}$ with $\xi = \{s, n\}$ for stressed and normal. When $\varepsilon^{s} < \varepsilon^{n}$ hedgers dump assets on investors in the stressed state. The positive constant S is the asset's aggregate supply, which is fixed in our asset pricing context. As x and y are the aggregate counterparts of demand, the last equation represents market clearing.

The only non-standard element of the minimal asset demand system (1) is on the investor's side: both the slope, b^{ξ}, and the intercept, a^{ξ} are indexed by the state $\xi = \{s, n\}$. That is,

[5] In this regard our paper is also related to Berk and Van Binsbergen (2025), who show that the magnitude of the index inclusion effect in recent years (Greenwood and Sammon, 2024) is consistent with a classic CAPM setting, once carefully calibrated to take into account of the wealth share of index investors and the persistence of their demand shift due to index inclusion. We offer a more detailed discussion on this issue in Section 6.3.

we allow these parameters to change with the demand shock.

Demand slope and measured slope. We think of the slope of an agent's demand as her optimal quantity response to unit change in expected returns. Our focus is on the slope of the investors' asset demand. Suppose that an investor is asked how much she would increase her demand for the asset if $\mathbb{E}(R)$ would increase to $\mathbb{E}(R)+\delta$ where δ is an arbitrary constant. In the demand system in (1), this gives the *conceptual* slope of investors' demand curve in state ξ (we will use $\mathcal{C}$ for *c*onceptual):

$$\mathcal{C}^{\min} \equiv \frac{\hat{y}^h - \hat{y}}{\delta} = b^\xi, \tag{2}$$

where $\hat{y}^h$ is the demand under the *h*ypothetical increase in expected excess return.

Now suppose instead that we observe the equilibrium under the two realizations $\varepsilon = \{\varepsilon^n, \varepsilon^s\}$. By only shocking the x-demand-curve, we might be able to trace out the slope of y-demand-curve. In fact, the measured equilibrium ratio of changes in quantities and returns, that is, the *measured slope* is (we will use $\mathcal{M}$ for *m*easured)

$$\mathcal{M}^{\min} \equiv \frac{y^s - y^n}{\mathbb{E}(R^s) - \mathbb{E}(R^n)} = \underbrace{b^n}_{\substack{\text{demand supply} \\ \text{in } \xi = n}} + \underbrace{\frac{(b^s - b^n)[\mathbb{E}(R^s) - r]}{\mathbb{E}(R^s) - \mathbb{E}(R^n)}}_{\text{endogenous risk effect}} + \underbrace{\frac{a^s - a^n}{\mathbb{E}(R^s) - \mathbb{E}(R^n)}}_{\text{amplified hedging effects}}. \tag{3}$$

Clearly, if the slope and the intercept of investors' demand do not respond to the shock ε, that is, $b^s = b^n$ and $a^s = a^n$, this strategy identifies slope b^n. This is the text-book case.

However, as we will argue in rest of the paper, when hedgers dump assets on investors, investors' aggregate risk-exposure, y, changes, which tilts and shifts their demand curve in general. The resulting second and third terms bias the measurement. We refer to the second term as the endogenous risk effect; as we will show in a dynamic asset pricing model the tilt $b^s - b^n$ is related to the change in endogenous return volatility and co-movement, akin to the amplification effect in standard heterogeneous agent models in macroeconomics and finance (e.g. Longstaff and Wang, 2012; He and Krishnamurthy, 2013; Brunnermeier and Sannikov, 2014; Kondor and Vayanos, 2019). We will also explain how the shift, $a^s - a^n$, is related to intertemporal hedging amplified by general equilibrium forces.

Importantly, we will demonstrate that even if the shock to hedgers is small, implying in equilibrium only a small tilt $b^s - b^n$ or a small shift $a^s - a^n$, the second and third components still might be sizable. This is so, because the denominator is often the same order as the numerators in the last two terms of (3) in equilibrium. Intuitively, when the supply shock changes the return covariance only marginally, it also tends to change equilibrium returns at the same order of magnitude.

3 A Model of Dynamic Asset Pricing

We build a dynamic asset pricing model with multiple assets and heterogeneous investors based on Kondor and Vayanos (2019). Our aim is to study how returns and holdings of a particular group of investors change when an exogenous shock shifts their residual supply curve. To this goal, in this section we first set up the model, and then present our main objects of interest in the static benchmark.

3.1 The Dynamic Setup and Definitions

Time is continuous and infinite, indexed by $t \in [0, \infty)$.

Assets. There is a risk-free asset that yields a constant instantaneous rate of return $r > 0$, which is taken as exogenous in the model; that is, we do not clear the goods market.[6]

There are I long-lived risky assets with cumulative cash flows D_t following

$$dD_t = \bar{D}dt + \sigma^\top dB_t, \tag{4}$$

where $\bar{D}$ is a constant $I \times 1$ vector, σ is a constant and invertible $I \times I$ matrix, and $\top$ denotes transpose. Uncertainty of cash flows is described by the I-dimensional Brownian motion B_t. Each risky asset is in fixed supply, and we denote by S the $I \times 1$ vector consisting of asset supplies measured in terms of number of shares. We let $\Sigma \equiv \sigma^\top \sigma$.

Agents. The economy is populated by two types of agents: hedgers and investors. Without loss of generality, we assume that there is a unit measure continuum of each type of agents.

For simplicity, we assume that hedgers are short-lived and maximize a mean-variance objective over their instantaneous changes in wealth. Also for simplicity, we assume that each generation leaves their wealth to the next generation without consumption.[7] That is, a hedger maximizes

$$\mathbb{E}_t[dv_t] - \frac{\alpha}{2}\mathrm{Var}_t(dv_t),$$

where v_t is the hedger's time t wealth. $\alpha > 0$ is hedgers' absolute risk aversion coefficient over wealth.

[6] By fixing interest rate to be exogenously given as in the tradition of the asset demand system literature, our analysis represents the minimal departure from Koijen and Yogo (2019); Gabaix and Koijen (2022) and therefore sharpens our economic message.

[7] As we explain below, in our model short-lived hedgers' asset demand just serves as shifter to identify the demand slope for the long-lived investors. As a result, the exact modeling of hedgers is less important for our purpose. For instance, the alternative assumption that each generation of hedgers consume their wealth and the next generation arrives at the economy with some exogenous constant wealth would not change the analysis significantly. Kondor and Vayanos (2019) solves a version of this model with long-lived hedgers.

On the other hand, investors are long-lived and maximize their discounted utility over intertemporal consumption. That is, an investor maximizes

$$\mathbb{E}_t\Big[\int_t^\infty e^{-\rho(\tau-t)}u(\hat{c}_\tau)d\tau\Big],$$

where $\hat{c}_\tau$ is the individual investor's time τ consumption, and ρ is investors' time discount rate. We assume that $\rho > r$ to ensure that investors do not accumulate infinite wealth over time. We denote by w_t investors' aggregate time-t wealth, which is one of the state variables in this economy.

Our paper focuses on Constant Relative Risk Aversion (CRRA) investors, with the following two subcases.[8]

Case 1 (Log investors)**.** *Investors' instantaneous utility* $u(\hat{c}_\tau) = \log \hat{c}_\tau$*.*

Case 2 (General CRRA investors)**.** *Investors' instantaneous utility* $u(\hat{c}_\tau) = \frac{\hat{c}_\tau^{1-\gamma}}{1-\gamma}$ *for* $\gamma \neq 1$*.*

Hedging demand and states. Each hedger receives a random endowment $u_t^\top dD_t$ from time t to $t+dt$, where u_t is an $I \times 1$ vector and parametrizes the background risk in the economy. In our main model, u_t is modeled to follow a Markov switching process. The economy is either in a normal state or a shock state. We use $\xi \in \{n, s\}$ to indicate the state of the economy, with n denoting the normal state and s denoting the shock state; so $u_t \in \{u^n, u^s\}$ depending on the state of the economy. For expositional reasons we assume $u^s \geq u^n$, with u^n and u^s being $I \times 1$ vectors.

The state of the economy switches from the normal state to the shock state with Poisson intensity $\lambda^n \geq 0$, and from the shock state to the normal state with intensity $\lambda^s \geq 0$.

Equilibrium definition. Throughout we focus on symmetric equilibrium where economic agents are taking the same policy within each group (hedgers or investors). Denote by X_t^ξ and y_t^ξ, each $I \times 1$ vectors, the aggregate asset demand from hedgers and investors. The endogenous price process is P_t^ξ, also with dimension of $I \times 1$. Market clearing requires that the induced asset demand from both types of agents clear the exogenous asset supply at any time, that is,

$$y_t^\xi w_t^\xi = \mathrm{diag}(P_t^\xi)S - X_t^\xi. \tag{5}$$

It is useful to think about the right hand side as the residual supply curve from investors' point of view. Note that investors' aggregate demand y_t^ξ is defined as a share of their aggregate wealth, while X_t^ξ is the dollar value of hedger's aggregate demand in state ξ and time t.

[8] In Appendix C, we also work out the case of CARA investors with $u(\hat{c}_\tau) = -e^{-\frac{\gamma}{r}\hat{c}_\tau}$. That specification gives analytical characterization for various of our objects of interest.

Prices and returns. We conjecture and later verify that the structure of the equilibrium price vector P_t^ξ, for $\xi \in \{s, n\}$, is

$$P_t^\xi(w_t^\xi) = \frac{\bar{D}}{r} - \pi^\xi(w_t^\xi), \tag{6}$$

where $\pi^\xi(w_t^\xi)$ is an $I \times 1$ vector describing the time-varying risk premium when investors' aggregate wealth is w_t^ξ. That is, the conjecture implies that the aggregate wealth of the investors, w_t^ξ, and the shock state for hedgers, ξ, are the only two state variables of the model. As the jumps in hedging demand shocks affect equilibrium prices, we conjecture that for the endogenously determined drift μ_{Pt}^ξ, volatility σ_{Pt}^ξ, and jump $J_t^\xi \equiv P_t^{-\xi} - P_t^\xi$ we can write

$$dP_t^\xi = \mu_{Pt}^\xi dt + \sigma_{Pt}^{\xi\top} dB_t + J_t^\xi dN_t^\xi, \tag{7}$$

where $\{N_t^\xi : t \geq 0\}$ is a Poisson counting process with intensity λ^ξ. It will be useful to map the change in the price vector, dP_t^ξ to the (percentage) asset return vector defined as

$$dR_t^\xi \equiv \mathrm{diag}(P_t^\xi)^{-1}\left(dD_t + dP_t^\xi\right); \tag{8}$$

throughout, we restrict the parameter space to ensure that P_t^ξ is strictly positive, and hence dR_t^ξ is well defined. Then the endogenous (percentage) asset return vector dR_t^ξ has a drift $\mu_{Rt}^\xi \equiv \mathrm{diag}(P_t^\xi)^{-1}(\bar{D} + \mu_{Pt}^\xi)$ and volatility $\sigma_{Rt}^\xi \equiv (\sigma_{Pt}^\xi + \sigma)\,\mathrm{diag}(P_t^\xi)^{-1}$ defining the return covariance matrix, $\Omega_t^\xi \equiv (\sigma_{Rt}^\xi)^\top \sigma_{Rt}^\xi$ and a jump component $J_{Rt}^\xi \equiv \mathrm{diag}(P_t^\xi)^{-1}(P_t^{-\xi} - P_t^\xi)$. With a slight abuse of notation, we shall write

$$\mathbb{E}_t(R^\xi) \equiv \frac{\mathbb{E}_t(dR_t^\xi)}{dt} \quad \text{and} \quad \Delta\mathbb{E}_t(R) \equiv \frac{\mathbb{E}_t(dR_t^s)}{dt} - \frac{\mathbb{E}_t(dR_t^n)}{dt} \tag{9}$$

for the instantaneous expected return in state ξ and the change in instantaneous expected return when the shock hits.

Hedgers' demand curve and the residual supply shock. Given the dynamics of returns, hedgers' wealth follows

$$\begin{aligned} dv_t^\xi = &\Big[rv_t^\xi + X_t^{\xi\top}\left(dR_t^\xi - r\mathbf{1}dt\right) + \underbrace{u^{\xi\top}\bar{D}}_{\text{expected endowment dividends}}\Big]dt + \\ &\Big[(\sigma_{Rt}^\xi)X_t^\xi + \underbrace{\sigma u^\xi}_{\text{endowment shocks}}\Big]^\top dB_t + X_t^{\xi^\top} J_{Rt}^\xi dN_t^\xi, \end{aligned} \tag{10}$$

where $\mathbf{1}$ is an $I \times 1$ vector of ones. Given the hedger's wealth and their mean-variance utility, the first-order condition implies

$$X_t^\xi = \left[\Omega_t^\xi + \lambda^\xi J_{Rt}^\xi J_{Rt}^{\xi\top}\right]^{-1} \left[\frac{\mathbb{E}_t(R^\xi) - r\mathbf{1}}{\alpha} - (\sigma_{Rt}^\xi)^\top \sigma u^\xi\right]. \tag{11}$$

This is hedgers' demand. As it is apparent, when u^n switches to $u^s \geq u^n$, hedgers demand curve shifts downward. This translates to an upward shift in investors' residual supply curve $\mathrm{diag}(P_t^\xi)S - X_t^\xi$, and we expect that the state s corresponds to positive flows from hedgers to investors in equilibrium.

A one-off residual supply shock. Instead of the general case with arbitrary λ^s and λ^n, it is convenient to focus on the case of $\lambda^s = \lambda > 0$ while $\lambda^n \to 0$. Intuitively, this corresponds to an unexpected (MIT) shock pushing the system to the stressed state which is expected to revert to the normal state with intensity λ with no further shifts expected. This is a *one-off shock* with persistence $\frac{1}{\lambda}$. Considering a λ^n bounded away from zero would have the expected effect on the quantitative results, without affecting the qualitative picture.

The measured slope There is a long tradition in economics to recover the slope of a demand curve of some agents by observing how quantities and prices are affected by shocks to the (residual) supply. We define a corresponding measure in our economy as follows.

Definition 1. *Consider a dynamic equilibrium with a one-off shock with persistence $\frac{1}{\lambda}$. Measured slope is defined as*

$$\mathcal{M}\left(w_t^n; \lambda, u^n, u^s\right) \equiv \frac{[y_t^s - y_t^n]_i}{[\Delta \mathbb{E}_t\left(R\right)]_i}. \tag{12}$$

Note that, to ease the exposition, we define measured slope with respect to a one-off shock. That is, we take the normal state as base, and measure how an unexpected residual supply shock with persistence $\frac{1}{\lambda}$ changes investors' demand for asset i for each unit of change in its expected returns.

Road map. Our paper aims to understand whether the measured slope recovers the slope of investor's demand curve. Conceptually, we think of the latter as the representative investor's optimal quantity response per a unit change in expected returns. The analysis in the most general case of our model requires a general formal definition of the slope of investor's demand which we will provide in Section 5.2. Before that, we build up the intuition of why and when it is mismeasured by the measured slope through a series of examples and special cases.

3.2 The Static Benchmark

Just as in the dynamic model, in this static benchmark we have I assets and two groups of agents. For simplicity, we keep hedgers' demand general. We assume that for a price vector, P, and a vector of demand shocks u, hedgers' aggregate demand as share of their wealth is given by $x = x(P, u)$ with $x(P, u) : \mathbb{R}^I \times \mathbb{R}^I \to \mathbb{R}^I$ being an arbitrary function.

Following Campbell and Viceira (2002), we assume that the risky assets pay log-normally distributed dividends $\log D \sim N(\bar{D}, \Sigma)$ where S, $\bar{D}$ and D are $I \times 1$ while Σ is $I \times I$. Then, following Campbell and Viceira (2002) and Koijen and Yogo (2019) in relying on the log-linearized approximation, we write investors' demand curve in terms of portfolio shares as

$$\hat{y} = \frac{1}{\gamma}\Sigma^{-1}\left(\mathbb{E}(R) - r\mathbf{1} + \frac{1}{2}\operatorname{diag}(\Sigma)\right) \tag{13}$$

where $\mathbb{E}(R) = \bar{D} - \log P$ is the expected log gross return vector.Comparing (13) to (1) maps b to the matrix $\frac{1}{\gamma}\Sigma^{-1}$ while a to $\frac{1}{2\gamma}\Sigma^{-1}\operatorname{diag}(\Sigma)$. Importantly, in this static benchmark, both depend on deep parameters only, hence they are not sensitive to the state of the economy.

Static demand slope $\mathcal{C}^{\mathbf{stat}}$. For didactic purposes, we map the vector-to-vector slope $\frac{1}{\gamma}\Sigma^{-1}$, into an asset specific slope as follows. Suppose an investor is asked how much she would increase her demand for asset i if the vector $\mathbb{E}(R)$ would change to $\mathbb{E}(R) + \delta$ where δ is an arbitrary vector. The static slope will measure the response by comparing the change in the investor's portfolio share to the change in the implied return, that is

$$\mathcal{C}^{\text{stat}}(\delta) \equiv \frac{\left[\hat{y}^h - \hat{y}\right]_i}{[\delta]_i} = \frac{1}{\gamma}\left[\Sigma^{-1}\right]_{ii} + \frac{1}{\gamma}\frac{\sum_{j\neq i}\Sigma_{ij}^{-1}[\delta]_j}{[\delta]_i}, \tag{14}$$

where $\hat{y}^h$ is the investor's portfolio share under the hypothetical, just like we defined in (2).

The first element in (14) is the own-slope characterizing the investor's response to a change in the return of the same asset only. The second element is due to the cross-sensitivities $\frac{1}{\gamma}\Sigma_{ij}^{-1}$ characterizing the response due to changes in the returns of other assets. If only δ_i were different from 0, or if cash-flows were uncorrelated, or if there were a single asset, the inverse of return variance (scaled by risk-tolerance) would describe the slope of the demand curve. However, in general, for a complete characterization of the static slope of any investor's demand curve, we require the knowledge of the full slope matrix of $\frac{1}{\gamma}\Sigma^{-1}$.

We highlight one important take-away from the static benchmark demand slope in (14), which will be crucial in understanding our result in a dynamic setting. While $\mathcal{C}^{\text{stat}}(\delta)$ depends on the direction of the change in returns δ, the demand curve as a function of δ is independent of the demand from the other agents (i.e., hedgers).

Note that we define the slope as the quantity response to a unit change of expected return, not to a unit change of price. In a static setting, these two definitions would coincide, because with exogenous liquidating dividends a unit drop of today's price translates one to one to a unit increase of expected return by holding one share of asset today. However, as we will see, in a dynamic setting, the persistence of shocks and future risk matters, hence this translation no longer holds. Because researchers in financial economics think of demand for an asset as a function of its return characteristics, we believe this distinction is conceptually important.

Also, in contrast with the empirical literature, we do not normalize our measure to transform the slope to an elasticity. As we explain in HHHKL, this is so, because *natural units* differ across modeling approaches. For instance, in the logit framework of Koijen and Yogo (2019), a percentage change in demand as a function of a percentage change in return gives the most canonical form for the demand curve, while in our CRRA framework the portfolio share as a function of percentage (or log) returns does. In fact, since Relative Risk Aversion can be defined in terms of the investor's value function, the portfolio share indeed serves the natural unit of analysis within the general framework of the classic Merton (1971).

Static measured slope $\mathcal{M}^{\text{stat}}$. Now we can compare $\mathcal{C}^{\text{stat}}(\delta)$ to the measured static slope $\mathcal{M}^{\text{stat}}(\delta)$, which is the equilibrium effect of a shift in residual demand on the position of investors. Demand shocks change the equilibrium through the market clearing condition

$$y + x = \operatorname{diag}(P)S,$$

where we normalized the wealth of each group to 1. Hence, $\Delta\mathbb{E}_t(R) \equiv (\bar{D} - \log P^s) - (\bar{D} - \log P^n)$ is the change in expected (log) return due to the shock.

As a shift in u leaves $-\frac{(1+r)}{\gamma}\Sigma^{-1}$ unchanged, using (13), we have

$$\mathcal{M}^{\text{stat}}(u^n, u^s) \equiv \frac{[y^s - y^n]_i}{\Delta\mathbb{E}(R)_i} = \frac{1}{\gamma}\left(\left[\Sigma^{-1}\right]_{ii} + \frac{\sum_{j\neq i} \Sigma^{-1}_{ij} \left[\Delta\mathbb{E}(R)\right]_j}{\left[\Delta\mathbb{E}(R)\right]_i} \right). \tag{15}$$

Comparing (14) and (15) shows that the measured slope is identical to the static slope with respect to a carefully chosen δ.[9] Indeed, one can easily show that

$$\mathcal{M}^{\text{stat}}(u^n, u^s) = \mathcal{C}^{\text{stat}}(\delta)|_{\delta=\Delta\mathbb{E}(R^s)}. \tag{16}$$

[9] Fuchs et al. (2025) refers to the second element in the bracket of expression (15) as the spill-over. One of the main insight from their paper is that in general $\mathcal{C}^{\text{stat}}(\delta) \neq \mathcal{M}^{\text{stat}}$ unless $\delta = k \cdot \mathbf{1}_i$, which is proportional to i^{th} element of unit vector (k is a scalar). That is, measured elasticity is not own price elasticity because supply shocks affect all expected returns. Our requirement of $\delta = \Delta\mathbb{E}(R^s)$ in (16) represents the implicit assumption that the econometrician understands this point, hence all the effects we discuss in the rest of the paper are over and above of those originating from this spillover.

Therefore a shift in residual supply recovers the slope of static demand.

The main insight of the paper is that this principle does not hold in a general dynamic framework.[10]

4 Log Investors: The Anatomy of the Tilt

To introduce our effects in steps, we start the analysis of our dynamic model in some special cases. We consider log investors, a one-off shock in residual supply, and two extreme cases: when the shocked state is fully persistent, $\lambda \to 0$, and when it is fully transitory, $\lambda \to \infty$. In the former, after the shift in hedgers' demand, investors expect to remain in the shock state indefinitely. In the latter, investors expect the shock state to last only for an instant and return to the normal state almost immediately. These cases not only provide simplified intuition for the tilt but also allows for some analytical results.

4.1 Equilibrium with Log Investors

Before we study the limiting economy with one-off shock, we derive the equilibrium under general $\{\lambda^n, \lambda^s\}$, following standard arguments. In the main text we provide a sketch only while the details are available in Appendix A.1.1.

As in any general equilibrium model with infinitesimal agents, we need to distinguish between an investor's individual wealth $\hat{w}_t$ and investors' aggregate wealth w_t. Each individual controls $\hat{w}_t$, which coincides with the investor sector's aggregate wealth w_t only in equilibrium; in contrast, w_t serves as a state variable in this model, which each individual takes as given. The conjectured value function, which depends on both $\hat{w}_t^\xi$ and w_t^ξ, is

$$V^\xi(\hat{w}_t^\xi, w_t^\xi) = \frac{1}{\rho} \log \hat{w}_t^\xi + \bar{q}^\xi(w_t^\xi). \tag{17}$$

As it is conventional with CRRA (power) utility, we define an investor's demand in terms of her chosen portfolio share, $\hat{y}_t^\xi$. Let y_t^ξ be the aggregate counterpart of $\hat{y}_t^\xi$, and the corresponding consumption variables are c_t^ξ and $\hat{c}_t^\xi$, respectively. Then each investor's wealth dynamics

[10] Note that most empirical studies would not attempt to identify each element of the slope matrix $\frac{1}{\gamma}\Sigma^{-1}$. Instead, as HHHKL explain, there are implicit or explicit assumptions on the structure of the slope matrix helping to identify some combination of elements of interest. For instance, studies based on cross-sectional variation typically identify relative elasticity, the response in demand for one asset relative to another one with the same observables to a change in the relative price of these assets. HHHKL also explain how the rest of the matrix might be recovered using time-series variation. In this paper, we abstract away from the implementation problems arising from the dimensionality of the slope matrix and focus only on the higher level question—What if this matrix is not invariant to supply shocks?

is

$$
\begin{aligned}
d\hat{w}_t^\xi &= r\hat{w}_t^\xi dt + \hat{w}_t^\xi \hat{y}_t^{\xi\top}\left(dR_t^\xi - r\mathbf{1}dt\right) - \hat{c}_t^\xi dt \\
&= \left[(1-\hat{y}_t^{\xi\top}\mathbf{1})r\hat{w}_t^\xi - \hat{c}_t^\xi + \hat{w}_t^\xi \hat{y}_t^{\xi\top}\mu_{Rt}^\xi\right]dt + \hat{w}_t^\xi \hat{y}_t^{\xi\top}(\sigma_{Rt}^\xi)^\top dB_t + \hat{w}_t^\xi \hat{y}_t^{\xi\top} J_{Rt}^\xi dN_t^\xi.
\end{aligned}
\tag{18}
$$

From (18), the total wealth dynamic for investors aggregates to

$$
dw_t^\xi = \left[(1-y_t^{\xi\top}\mathbf{1})rw_t^\xi - c_t^\xi + w_t^\xi y_t^{\xi\top}\mu_{Rt}^\xi\right]dt + w_t^\xi y_t^{\xi\top}(\sigma_{Rt}^\xi)^\top dB_t + w_t^\xi y_t^{\xi\top} J_{Rt}^\xi dN_t^\xi, \tag{19}
$$

which gives the drift and diffusion of aggregate wealth, $\mu_t^{w,\xi}$ and $\sigma_t^{w,\xi}$. Then, substituting the conjectured value function into the representative investor's HJB equation and deriving the first-order conditions give the familiar optimal consumption rule for a log agent

$$
\hat{c}_t^\xi = \rho\hat{w}_t^\xi, \tag{20}
$$

and the demand function for risky assets. The next proposition spells out the latter for the following two cases of residual supply shock: i) one-off and fully persistent with $\lambda \to 0$; and ii) fully transitory with $\lambda \to \infty$.[11]

Proposition 1. *Suppose that investors have log utility and there is a one-off residual supply shock. Then in both limiting cases $\lambda \to 0$ and $\lambda \to \infty$, the optimal portfolio policy of log investors has the form of*

$$
\hat{y}_t^\xi = (\Omega_t^\xi)^{-1}(\mathbb{E}_t(R^\xi) - r\mathbf{1}). \tag{21}
$$

Note the simple form of investors' demand function (21). Similarly as we noted for our static benchmark, the slope of investors' demand is captured by the inverse of the covariance matrix of returns. Indeed, analogously to the static case we can define the slope of demand of the log investors as follows. Suppose in state n, the investor is asked how much she would increase her demand for asset i if the vector $\mathbb{E}_t(R)$ would change to $\mathbb{E}_t(R) + \delta$ where δ is an arbitrary vector. The log slope will measure the response by comparing the change in the investor's portfolio share to the change in the implied return, that is

$$
\mathcal{C}^{\log}(w_t^n;\delta) \equiv \frac{\left[\hat{y}_t^h - \hat{y}_t^n\right]_i}{[\delta]_i} = \left[(\Omega_t^n)^{-1}\right]_{ii} + \frac{\sum_{j\neq i}[(\Omega_t^n)^{-1}]_{ij}\,[\delta]_j}{[\delta]_i}, \tag{22}
$$

where $\hat{y}_t^h$ is the investor's portfolio share under the hypothetical scenario with a higher ex-

[11]The case of $\lambda \to 0$ is easy to understand since in this limit it is as if the economy never reverts back to the normal state from the shocked state and vice-versa. The case of $\lambda \to \infty$ is a bit more subtle, as it involves the equilibrium property of the price path; for some intuition, see footnote 17.

pected return $\mathbb{E}_t(R) + \delta$. An inconsequential difference between (14) and (22) is that log investors have a constant relative risk-aversion of 1, hence the γ parameter can be omitted.

In contrast, the consequential difference is that the exogenous Σ, which is the covariance matrix of the logarithm of date-1 dividends as in the setting of Campbell and Viceira (2002), is replaced by the endogenous Ω_t^ξ, the covariance of equilibrium asset returns. To see the implication, let us write the measured slope as

$$\begin{aligned}\mathcal{M}^{\log}(w_t^n; u^s, u^n) &\equiv \frac{[y_t^s - y_t^n]_i}{[\Delta \mathbb{E}_t\,(R)]_i} \\ &= \mathcal{C}^{\log}(w_t^n; \delta)|_{\delta = \Delta\mathbb{E}_t(R)} + \frac{\left[\left((\Omega_t^s)^{-1} - (\Omega_t^n)^{-1}\right)(\mathbb{E}_t\,(R^s) - r\mathbf{1})\right]_i}{[\Delta \mathbb{E}_t\,(R)]_i},\end{aligned} \tag{23}$$

where we have used (21).[12] Unlike in the static case, in our dynamic equilibrium with log investors (either perfectly persistent shocks $\lambda \to 0$ or transitory shocks $\lambda \to \infty$), shifts in residual supply do not recover the slope of investors' demand, $\mathcal{C}^{\log}$. The difference stems from the fact that when hedgers dump assets on investors, the persistent change in the allocation of risk changes the covariance matrix of returns. Hence, the change in residual supply tilts investors' demand.[13] Just as in equation (3), the resulting second term in (23) is the endogenous risk effect.

To build more intuition on the sources and direction of the tilt, we consider two examples. First, we consider the case of a single risky asset. In the second example, we consider multiple assets out of which the shift in residual supply affects only two, but in exactly opposite ways. As it turns out, we obtain sharp analytical characterizations of the tilt in two scenarios: in the first case, as $\lambda \to \infty$; and in the second case, as $\lambda \to 0$.

4.2 Single Risky Asset

With a single risky asset and log investors, the slope of demand $\mathcal{C}^{\log}$ simplifies to inverse of return variance, $(\sigma_{Rt}^\xi)^{-2}$. In state s, when hedgers demand less of the risky asset for given expected return, investors hold a riskier portfolio. Therefore, we expect the volatility of their wealth to increase, which, through (6), is expected to increase the volatility of the risky asset. This should imply that the measured slope, $\mathcal{M}^{\log}$ to be smaller than $\mathcal{C}^{\log}$.

It is tempting to conjecture that at least when the hedging shock is extremely short-lived, so that $\lambda \to \infty$, this effect diminishes. The following statement proves that this is never the case: $\mathcal{M}^{\log}$ is strictly smaller than $\mathcal{C}^{\log}$ even in this limit.

[12] In the Appendix A.2 we show that $\lim_{\lambda\to\infty} \Delta\mathbb{E}_t(R) > 0$ hence the second term in (23) is bounded in this limit.

[13] Note that our analysis on log investors provides a sharp comparison to Koijen and Yogo (2019) who also consider (myopic) log investors. The difference stems from the fact that their Assumption 1 assumes away the possibility that the residual supply shock affects the volatility of assets. In contrast, we show that general equilibrium forces imply that this is likely to be the case.

Proposition 2. *Consider an economy with log investors, single risky asset $I = 1$, and the demand shift shock is transitory $\lambda \to \infty$. In this economy, the variance of the asset increases, $\sigma^n_{Rt} < \sigma^s_{Rt}$, hence the endogenous risk effect is negative,*

$$\mathcal{M}^{log}(w^n_t; u^n, u^s) - \mathcal{C}^{log}(w^n_t; \delta)|_{\delta=\Delta\mathbb{E}_t(R)} = \frac{\left[\frac{1}{(\sigma^s_{Rt})^2} - \frac{1}{(\sigma^n_{Rt})^2}\right](\mathbb{E}_t(R^s) - r)}{\Delta\mathbb{E}_t(R)} < 0. \tag{24}$$

Furthermore, the endogenous risk effect is bounded away from zero even if the residual supply shock is diminishingly small:

$$\lim_{u^n \to u^s} \left(\mathcal{M}^{log}(w^n_t; u^n, u^s) - \mathcal{C}^{log}(w^n_t; \delta)|_{\delta=\Delta\mathbb{E}_t(R)}\right) < 0.$$

Proposition 2 states that in our economy, as hedgers liquidate some of their positions due to the shock, the endogenous return volatility goes up. Hence, the endogenous risk effect is negative and the measured slope is flatter than its conceptual counterpart. The economic intuition is closely related to the well-known amplification effect in various heterogeneous agent models (e.g. Longstaff and Wang, 2012; He and Krishnamurthy, 2013; Brunnermeier and Sannikov, 2014; Kondor and Vayanos, 2019).[14] Intuitively, if investors hold more of the risky asset in equilibrium due to hedgers' shock, $y^s_t > y^n_t$, it implies that a future adverse shock, $dD_\tau < 0$ results in a larger drop in investors' aggregate wealth w_τ, increasing risk premium more; and this leads to a larger decrease in price, which amplifies the initial drop in aggregate wealth. As this amplification effect is stronger when $y^s_t > y^n_t$, the shock implies a volatility hike: $\sigma^n_{Rt} < \sigma^s_{Rt}$.

To see this argument formally, from the definition of return volatility (7)-(8) we have $\sigma^\xi_{Rt} = (\sigma + \sigma^\xi_{Pt})/P^\xi_t$, where the first (second) term captures the exogenous (endogenous) volatility of Brownian dividends (asset price). The amplification mentioned above applies to the second component regarding the endogenous price volatility σ^ξ_{Pt} of the risky asset, which is determined in equilibrium by (using Ito's lemma on (6) and matching volatility terms):

$$\underbrace{\sigma^\xi_{Pt}}_{\text{price volatility}} = \underbrace{P^{\xi\prime}(w_t)}_{>0,\text{ price loading on wealth}} \times \underbrace{\overbrace{\text{Investors' Holding}_t}^{\text{increasing in } y^\xi_t} \cdot \left(\sigma + \sigma^\xi_{Pt}\right)}_{\text{wealth } w_t\text{'s exposure to risky asset}}. \tag{25}$$

Therefore the price volatility of the risky asset increases when investors increase their hold-

[14] In He and Krishnamurthy (2013) and Brunnermeier and Sannikov (2014) intermediaries play the role of investors. In He and Krishnamurthy (2013) the amplification is due to the endogenous equity constraint induced by moral hazard (of intermediaries). To see the parallel between Brunnermeier and Sannikov (2014) and our set up, consider the $\gamma \to 0$ limit in our case. Then, in both models intermediaries/investors are the efficient holders of risk. However, as they foresee that they cannot operate when their wealth is 0, they are willing to hold less and less risk as their capital shrinks. (For a larger γ this effect is stronger by risk-aversion.) This is why, the two models imply similar patterns of endogenous risk and amplification.

ings. Finally, when $\lambda \to \infty$, because the endogenous risky asset holdings of investors across two states are determined by the nature of u shocks and therefore differ, while risky asset prices converge in this limiting case, the return volatility of the risky asset remains strictly higher in the shock state than in the normal state. This delivers our desired negative endogenous risk effect.[15]

Perhaps it is worth noting that the sign of the endogenous risk effect is independent of the sign of the supply shock. If hedgers were to buy assets from investors in the stressed state (rather than sell to them), then both the numerator and the denominator on the right-hand side of (24) would flip signs. Namely, higher demand from hedgers would increase prices, decreasing the expected return, while the lower resulting risk-exposure of investors would decrease endogenous volatility. Therefore, the negative endogenous risk effect would still imply that the measured slope is too flat.

We finish this section with an illustration of the difference due to this effect in the fully persistent ($\lambda = 0$) and fully transitory cases ($\lambda = \infty$). Placing the change of expected return $\Delta\mathbb{E}_t(R)$ on the x-axis, Panel A and B of Figure 1 show $\mathcal{M}^{\log}$ and $\mathcal{C}^{\log}(\delta)|_{\delta=\Delta\mathbb{E}_t(R)}$ for various values of u^s for these two cases, with parameters calibrated to Gabaix and Koijen (2022) who study the macro elasticity.[16] While the difference between $\mathcal{M}^{\log}$ and $\mathcal{C}^{\log}$ is significantly lower when the shock is transitory, in line with Proposition 2, the measured slope is too flat and the mismeasurement is clearly bounded away from zero even for small shocks.

4.3 Multiple Assets: Index Inclusion

Next, we consider a multi-asset example, to see how moving from return variance to the covariance matrix affects our intuition.

We picture a market with a large number of fundamentally identical assets separated into two groups: red and blue. The only difference between these assets is in the corresponding component of hedgers' demand shifter, u^ξ; that is, hedgers hold red assets in larger proportion due to the difference in their background risk. We imagine that when the state switches from n to s, hedgers treat the first asset $i = 1$ (which was red) as they used to treat the last asset $i = I$ (which was blue) and vice-versa. The shock does not affect any other asset. Essentially, in state s hedgers demand decreases for the first asset but increases for the last asset. This captures the essence of index inclusion, with "blue" being part of index.

More precisely, suppose that $\Sigma = \sigma^2\mathbf{I}$, $S = s\mathbf{1}$, $\bar{D} = \bar{d}\mathbf{1}$ where $\mathbf{I}$ is the identity matrix,

[15] We show in the Appendix that when $\lambda \to \infty$, we have $P^s(\cdot) \to P^n(\cdot)$ and $w^n \to w^s$; intuitively, price path should be continuous under immediate (and almost certain reversal). However, the endogenous risky asset holdings of investors differ across two states even when $\lambda \to \infty$. Because $P^{n\prime}(w) \to P^{s\prime}(w) > 0$, the return volatility of the risky asset is strictly higher in the shock state than in the normal state.

[16] The details of the calibration and the baseline parameter values are discussed in Appendix E.3. As we explain there, $\lambda \approx 1$ would match the persistence of flows in Gabaix and Koijen (2022).

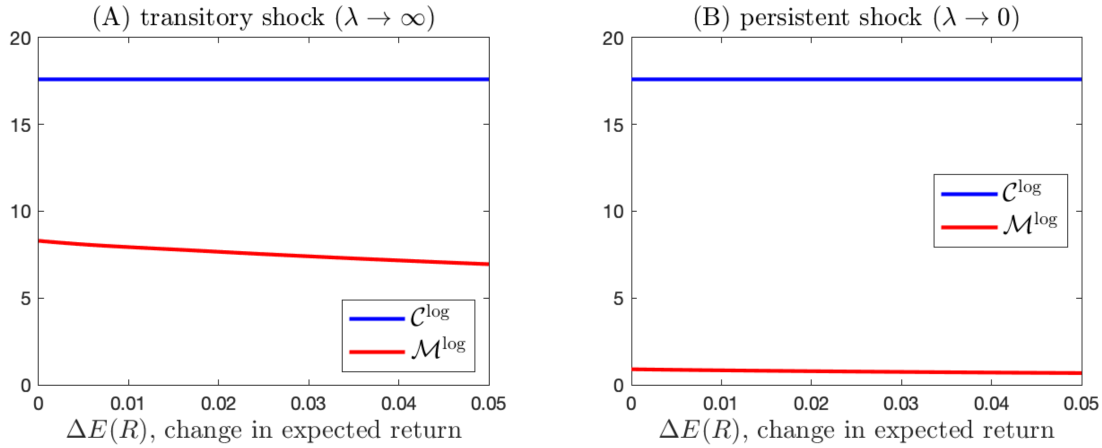

Figure 1: Measured slope of a log investor, $\mathcal{M}^{\log}$, and the corresponding conceptual slope, $\mathcal{C}^{\log}(\delta)|_{\delta=\Delta\mathbb{E}_t(R)}$ with one risky asset. Panel A plots $\mathcal{C}^{\log}$ and $\mathcal{M}^{\log}$ for a transitory shock ($\lambda \to \infty$), and Panel B plots the same objects for a persistent shock ($\lambda \to 0$). We change the u^s shock to get varying incremental expected return (in percentage) $\Delta\mathbb{E}_t\left(R\right)$. We plot our measures against the implied change in percentage incremental expected return, with our benchmark parametrization: $\bar{D} = 0.415$, $\sigma = 0.57$, $\alpha = 1$, $r = 0.03$, $\rho = 0.08$, $S = 1$, $u^n = 0$ and $w = 0.9021$. The choice of parameter values are discussed in Appendix E.3.

and s, σ^2, and $\bar{d}$ are arbitrary scalars. The demand shifter for hedgers are modeled as

$$[u^n]_i = \begin{cases} v & \text{for } i < I_1, \\ v + \Delta v & \text{for } i \geq I_1, \end{cases}$$

and

$$[u^s]_i = \begin{cases} v + \Delta v & \text{for } i = 1, \\ v & \text{for } i = I, \\ [u^n]_i & \text{otherwise}, \end{cases}$$

with v and Δv being some positive constants. Here, the first I_1 assets are initially blue.

To sharpen our theoretical results, we consider the limit when the difference in hedgers' exposure to blue and red assets, and consequently, the change in their demand captured by Δv in (4.3), is diminishingly small. One might expect that the negligible effect of this small local shock on the equilibrium outcome implies that the informativeness of shifts to residual supply should perhaps be restored. We show that this is not the case. The next statement formally defines the example and states the result.

Proposition 3. *Consider the limiting economy when $\lambda \to 0$. In this economy, for sufficiently small Δv, both the first asset's variance and its covariance with any other asset increase due* *to the shock (except the I^{th} asset due to symmetry of index inclusion):*

$$\lim_{\Delta v \to 0} \frac{[\Omega_t^s - \Omega_t^n]_{1i}}{\Delta v} = \bar{F}(w_t)\,\sigma^2 \begin{cases} 2\frac{\bar{D}}{r}\frac{(S+v)^2}{(P_t)^3} & \text{for } i = 1, \\ \frac{\bar{D}}{r}\frac{S+v}{(P_t)^3} & \text{for } I > i > 1, \\ 0 & \text{for } i = I. \end{cases}$$

where $\bar{F}(\cdot)$ is a scalar function defined in the Appendix. The endogenous risk effect is negative so that $\mathcal{M}^{log}(w_t^n; u^n, u^s) < \mathcal{C}^{log}(w_t^n; \delta)|_{\delta = \Delta \mathbb{E}_t(R)}$, and is strictly bounded away from zero.

The first notable result of Proposition 3 is that the diminishingly small shock does not lead to diminishingly small difference in $\mathcal{C}^{\log} - \mathcal{M}^{\log}$; this is because both the denominator and the numerator of the second term in (23) is of order Δv. That is, while small shocks lead to small changes in equilibrium objects as expected, the mismeasurement depends on the ratio of the effect on the covariance and on expected returns.

We also state that just as in the single asset case, the measured slope is flatter than its conceptual counterpart. It is because as hedgers liquidate their positions in the first asset, both its endogenous return volatility and its endogenous covariance with all the unaffected assets go up. The intuition is similar to the single asset case, because in our model the endogenous volatility of all assets is due to investors' wealth shocks. As a result, when the first asset becomes blue, hence investors hold more of it, not only its volatility increases but also its comovement with every other asset.

5 The General Case: Intertemporal Hedging and Persistence

Now we turn to the general model with CRRA utility and residual supply shocks with arbitrary persistence, $1/\lambda$. First, we summarize the main properties of the equilibrium. Then we show how the investor's first-order condition defines a general concept of the slope of investors' demand, the *dynamic slope*. Finally, we show that when the residual supply shock hits, investor's demand curve not only tilts due to the endogenous risk effect, but also shifts due to additional effects stemming from general equilibrium forces connected to intertemporal hedging and the persistence of the shock.

5.1 Equilibrium in the General Case

We derive the equilibrium in our multi-asset dynamic model. As main ingredients, we already defined the conjectured price process, hedgers' demand, and investor's individual and aggregate wealth in equations (7) and (11) and (18)-(19).

Investors' demand. Given the dynamics of the individual and aggregate wealth of investors, an investor's value function in state $\xi \in \{n, s\}$ satisfies the Hamilton-Jacobi-Bellman (HJB) equation which we write in a general form:

$$\rho V^{\xi} = \max_{\hat{c}_t^{\xi}, \hat{y}_t^{\xi}} : \Big\{ u(\hat{c}_t^{\xi}) + V_{\hat{w}}^{\xi} \mu_t^{\hat{w},\xi} + \frac{1}{2} V_{\hat{w}\hat{w}}^{\xi} (\sigma_t^{\hat{w},\xi})^{\top} \sigma_t^{\hat{w},\xi} + V_{w}^{\xi} \mu_t^{w,\xi} + \frac{1}{2} V_{ww}^{\xi} (\sigma_t^{w,\xi})^{\top} \sigma_t^{w,\xi} \\ + V_{\hat{w}w}^{\xi} (\sigma_t^{\hat{w},\xi})^{\top} \sigma_t^{w,\xi} + \lambda^{\xi} (V^{-\xi,+} - V^{\xi}) \Big\}. \tag{26}$$

Here, V^{ξ} stands for the value function $V^{\xi}(\hat{w}_t^{\xi}, w_t^{\xi})$; $V^{-\xi,+}$ is the value function evaluated after the jump, with the superscript $+$ denoting post-jump while $-\xi$ denoting the other state once jump occurs:

$$V^{-\xi,+} \equiv V^{-\xi} \left(\hat{w}_t^{\xi} (1 + \hat{y}_t^{\xi\top} J_{Rt}^{\xi}), w_t^{\xi} (1 + y_t^{\xi\top} J_{Rt}^{\xi}) \right). \tag{27}$$

Denote $\Omega_t^{\xi} \equiv (\sigma_{Rt}^{\xi})^{\top} (\sigma_{Rt}^{\xi})$ to be the covariance matrix of returns. The first-order condition in $\hat{y}_t^{\xi}$ gives the familiar components of asset demand as in Merton (1971):

$$\hat{y}_t^{\xi} = \underbrace{\overbrace{\frac{V_{\hat{w}}^{\xi}}{\hat{w}_t^{\xi}(-V_{\hat{w}\hat{w}}^{\xi})}}^{\text{risk tolerance}} (\Omega_t^{\xi})^{-1} \left(\mathbb{E}_t(R^{\xi}) - r\mathbf{1} \right)}_{\text{myopic}} + \underbrace{\frac{V_{\hat{w}w}^{\xi}}{\hat{w}_t^{\xi}(-V_{\hat{w}\hat{w}}^{\xi})} w_t^{\xi} y_t^{\xi}}_{\text{hedging of Brownian}} + \underbrace{\frac{1}{\hat{w}_t^{\xi}} \frac{V_{\hat{w}}^{-\xi,+} - V_{\hat{w}}^{\xi}}{(-V_{\hat{w}\hat{w}}^{\xi})} (\Omega_t^{\xi})^{-1} \lambda^{\xi} J_{Rt}^{\xi}}_{\text{hedging for reversal, } J_{Rt}}. \tag{28}$$

The first part, which is myopic demand, is analogous to the static demand curve: the instantaneous expected return scaled by the product of the inverse of the covariance matrix and risk-tolerance. One could also map the term b^{ξ} in (1) in our example to $\frac{V_{\hat{w}}^{\xi}}{\hat{w}_t^{\xi}(-V_{\hat{w}\hat{w}}^{\xi})} (\Omega_t^{\xi})^{-1}$. As we will see, the shock to residual supply affects this term through the endogenous covariance matrix, and this endogenous risk effect is responsible for the tilt of the demand curve.

The second part is the intertemporal hedging demand with respect to the diffusion part of the aggregate state variable, w_t. When w_t affects the marginal value of a dollar, its correlation with the investor's individual wealth $\hat{w}_t$ affects her demand. The last part is a similar term but it represents hedging demand with respect to the reversal. When the residual supply curve reverts, prices jump, affecting investor's marginal value of a dollar and their individual wealth at the same time. Hedging this affects her demand.

These two hedging terms in (28) map to the term a^{ξ} in (1). As we will see, these terms are affected by the residual supply shock in a complex way, resulting in a shift of the demand curve.

Solving for equilibrium. We solve for the equilibrium prices after imposing that individual positions equal the aggregate position $\hat{y}_t^\xi = y_t$ and market clearing for each instant

$$X_t^\xi + w_t^\xi y_t^\xi = \mathrm{diag}(P_t^\xi)S. \tag{29}$$

The next proposition spells out the resulting dynamic demand function and the structure of equilibrium for the cases of log investors and CRRA investors with arbitrary state switching intensities $\{\lambda^s, \lambda^n\}$.

Proposition 4. *In Case 1 (log investors), the value function has the form of*

$$V^\xi(\hat{w}_t^\xi, w_t^\xi) = \frac{1}{\rho}\log \hat{w}_t^\xi + \bar{q}^\xi(w_t^\xi). \tag{30}$$

The optimal consumption-portfolio policies are

$$\hat{c}_t^\xi = \rho \hat{w}_t^\xi, \tag{31}$$

$$\hat{y}_t^\xi = (\Omega_t^\xi)^{-1}\Big(\mathbb{E}_t(R^\xi) - r\mathbf{1}\Big) + \lambda^\xi(\Omega_t^\xi)^{-1}\left(\frac{1}{1+\hat{y}_t^{\xi\top}J_{Rt}^\xi} - 1\right)J_{Rt}^\xi. \tag{32}$$

The ODE that determines $\bar{q}^\xi(w_t^\xi)$ is in the Appendix A.1.1.

In Case 2 (CRRA investors), the value function has the form of

$$V^\xi(\hat{w}_t^\xi, w_t^\xi) = \frac{(\hat{w}_t^\xi)^{1-\gamma}}{1-\gamma}q^\xi(w_t^\xi), \tag{33}$$

where $q^\xi(w_t^\xi)$ is the marginal value of wealth in state w_t^ξ. The optimal consumption-portfolio policies are

$$\hat{c}_t^\xi = q^\xi(w_t^\xi)^{-\frac{1}{\gamma}}\hat{w}_t^\xi; \tag{34}$$

$$\hat{y}_t^\xi = \frac{1}{\gamma}(\Omega_t^\xi)^{-1}\left(\mathbb{E}_t\left(R^\xi\right) - r\mathbf{1}\right) + \frac{1}{\gamma}\frac{q^{\xi\prime}(w_t^\xi)}{q^\xi(w_t^\xi)}w_t^\xi y_t^\xi + \frac{\lambda^\xi}{\gamma}(\Omega_t^\xi)^{-1}\left(\frac{q^{-\xi}\left(w_t^\xi\left(1+y_t^{\xi\top}J_{Rt}^\xi\right)\right)}{q^\xi(w_t^\xi)[1+\hat{y}_t^{\xi\top}J_{Rt}^\xi]^\gamma} - 1\right)J_{Rt}^\xi. \tag{35}$$

The ODEs determining $q^\xi(w_t^\xi)$ are given in the Appendix A.1.2.

Note that investors' demand in each of our special cases follows the general structure of (28). However, (32) illustrates that in the simpler log case there is no intertemporal hedging term for the Brownian shock. This is a well-known property pointed out by Merton (1971). At the same time, the intertemporal hedging term for the reversal remains.[17]

[17] Equation (32) also echoes the simple optimal myopic portfolio decision in Proposition 1. The subtlety

Importantly, because of the form of the equilibrium pricing vector (6), investors' portfolio choice changes not only with the state of the residual supply curve, ξ, but also with their aggregate wealth, w. Because poorer investors are less willing to absorb risk, aggregate investors' wealth affects the risk premium.

5.2 Dynamic Slope and Myopic Slope

How should we think about the concept of slope of our investors' demand function in this general dynamic environment? In a static benchmark with a single asset, the slope, which measures the response in the asset holding to a unit change in its return, is a scalar. As (13) illustrates, with multiple assets, a matrix describes how each component of the portfolio changes as the returns of assets do.

The general dynamic environment is more complex. As the investment opportunity set can predictably fluctuate, the investor will consider all current instantaneous (expected) returns as well as the distribution of future return paths when making her portfolio decision. Conceptually, one should measure the response in $\hat{y}_t$ for any marginal change at any point of these return paths. This is an infinite-dimension object.

The power of the Merton approach comes from summarizing the joint-distribution of all future price paths into a small number of simpler economic objects. As (28) illustrates, apart from the expected return vector and covariance matrix, the Merton investor's demand also responds to changes in the future investment opportunity sets in all periods $\tau > t$. But this also suggests that our heuristic approach in defining the slope of the investors' demand curve for our special cases, that is, $\mathcal{C}^{\min}$, $\mathcal{C}^{\text{stat}}$, and/or $\mathcal{C}^{\log}$ is not sufficient. When intertemporal hedging is present, the investor's response to an increase in expected returns from $\mathbb{E}_t(R)$ to $\mathbb{E}_t(R) + \delta$ will depend on how future price paths change beyond the next instant.

Based on the discussion above, we suggest two alternatives.

Myopic slope. When simplicity is preferred, one can focus on the myopic component of demand and consider the matrix $\frac{1}{\gamma}(\Omega_t^\xi)^{-1}$ as an approximation of the conceptual slope of the demand function. Then, analogously to the static conceptual demand (14), we define the measure of myopic conceptual slope of the CRRA investor in the normal state as

$$\mathcal{C}^{\text{myo}}(w_t^n;\delta) \equiv \underbrace{\frac{1}{\gamma}\left[(\Omega_t^n)^{-1}\right]_{ii}}_{\text{own-effect}} + \underbrace{\frac{1}{\gamma}\frac{\sum_{j\neq i}(\Omega_t^n)_{ij}^{-1}\left[\delta\right]_j}{\left[\delta\right]_i}\Big|_{\delta=\Delta\mathbb{E}_t(R)}}_{\text{cross-asset spillover}}.$$

of $\lambda \to \infty$ case is that in Appendix D we prove the associated return jump shrinks to zero, so that J_{Rt} $\lim_{\lambda\to\infty}\lambda J_{Rt}$ remains finite and the term inside the bracket (with the order of $\hat{y}_t^{\xi\top}J_{Rt}^\xi$) converges to zero.

Dynamic slope. The second approach is to parameterize the future path of returns beyond the next instant, capturing the idea that the investor cares about the persistence of the shock to expected returns, its size and relative magnitude across assets (i.e., its direction), and also the flow share of additional return versus the lump-sum when the shock reverses (i.e., its composition). Our headline measure is *the dynamic slope* defined as follows. We then characterize it fully in our equilibrium model.

Definition 2. *[Dynamic Slope] Suppose that an investor is offered to buy any asset i with a δ_i additional instantaneous expected return where δ is an arbitrary vector. In particular, she is asked to form demand $\hat{y}_t^h$ facing the dividend process*

$$dD_t^h = \bar{D}dt + \sigma^\top dB_t + (1-\chi)\operatorname{diag}(P_t^n)\delta dt + \chi\frac{\operatorname{diag}(P_t^n)\delta}{\lambda}dN_t^h \tag{36}$$

instead of (4) *where dN_t^h is a Poisson process with intensity λ. After the first jump the dividend process switches to* (4) *forever. Then dynamic slope is measured as*

$$\mathcal{C}^{dyn}(w_t^n;\theta) \equiv \frac{\left[\hat{y}_t^h - \hat{y}_t^n\right]_i}{\delta_i}. \tag{37}$$

where $\hat{y}_t^n$ refers to the equilibrium demand in a normal state where $\lambda^n = 0$, and the tuple $\theta \equiv \{\lambda, \delta, \chi\}$ parameterizes the persistence, direction and composition of the shock to return path, respectively.

The concept of dynamic slope is appealing for several reasons. First, it is simple to understand. It measures the optimal response of an investor's demand to a change in expected returns (implemented through a hypothetical dividend process) of given persistence, direction, and composition. Note that under our parameterization of (37), the change in expected return is always δ irrespectively of λ and χ.

Second, as we argue below, it allows for a change in return path sufficiently close to its equilibrium counterpart in our dynamic setting. In our model, a shift in residual supply governed by a Poisson process implies, apart from an increase in instantaneous expected returns, a higher flow return and an added lump-sum at reversal. The parameter χ governs the share of the higher return which is received as a flow relative to the share received as lump sum. The inverse of parameter λ governs the persistence of the improvement of the investment opportunity set. In the $\lambda \to \infty$ limit, dynamic slope captures the investor's response to a hike (or drop) in expected return for an instant. In the other extreme, the $\lambda \to 0$ case captures a permanent change in expected return.

Third, dynamic slope is closely tied to typical policy questions addressed in the growing literature of demand elasticity. For instance, the central question of quantitative easing (QE) is: to maintain the yield of certain bonds at z basis points below their non-intervention equilibrium level, how much must the Federal Reserve purchase as a fraction of outstanding bonds over a specific period? Dynamic slope provides the critical individual demand characteristic needed to answer this question: the change in the amount an investor is willing to hold in response to a change in return of given persistence.

Finally, observe that just as our definition for the static case, dynamic slope captures the quantity response to a given change in the expected return vector, not to a given change in the price vector. In the dynamic context, the distinction is critical. Clearly, as it has been point out by Davis et al. (2023) and van Binsbergen et al. (2025), the same price shock implies a much lower expected return shock if it is persistent than if it is transitory. Therefore, throughout our analysis we control for this by keeping the expected return shock constant in our comparative statics.

Dynamic slope as an optimization problem. To calculate dynamic slope, we first solve for the equilibrium as described in Proposition 4 with a constant residual supply curve, that is, $u = u^n$ and $\lambda^n = 0$. This defines the portfolio holdings $\hat{y}^n$ and value function $V^\xi(\hat{w}_t^\xi, w_t^\xi)$ when the economy is under the baseline normal state. Also, the equilibrium objects μ_t^w, σ_t^w represent the dynamics of the aggregate state variable which is not affected by the hypothetical dividend distribution (because the extra dividends are offered only to a given investor).

Then, we introduce $V^h(\cdot,\cdot)$, the value function of that investor who is receiving the hypothetical dividends (36). We conjecture that in the CRRA case, it has the form of

$$V^h(\hat{w}_t^h, w_t^n) = q^h(w_t^n;\theta)\frac{(\hat{w}_t^h)^{1-\gamma}}{1-\gamma}. \tag{38}$$

Note that the marginal value of a dollar $q^h(w_t^n;\theta)^{1-\gamma}$ depends on the parameters of the dividend process. Also, the HJB equation of this problem takes into account that when the offer ends so the dividend process reverts back to its equilibrium dynamics, this investor's value function jumps back to that under normal times.

Given these objects, we solve for the optimal portfolio choice $\hat{y}^h$ of the investor facing hypothetically higher dividends. Comparing to the investor's demand in normal times $\hat{y}^n$ and substituting both into the definition of dynamic slope, (37) gives a similar decomposition of dynamic slope as in the dynamic asset demand in (28). The following proposition summarizes the results both for the log and the CRRA cases, with details provided in Appendix A.4.[18]

[18] Recall that in Section 4.1 investigating $\mathcal{C}^{\log}$ for the limiting cases $\lambda \to 0$ and ∞, we have derived (22) directly from the corresponding demand curve (21) in Proposition 1. Does (40) converge to (22) in these limits? The limit of $\lambda \to \infty$, so that the term of "hedging against reversal" in (40) goes to zero, is immediate. For the $\lambda \to 0$ case, to ensure consistency we must require that $\chi = \lambda\frac{k(\lambda)}{\delta}$ with an arbitrary bounded function $k(\lambda)$. That is, formally, we are taking the limit $\lim_{\lambda\to 0}\mathcal{C}^{\mathrm{dyn}}(w_t;\lambda,\delta,\lambda\frac{k(\lambda)}{\delta})$ for convergence to (22). As we argue in the next section, the idea is that to match the concept of $\mathcal{C}^{\mathrm{dyn}}$ and $\mathcal{M}$ in equilibrium, we require

Proposition 5. *In Case 1 (log investors), demand under hypothetical dividend path* (36) *is*

$$\hat{y}_t^h = (\Omega_t^n)^{-1}(\mu_{Rt}^n - r\mathbf{1} + \delta) + \Big[\frac{\lambda}{\lambda + \hat{y}_t^{h\top}\chi\delta} - 1\Big](\Omega_t^n)^{-1}\chi\delta. \tag{39}$$

Hence, dynamic slope has only two components:

$$\begin{aligned}\mathcal{C}^{dyn}(w_t^n;\theta) &= \frac{[\hat{y}_t^h - \hat{y}_t^n]_i}{\delta_i} \\ &= \underbrace{[(\Omega_t^n)^{-1}]_{ii} + \frac{\sum_{j\neq i}\Big[(\Omega_t^n)^{-1}\Big]_{ij}[\delta]_j}{[\delta]_i}}_{\textit{myopic slope}} + \underbrace{\Big[\frac{\lambda}{\lambda + \hat{y}_t^{h\top}\chi\delta} - 1\Big]\frac{[(\Omega_t^n)^{-1}\chi\delta]_i}{[\delta]_i}}_{\textit{hedging against reversal}}.\end{aligned} \tag{40}$$

In Case 2 (CRRA investors) demand under hypothetical dividend path (36) *is*

$$\begin{aligned}\hat{y}_t^h =& \frac{1}{\gamma}(\Omega_t^n)^{-1}(\mu_{Rt}^n - r\mathbf{1} + \delta) + \frac{1}{\gamma}\frac{q^{h\prime}(w_t^n;\theta)}{q^h(w_t^n;\theta)}w_t^n y_t^n \\ &+ \frac{\chi}{\gamma}(\Omega_t^n)^{-1}\delta\left(\frac{q^n(w_t^n)}{q^h(w_t^n;\theta)}(1 + \hat{y}_t^{h\top}\frac{\chi\delta}{\lambda})^{-\gamma} - 1\right),\end{aligned} \tag{41}$$

where $q^h(w_t^n;\theta)$ is the investor's marginal value of wealth given the hypothetical D^h (with ODE for $q^h(w_t^n;\theta)$ given in the Appendix A.4.2), while $q^n(w_t^n)$ is the marginal value of wealth in equilibrium with constant u^n. The implied demand slope is as follows:

$$\begin{aligned}\mathcal{C}^{dyn}(w_t^n;\theta) =& \frac{[\hat{y}_t^h - \hat{y}_t^n]_i}{\delta_i} = \underbrace{\frac{1}{\gamma}[(\Omega_t^n)^{-1}]_{ii} + \frac{1}{\gamma}\frac{\sum_{j\neq i}\Big[(\Omega_t^n)^{-1}\Big]_{ij}[\delta]_j}{[\delta]_i}}_{\textit{myopic slope}} \\ &+ \underbrace{\frac{1}{\gamma}\Big[\frac{q^{h\prime}(w_t^n;\theta)}{q^h(w_t^n;\theta)} - \frac{q^{n\prime}(w_t^n)}{q^n(w_t^n)}\Big]\frac{[y_t^n w_t^n]_i}{[\delta]_i}}_{\textit{Brownian hedging}} \\ &+ \underbrace{\frac{1}{\gamma}\Big[\frac{q^n(w_t^n)}{q^h(w_t^n;\theta)}\Big(1 + \hat{y}_t^{h\top}\frac{\chi\delta}{\lambda}\Big)^{-\gamma} - 1\Big]\frac{[(\Omega_t^n)^{-1}\chi\delta]_i}{[\delta]_i}}_{\textit{hedging against reversal}}.\end{aligned} \tag{42}$$

Dynamic slope vs. myopic slope. Despite of the conceptual differences, our concepts of myopic and dynamic slopes are intimately related. The next proposition states that the dynamic concept is identical to the myopic one if the return path changes only for an instant.

$\chi = \lambda\frac{J_{Rt}^s}{\delta}$ to be the jump component of $\Delta\mathbb{E}_t(R)$ in equilibrium. Since $\lim_{\lambda\to 0} J_{Rt}^s$ is bounded in equilibrium, $k(\lambda)$ has to be bounded too when taking the limit of $\mathcal{C}^{\log}$.

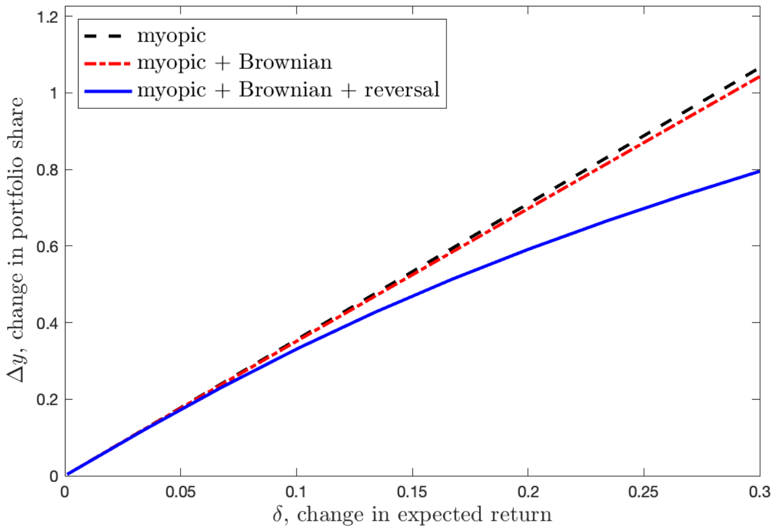

Figure 2: Change in demand as a function of change in expected return and its components. The blue solid line gives the investor's dynamic demand, which is the change in the investor's portfolio share Δy as a function of the change in the given instantaneous expected return δ. The black dashed line gives the myopic component, the red dash-dotted line gives the myopic demand plus the Brownian hedging component; so the gap between solid blue line and dash-dotted red line is the reversal hedging component. The parameter values are $\bar{D} = [0.27, 0.26]$, $\sigma = \text{diag}(0.485, 0.485)$, $\alpha = 1$, $\gamma = 3$, $r = 0.03$, $\rho = 0.08$, $s = [1, 1]$, $\lambda = 0.5$, and $\chi = 2/3$.

Proposition 6. *For CRRA investors, the limit of the dynamic slope is the myopic slope when the hypothetical dividend path is perfectly transitory:*

$$\lim_{\lambda \to \infty} \mathcal{C}^{dyn}\left(w_t^n; \theta\right) = \mathcal{C}^{myo}(w_t^n; \delta).$$

Figure 2 illustrates the differences between the myopic and dynamic slopes, under the calibrated parameters (see the details of calibration in Appendix E.1). The blue solid line plots how much the investor would change her demand as a function of the given change in instantaneous return δ in our dynamic setting. Hence, dynamic slope, $\mathcal{C}^{\text{dyn}}\left(w_t^n; \theta\right)$ approximates the slope of this curve. We also plot the black dashed line as the corresponding myopic component given in (42); and the red dash-dotted line which gives the myopic demand plus the Brownian hedging component. Note that, just like the static demand curve $\mathcal{C}^{\text{stat}}\left(\delta\right)$ in (14), the myopic component is linear in δ given in (42).

There are two immediate take-aways from Figure 2: First, the change of asset demand is non-linear so that the demand curve turns flatter for larger shocks. Second, the curve tends to be flatter compared to the (linear) myopic demand; this is due to the classic Brownian and reversal hedging components à la Merton (1971), which, as shown, are both negative.[19]

[19]For the interested reader, Appendix D discusses the economic intuition of why the Dynamic Slope tends

Figure 2 also illustrates that as long as the shock to expected returns, δ, is not too large, the intertemporal hedging effects tend to be small, hence the dynamic slope is close to its myopic component. This fact, and its simpler structure, might make the myopic slope an attractive alternative to dynamic slope as a concept to capture investors' demand sensitivity. Indeed, we shall demonstrate soon, that whichever of them the econometrician wish to recover based on shifts to residual supply, the problems remain the same.

It is important to emphasize that this does not imply that the effect of intertemporal hedging can be ignored. In terms of our minimal example, the quantitatively small intertemporal hedging corresponds to a small $a(\cdot)$. However, as the last term in (3) illustrates, if the investor's demand curve shifts, the mismeasurment of its slope is not proportional to the absolute size of $a(\cdot)$; it instead depends on the ratio between the shift size and the corresponding change in expected return, $\frac{a(y^s)-a(y^n)}{\mathbb{E}(R^s)-\mathbb{E}(R^n)}$. We will demonstrate below that this ratio can be large, even when the difference between dynamic slope and myopic slope is small.

5.3 Measured Slope versus Its Conceptual Counterparts

Per Definition 1, the measured slope of asset i is simply the ratio of the change in the representative investor's portfolio $[y^s - y^n]_i$ to the change in its expected return $[\Delta\mathbb{E}_t(R)]_i$ as the equilibrium responds to a one-off residual supply shock. With CRRA utility, the optimal portfolio policy (35) implies that the measured slope is

$$\begin{aligned}\mathcal{M}\left(w_t^n;\lambda,u^n,u^s\right) = & \underbrace{\mathcal{C}^{\text{myo}}(w_t^n;\delta)|_{\delta=\Delta\mathbb{E}_t(R)} + \frac{1}{\gamma}\frac{\left[\left((\Omega_t^s)^{-1}-(\Omega_t^n)^{-1}\right)\left(\mathbb{E}_t\left(R^s\right)-r\mathbf{1}\right)\right]_i}{\left[\Delta\mathbb{E}_t\left(R\right)\right]_i}}_{\text{myopic measured slope}} + \\ & + \underbrace{\frac{1}{\gamma}\frac{\left[\frac{q^{s\prime}(w_t^s)}{q^s(w_t^s)}w_t^s y_t^s - \frac{q^{n\prime}(w_t^n)}{q^n(w_t^n)}w_t^n y_t^n\right]_i}{\left[\Delta\mathbb{E}_t\left(R\right)\right]_i}}_{\text{measured Brownian hedging}} + \underbrace{\frac{\lambda}{\gamma}(\Omega_t^s)^{-1}\left(\frac{q^n\left(w_t^s\left(1+y_t^{s\top}J_{Rt}^s\right)\right)}{q^s(w_t^s)[1+\hat{y}_t^{s\top}J_{Rt}^s]^\gamma}-1\right)J_{Rt}^s}_{\text{measured hedging against reversal}}.\end{aligned} \tag{43}$$

For easy comparison with Dynamic slope, we add "measured" to the label of each components.

Now we are ready to confront the main question of our paper in the general setting: to what extent an econometrician with an instrument capturing a shift in the residual supply curve can successfully measure the slope of an investor's demand curve? Under what conditions does the measured slope recover its conceptual counterparts, the Dynamic slope or the Myopic slope?

to be flatter than its myopic counterpart.

Measured slope versus myopic slope. Note that Expression (43) immediately provides an intuitive decomposition for the difference between measured and myopic slope, $\mathcal{M}\left(w_t^n; \lambda, u^n, u^s\right) - \mathcal{C}^{\text{myo}}(w_t^n; \delta)|_{\delta=\Delta\mathbb{E}_t(R)}$. The second term (43) is the endogenous risk effect, while the third and fourths terms are the effect of amplified intertemporal hedging and measured hedging against reversal effects.

Measured slope versus dynamic slope. Now we turn to the comparison of $\mathcal{M}$ to $\mathcal{C}^{\text{dyn}}$. To give this exercise a fair chance, we pick i) $\delta = \Delta\mathbb{E}_t(R)$ to match the size and direction of the expected return shock due to the shift in supply; ii) $\chi = \frac{\lambda J^s_{Rt}}{\delta}$ to match the composition; and iii) λ to match the persistence of the shock.

We now show that even if we match the parameters of these objects as closely as possible, $\mathcal{C}^{\text{dyn}}$ and $\mathcal{M}$ in general differ because $\mathcal{M}$ takes into account the adjustment of the dynamic equilibrium to the supply shock. Just as before, we can trace back the difference between $\mathcal{C}^{\text{dyn}}$ and $\mathcal{M}$ to three distinct sources.

Endogenous risk effect. First, recall that the myopic component of the slope of demand, which is the product of risk-tolerance and the inverse of the covariance matrix of returns Ω_t, is analogous to the static case. As the form (28) illustrates, both of these elements are equilibrium objects for general utility functions. In our CRRA setting, (33) implies the risk-tolerance to be an exogenous parameter $\frac{V_w(\cdot)}{wV_{ww}(\cdot)} \equiv \frac{1}{\gamma}$. However, the covariance matrix of returns is endogenous because the shift in residual supply can change the riskiness of the asset's endogenous price and its covariance with other assets, even if cash-flows do not change.

As the slope of the dynamic demand curve is conceptually measuring how a single agent reacts to changes in returns, our measure $\mathcal{C}^{\text{dyn}}$ is not affected by these general equilibrium effects. Therefore, the myopic component of $\mathcal{C}^{\text{dyn}}$ will differ from that of $\mathcal{M}$. The following corollary states the result.

Corollary 1. *For a CRRA or log investor, the myopic slope differs from myopic measured slope because the return covariance matrix changes, i.e. there is an endogenous risk effect:*

$$\textit{myopic measured slope} - \textit{myopic slope} \; = \frac{1}{\gamma} \frac{\left[\left((\Omega_t^s)^{-1} - (\Omega_t^n)^{-1}\right)\left(\mathbb{E}_t\left(R^s\right) - r\mathbf{1}\right)\right]_i}{\left[\Delta\mathbb{E}_t(R)\right]_i}; \qquad (44)$$

where $\gamma = 1$ for the log investor.

As we discussed in Section 4 right after Proposition 2, the economic intuition behind the endogenous risk effect is closely related to standard amplification effects in heterogeneous agent models. Ω_t^s differs from Ω_t^n because all investors increase their holdings of the asset when hedgers liquidate following the shock. The investors' increased exposure to the fundamental dividend shocks tend to imply an increased volatility ((25)) and increased comovement with the other assets (Proposition 3). Hence, given the inverse operator, this effects tend to be negative, making the measured slope flatter than the dynamic slope.

Note the similarity of (44) with the second term in (3) which we labeled as the endogenous risk effect due to the tilt of the demand curve. Just as in our minimal example, the size of this effect depends on the ratio of the tilt, i.e., the change in the (inverse) covariance matrix and the corresponding change in expected returns. Both are endogenously determined. Because of this, as we will see in our calibrated examples, the endogenous risk effect could be large even if the change in covariance matrix (the numerator) is relatively small.

The effect of amplified intertemporal hedging. We now move on to the intertemporal hedging components, which also differ between dynamic slope and measured slope. In particular, the difference of intertemporal hedging with respect to the Brownian shock in (42) and (43) gives the expressions in the next corollary.

Corollary 2. *The amplified intertemporal hedging effect is:*

$$\textit{measured Brownian hedging in } \mathcal{M} - \textit{Brownian hedging in } \mathcal{C}^{dyn}\Big|_{\delta=\Delta\mathbb{E}_t(R),\chi=\frac{\lambda J^s_{Rt}}{\delta}} = \tag{45}$$

$$= \frac{\frac{q^{s\prime}(w^s_t)}{q^s(w^s_t)}\left[w^s_t y^s_t\right]_i - \frac{q^{h\prime}(w^n_t;\theta)}{q^h(w^n_t;\theta)}\left[w^n_t y^n_t\right]_i}{\left[\Delta\mathbb{E}_t\left(R\right)\right]_i} \tag{46}$$

for the Case 2 (CRRA investors) and 0 for Case 1 (log investors).

There are two sources of this difference. The first is that the shape of the value functions under the hypothetical and under the shocked state, that is, $q'(\cdot)/q(\cdot)$, are different. This effect tends to be quantitatively small in our calibration exercises, since in both cases the value functions respond to a similar change in expected returns.

The second difference is more quantitatively important. Unlike $\mathcal{C}^{\mathrm{dyn}}$, the general equilibrium effect enters $\mathcal{M}$: in equilibrium, all investors react to the shift in residual demand, and therefore, aggregate positions change from $w^n_t y^n_t$ to $w^s_t y^s_t$. Each investor understands that if others have a larger position of the asset that will influence the covariance of her wealth with future investment opportunities. In particular, when the portfolio of investors as a group performs well, the resulting increase in aggregate wealth implies larger risk-bearing capacity and lower risk premia. Investment opportunities are becoming worse. An investor with $\gamma > 1$ wants to hedge against that by increasing its exposure to these states, that is, a larger component of y implies a larger component of $\hat{y}$.

Linking the above observation back to our minimal example in Section 2, this is equivalent to having an endogenous intercept a^ξ that is increasing in investors' aggregate risk exposure y. Hence, $a^s > a^n$, so the investor's demand curve shifts upward.[20] We call it the "amplified

[20]This general equilibrium effect is somewhat similar to the standard aggregate demand effect in macroe-

intertemporal hedging effect" and, for $\gamma > 1$, it makes the measured slope steeper than the dynamic slope, going against the endogenous risk effect.

Hedging against reversal effect. Finally, noticing that $\lambda J^s_{Rt} = \chi \Delta \mathbb{E}_t(R)$, the next corollary gives the last difference between $\mathcal{C}^{\text{dyn}}$ and $\mathcal{M}$ due to intertemporal hedging demand for the reversal. Again, the difference comes from that our measured slope is affected by the equilibrium change in all price paths due to the supply shock.

Corollary 3. *The hedging effect against the reversal effect for CRRA investors is:*

$$\textit{hedging against reversal in } \mathcal{M} - \textit{hedging against reversal in } \mathcal{C}^{dyn}\Big|_{\delta=\Delta\mathbb{E}_t(R),\chi=\frac{\lambda J^s_{Rt}}{\delta}} =$$
$$= \frac{\frac{1}{\gamma}\left\{(\Omega^s_t)^{-1}\left[\frac{q^n(w^s_t(1+y^{s\top}_t J^s_{Rt}))}{q^s(w^s_t)(1+\hat{y}^{s\top}_t J^s_{Rt})^\gamma} - 1\right] - (\Omega^n)^{-1}\left[\frac{q^n(w^n_t)}{q^h(w^n_t;\theta)(1+\hat{y}^{h\top}_t \frac{\chi\delta}{\lambda})^\gamma} - 1\right]\right\}[\chi\Delta\mathbb{E}_t(R)]_i}{[\Delta\mathbb{E}_t(R)]_i}. \tag{47}$$

For the log investors $\gamma = 1$ and $q^h(\cdot) = q^n(\cdot) = q^s(\cdot) \equiv 1$.

Summary It is clear that shocks to residual supply exactly recover the slope of CRRA investors' demand only if the sum of the above three objects—endogenous risk effect, amplified intertemporal hedging effect, and hedging against reversal effect—is exactly zero.

In the next subsection, through two applications, we demonstrate that there is no reason that any of these terms should be negligible under reasonable parameter values—to the opposite, they can be quantitatively substantial. Hence, generically, we should not expect that shifts in residual supply recover the slope of dynamic demand.

6 Applications and Discussions

In this section we perform a quantitative study on the difference between the measured slope and its conceptual counterpart, with the help of two applications. We then discuss the connection between our model to the reduced-form factor-based asset demand system in HHHKL, and the way forward given our theoretical results regarding the wedge between these two objects.

6.1 Two Applications: Factor Elasticity and Micro Elasticity

We develop two applications to illustrate our results. These examples are deliberately set in markedly different contexts to highlight the generality of our quantitative findings.

conomic models; for instance, in the Marginal Propensity to Consumption literature, one dollar of additional individual consumption will translate to some endogenous increase in individual wealth, which induces more consumption.

(A) Factor elasticity			
Para.	Description	Value	Targeted Moments
$\bar{D}$	Expected cash flow of assets	[0.27, 0.26]	historical returns
σ	Cash flow volatility	diag(0.485, 0.485)	historical return volatilities
α	Hedgers' absolute risk aversion	1	normalization
γ	Investors' relative risk aversion	3	literature
r	Risk-free rate	0.03	historical average interest rate
ρ	Investors' time discount rate	0.08	investor holdings
S	Asset supply	[1, 1]	normalization
u^n	Hedgers' endowment in normal state	[0, 0]	normalization
u^s	Hedgers' endowment in shock state	[0.15, 0]	fund flows Ben-David et al. (2021)
λ	Intensity of state transitions	0.5	persistence Ben-David et al. (2021)

(B) Micro elasticity			
Para.	Description	Value	Targeted Moments
$\bar{D}_i$	Expected cash flow of asset i	0.0369	historical returns
σ_i	Cash flow volatility of asset i	0.0692	historical return volatilities
κ	Cash flow correlation of i and j	0.3	idiosyncratic volatility
α	Hedgers' absolute risk aversion	1	normalization
γ	Investors' relative risk aversion	3	literature
r	Risk-free rate	0.03	historical average interest rate
ρ	Investors' time discount rate	0.08	investor holdings
S_i	Asset supply of asset i	0.0446	normalization $(1/\sqrt{I})$
v_1	Endowment to index assets	0	normalization
v_2	Endowment to non-index assets	0.0107	fund flows GS (2024)
λ	Intensity of state transitions	0.2	persistence GS (2024)

Table 1: Parameters and Calibration Details. The table reports the parameter values used in the calibration exercises. Panel (A) presents the parameters for the factor elasticity calibration, while Panel (B) shows the parameters for the micro elasticity calibration. GS (2024) refers to Greenwood and Sammon (2024).

Factor elasticity. Our first calibration focuses on the effect of a relative demand shock on a few large factors. To this end, we calibrate to the study of Ben-David et al. (2021) focusing on factor-level elasticities. Our baseline parameters are obtained by calibrating our model to asset return characteristics and fund flows patterns in Ben-David et al. (2021). They show that when investment styles are highly correlated with Morningstar ratings, changes in fund ratings result in style-level fund flows which lead to price effect. In our calibration, we set $I = 2$. We interpret the two assets as two portfolios of different investment styles. In particular, we consider the first asset as the small value portfolio and the second asset as the large growth portfolio. Our choice of parameters is summarized in Table 1 Panel A. For instance, the persistence is 2 in our factor elasticity calibration, matching the empirical half life of fund flows as reported by Ben-David et al. (2021).

Micro elasticity. In the second application, we are interested in index inclusion so that the residual supply shock affects only a few assets out of many, by calibrating our model to the recent study of Greenwood and Sammon (2024). Similar to our multi-asset setting in Section 4.3, we picture a market with a large number of fundamentally identical assets with the only difference in hedgers' endowment. In particular, suppose that in the normal state, hedgers' endowment for the first I_1 assets differs from the rest:

$$[u^n]_{i=1}^{I_1} = \upsilon_1, \ [u^n]_{i=I_1+1}^{I} = \upsilon_2.$$

where $\upsilon_1 \neq \upsilon_2$ are scalars. Our interpretation is that the first I_1 assets are in an index. We are interested in a shock for which $[u^n]_{i=2}^{I-1} = [u^s]_{i=2}^{I-1}$ while $[u^s]_1 = \upsilon_2$, $[u^s]_I = \upsilon_1$. That is, the first asset is excluded from and the last asset is included in the index. Just as in the example of Section 4, we intentionally design this example, so that the shock itself has a minimal effect on the structure of the economy—effectively, we only swap the labels of the first and the last assets. However, the main difference here is that instead of focusing on diminishingly small, but infinitely persistent shock, we calibrate its size and persistence (and each of the other parameters) to a real-world example. Our choice of parameters is summarized in Table 1 Panel B; for instance, we set the persistence to be 5 ($\lambda = 0.2$), which targets the conditional probability of index inclusion or deletion. For detailed explanations of our calibration exercises, see Appendices E.1 and E.2.[21]

What do we learn from two calibrated applications? Table 2 reports our main calibration results. Panel A is for factor elasticity calibration while Panel B is for micro elasticity. In each panel the colored (with bold font) column contains our benchmark parametrization, while the other columns show how our effects vary with the persistence of the demand shift.[22] For each set of parameters, we calculate the myopic slope, the dynamic slope and the measured slope, and decompose the difference to our three effects.

There are a number of common takeaways across the two calibrations. First, while the dynamic slope is always smaller than myopic slope, we find that the difference is quantitatively small. This is consistent with the numerical illustration in Panel A and B in Figure 2. That is, whether one considers the myopic or the dynamic slope as the more intuitive measure for

[21] In Appendix E, we rescale measured slope, $\mathcal{M}$, to make it comparable to standard estimates of price elasticity as in Gabaix and Koijen (2022). Under our baseline parameters, our factor elasticity calibration implies a price elasticity of 1.4. Ben-David et al. (2021) finds 0.2. The index-inclusion calibration yields an implied price elasticity of 399, which is substantially lower than the commonly cited value of around 6250 from Petajisto (2009). For comparison, the empirical micro elasticity estimated by Greenwood and Sammon (2024) is approximately 2.7. These results are in line with a broader observation that we do not dispute in this paper: that frictionless asset pricing models, in their simplest form, face challenges in rationalizing the empirically small price elasticity.

[22] For each column we recalibrate the parameters u^{ξ} to ensure that the implied expected return shocks remain constant at $\Delta\mathbb{E}_t(R) = 0.025$.

(A) Factor elasticity					
	Persistence ($1/\lambda$)				
	0.20	0.50	1.00	**2.00**	5.00
Myopic slope	3.19	3.19	3.20	**3.21**	3.23
Dynamic slope	3.18	3.17	3.17	**3.17**	3.17
Measured slope	2.14	1.95	1.67	**1.24**	0.46
Measured slope/Dynamic slope	67%	61%	53%	**39%**	14%
Dynamic minus measured (DmM)	1.04	1.23	1.50	**1.93**	2.71
Shock to risk/DmM	134%	127%	120%	**112%**	104%
Diff hedging brownian/DmM	-34%	-27%	-20%	**-13%**	-6%
Diff hedging reversal/DmM	0%	0%	1%	**1%**	2%

(B) Micro elasticity					
	Persistence ($1/\lambda$)				
	0.20	0.50	1.00	2.00	**5.00**
Myopic slope	8.87	8.58	8.32	7.96	**7.81**
Dynamic slope	8.87	8.58	8.32	7.96	**7.81**
Measured slope	6.72	6.14	5.49	4.60	**3.42**
Measured slope/Dynamic slope	76%	72%	66%	58%	**44%**
Dynamic minus measured (DmM)	2.15	2.44	2.83	3.36	**4.39**
Shock to risk/DmM	166%	153%	141%	129%	**116%**
Diff hedging brownian/DmM	-66%	-53%	-41%	-29%	**-16%**
Diff hedging reversal/DmM	0%	0%	0%	0%	**0%**

Table 2: Comparative statics with respect to persistence $1/\lambda$. The table shows comparative statics with respect to shock persistence $1/\lambda$, for both the factor elasticity calibration (Panel A) and the micro elasticity calibration (Panel B). For the factor elasticity calibration, the colored (with bold font) column contains our benchmark parametrization: $\bar{D} = [0.27, 0.26]$, $\sigma = \text{diag}(0.485, 0.485)$, $\alpha = 1$, $\gamma = 3$, $r = 0.03$, $\rho = 0.08$, $S = [1, 1]$, $u^n = [0, 0]$, $u^s = [0.15, 0]$, $\lambda = 0.5$ and $w = 2.11$. The remaining columns show how our effects vary with the shock persistence, with u^s recalibrated such that the implied expected return shocks remain constant at $\Delta\mathbb{E}_t(R) = 0.025$. For the micro elasticity calibration, the colored (with bold font) column contains our benchmark parametrization: $\bar{D}_i = 0.0369$, $\sigma_i = 0.0692$, $\kappa = 0.3$, $\alpha = 1$, $\gamma = 3$, $r = 0.03$, $\rho = 0.08$, $S_i = 0.0446$, $v_1 = 0$, $v_2 = 0.0107$, $\lambda = 0.2$ and $w = 4.75$. The remaining columns show how our effects vary with the shock persistence, with v_2 recalibrated such that the implied expected returrn shocks remain constant at $\Delta\mathbb{E}_t(R) = 0.01$.

investor demand elasticity seems to have little practical significance.

Second, in all our examples, a shift in residual supply identifies a significantly flatter demand curve than investors' dynamic slope. In our baseline parameterizations, the measured slope is approximately 40% of the dynamic (and myopic) slope. This is because, just as in our minimal demand system, a shock that shifts residual supply also changes the shape of investors' demand function as general equilibrium effects alter the covariance of returns and induce changes in hedging demand. The fraction decreases with the persistence of the identifying shock (shrinking to 14% in our first example), meaning that the empirical underestimation of our conceptual demand slope is more pronounced when the identifying shock is more persistent. This is intuitive as a more persistent shock should have larger effect on the dynamic equilibrium. (However, recall that in Proposition 2 we show that the under-measurement persists even for pure transitory shocks.)

Third, the decomposition of the difference between demand slope and measured slope shows that the endogenous risk effect dominates the others. When hedgers demand less of the asset, investors have to hold a larger share. Therefore, shock to the aggregate wealth of this sector will have a larger effect on its return. From the point of view of individual investors, the assets are getting riskier and covary more with other stocks which decreases the slope of their demand. In fact, because the amplified intertemporal hedging component is going against the endogenous risk effect whenever $\gamma > 1$, in all the highlighted cases the endogenous risk effect is more than the 100% of the total under-measurement.[23] This share is shrinking with the persistence of the shock ranging from 104% to 166% across our examples.

Fourth, as intertemporal hedging is amplified in general equilibrium, this term plays a significant role in explaining the difference between dynamic slope and measured slope. We observe that while the direction of the effect is in line with the direction of Merton's intertemporal hedging, it is several magnitudes higher once the general equilibrium effect is taken into account. Recall from expression (46) that the size of this effect is mainly driven by the ratio of the shift in the equilibrium aggregate position to the shift in expected returns: $\frac{w^s y^s - w^n y^n}{[\Delta \mathbb{E}_t(R)]_i}$. In absolute terms, our results show that this effect can be responsible for up to 34% and 66% of the total mismeasurement in our two applications, respectively. Its absolute magnitude is especially large for less persistent shocks.

Finally, the component with respect to the intertemporal hedging for the reversal of the supply shock tends to be relatively small.[24]

6.2 Demand in Different Units and Connection to HHHKL

We follow the tradition of Merton (1971) and adopt CRRA preferences to link investors' portfolio share demand to asset returns, as in (35). Since the key identification challenge arises from the endogenous adjustment of investors' demand curve in response to supply shocks within a dynamic general equilibrium setting, it is evident that the issue highlighted by our analysis does not depend on the particular 'units' used when deriving individual investors' asset demand.

As an illustration, we switch here to the CARA-normal tradition to express asset demand

[23] In Appendix E, for both of our applications, we provide calibrations with $\gamma < 1$. In those cases, the endogenous risk effect is responsible for less than 100% of the total as our two main effects resulting in mismeasurements of the same sign.

[24] In Appendix E, we provide a sensitivity analysis with respect to the size of the shock to demonstrate that this effect is highly non-linear. In fact, it dominates the amplified hedging effect when the shock is large.

in terms of units of assets as a function of their prices. Let $\hat{Y}_t^\xi$ denote the individual investor's demand in units of assets—as opposed to $\hat{y}_t^\xi$ which is individual investor's portfolio share—when the economy is in state ξ, and let Y_t^ξ be the aggregate counterpart. Then, we can rewrite the optimal asset demand in (35) as

$$\begin{aligned}\underbrace{\hat{Y}_t^\xi}_{D_t} &= \underbrace{\frac{-r}{\gamma^A(w_t^\xi)}(\Pi_t^\xi)^{-1}}_{\mathcal{E}_t}\underbrace{P_t^\xi}_{P_t} \\ &+ \underbrace{\frac{1}{\gamma^A(w_t^\xi)}\frac{q^{\xi\prime}(w_t^\xi)}{q^\xi(w_t^\xi)}Y_t^\xi + \frac{1}{\gamma^A(w_t^\xi)}(\Pi_t^\xi)^{-1}\left[\bar{D} + \mu_{Pt}^\xi + \frac{\lambda^\xi q^{-\xi}(w_t^\xi + Y_t^{\xi\top}J_t^\xi)}{q^\xi(w_t^\xi)\left(1+\frac{1}{\hat{w}_t^\xi}\hat{Y}_t^{\xi\top}J_t^\xi\right)^\gamma}J_t^\xi\right]}_{e_t}.\end{aligned} \tag{48}$$

Here, we define $\gamma^A(w_t^\xi) \equiv \gamma/w_t^\xi$ as the investor's absolute risk aversion, $\Pi_t^n \equiv (\sigma_{Pt}^n+\sigma)^\top(\sigma_{Pt}^n+\sigma)$ as the dollar return variance-covariance matrix, and $J_t^\xi \equiv P_t^{-\xi} - P_t^\xi$ as the jump in price as the economy transitions from state ξ to state $-\xi$.

The labeling of each term in (48) shows how the dynamic demand curve would map to the standard formulation of demand elasticity estimation (with the notation of HHHKL):

$$D_t = \mathcal{E}_t P_t + e_t,$$

where D_t is the vector of investor's demand, P_t is the price vector, $\mathcal{E}_t$ is the matrix of own and cross elasticities while e_t is a vector of demand shifters (not necessarily zero-mean).

This mapping illustrates the complementary insights in HHHKL and our work. HHHKL shows that under the assumption of *homogeneous substitution conditional on observables* there is an intuitive estimation procedure using both cross-sectional and time-series information to estimate the elasticity matrix $\mathcal{E}$, if the econometrician can find the instrumental variables satisfying the standard conditions for causal inference.

Namely, those instruments should shift the price vector without affecting the elasticity matrix $\mathcal{E}_t$ which the econometrician aims to estimate, and the demand shifter e_t that the econometrician cannot observe. The main insight of our paper is that standard instruments based on the shift in residual supply will not meet these conditions once we consider a dynamic environment. This is apparent in (48) as many elements of e_t and $\mathcal{E}_t$ are indexed by the state of residual supply ξ.[25]

To close the argument, in the rest of this section, we show that demand curve (48) does satisfy the assumption of homogeneous substitution conditional on observables. That is, the

[25] Note that this rewrite also illustrates that our insights apply to the estimation of demand elasticities with respect to prices just the same as to the estimation of elasticities with respect to expected return.

HHHKL estimation procedure, if one finds instruments satisfying the standard conditions, still remains valid.

For simplicity, we consider i.i.d. underlying cash flow shocks, so that the exogenous exogenous diffusion matrix in (4) for the I assets is $\sigma = \bar{\sigma}\mathbf{I}$, with $\bar{\sigma}$ being a scalar and $\mathbf{I}$ an $I \times I$ identity matrix. (We consider a factor structure for σ in Appendix B). Following the same logic as in (25) in Section 4.2, we can derive the endogenous dollar return diffusion matrix of the assets to be

$$\underbrace{\sigma_{Pt}^{\xi}}_{I\times I} = \frac{\bar{\sigma}}{1 - \underbrace{P^{\xi\prime}(w_t^{\xi})^\top Y_t^{\xi}}_{\text{scalar}}} \cdot \underbrace{Y_t^{\xi}}_{I\times 1} \underbrace{P^{\xi\prime}(w_t^{\xi})^\top}_{1\times I}. \tag{49}$$

As a result, the endogenous dollar variance-covariance matrix equals

$$\Pi_t^{\xi} \equiv (\sigma_{Pt}^{\xi} + \sigma)^\top (\sigma_{Pt}^{\xi} + \sigma) = \bar{\sigma}^2 \left[\mathbf{I} + \frac{P^{\xi\prime}(w_t^{\xi}) Y_t^{\xi\top}}{1 - P^{\xi\prime}(w_t^{\xi})^\top Y_t^{\xi}}\right] \left[\mathbf{I} + \frac{Y_t^{\xi} P^{\xi\prime}(w_t^{\xi})^\top}{1 - P^{\xi\prime}(w_t^{\xi})^\top Y_t^{\xi}}\right]. \tag{50}$$

In Appendix B, we show that once we define the asset characteristic matrix to be

$$\boldsymbol{\beta} \equiv \begin{bmatrix} Y_t^{\xi} & P^{\xi\prime}(w_t^{\xi}) \end{bmatrix}_{I\times 2} \tag{51}$$

and the substitution matrix to be

$$\mathcal{E}_{sub} \equiv \begin{bmatrix} P^{\xi\prime}(w_t^{\xi})^\top P^{\xi\prime}(w_t^{\xi}) & -1 \\ -1 & 0 \end{bmatrix}_{2\times 2}, \tag{52}$$

then we have the exact form of demand system studied in HHHKL:

$$\mathcal{E}_t \equiv \left[\frac{\partial \hat{Y}^{\xi}}{\partial P^{\xi}}\right]_{I\times I} = -\frac{r}{\gamma^A(w_t^{\xi})\bar{\sigma}^2} \left(\mathbf{I} + \boldsymbol{\beta}\mathcal{E}_{sub}\boldsymbol{\beta}^\top\right). \tag{53}$$

In the language of HHHKL, in our model each asset has two (endogenous) characteristics, as suggested by (51): its aggregate holdings by investors Y_t^{ξ}, and its price loading on the aggregate investor wealth $P^{\xi\prime}(w_t^{\xi})$. And, we have assumed away any exogenous factor structure for the dividend process; if we were to assume a K-factor structure for the dividend diffusion σ, then the total dimension of asset characteristics will be $K + 2$, as shown in Appendix B.

6.3 The Way Forward

Our paper highlights a conceptual challenge for estimating simple asset demand systems in light of standard forces in dynamic asset pricing. A persistent shift in residual supply changes the equilibrium allocation of risk which affects the equilibrium return and covariance properties of assets. As investors' demand curves for the asset with the altered return and covariance properties are different, it implies that the identifying shock changes the estimand: a violation of the exclusion restriction.

How do researchers move forward in light of these observations? We discuss several potential routes.

Finding small and transitory residual supply shocks? One possibility is to focus on economies where the mismeasurement is smaller. We show that if the shift in residual supply is small and transitory, the difference between measured slope and dynamic (or myopic) slope is significantly muted. However, as shown in Section 4, this approach cannot eliminate the problem because Proposition 2 demonstrates that the bias remains even in the limit.

Adding control variables? One question is whether carefully chosen controls can alleviate this problem. To investigate, we follow the insight of Merton (1971) and write the demand curve in a more general way. As we have explained, the Merton approach represents the demand curve as a function of the instantaneous expected return, the instantaneous return covariance matrix, and the intertemporal hedging demand stemming from the covariance of marginal utility and future investment opportunities.

Considering the case of a single risky asset, and with a slightly loose notation we write the demand curve for the representative investor as

$$\hat{y}_t^{\xi} = D(\mathbb{E}_t(R^{\xi}), (\sigma_{Rt}^{\xi})^2, IHD_t^{\xi}). \tag{54}$$

Here, $D(\cdot,\cdot,\cdot)$ is a general function and the last argument, IHD_t^{ξ}, denotes the intertemporal hedging component. Then, in differences and after linearization, we can write

$$\Delta\hat{y}_t^{\xi} = D_1'\Delta\mathbb{E}_t(R^{\xi}) + D_2'\Delta(\sigma_{Rt}^{\xi})^2 + D_3'\Delta IHD_t^{\xi},$$

where D_1', D_2', D_3' are the three partial derivatives of D in (54).

Can the standard approach of instrumenting $\Delta\mathbb{E}_t(R^{\xi})$, augmented by the inclusion of controls for the volatility of the asset σ_{Rt}^{ξ} along with a proxy for variation in intertemporal hedging, yield a consistent estimate of the demand curve D, or, at least the slope with respect to $\mathbb{E}_t(R^{\xi})$, D_1'? Our observations in this paper imply that as long as the instrument is a shift in residual supply, for instance, $\xi = n$ changing to $\xi = s$ as in our model, then this approach leads to a bad control problem (Angrist and Pischke, 2009). More precisely, because of the tilt and shift, any candidate control variables for $(\sigma_{Rt}^{\xi})^2$ and IHD_t^{ξ} are likely to be affected by the instrument, leading to an endogeneity bias which potentially distorts the estimate of the slope coefficient corresponding to D_1'.

A structural approach to model equilibrium adjustment? Perhaps, a more promising avenue might be to impose more structure and separately model and instrument each argument of the demand function affected by the demand shift. Our fully solved model, especially the demand function of our CRRA investor characterized in (35) and the structure of equilibrium covariance matrix (A.148) derived in Appendix A.1 offers some guidance along this direction. The trade-off is as usual: this route requires taking a strong stance on the form of market environment, together with each participant's incentives and constraints.

Why not focus on equilibrium measured slope $\mathcal{M}$? Another more reduced-form route is to step back from the view that asset markets can be seen as a demand system with classic supply and demand curves as defined in ECON101. For instance, if the Fed wants to know the effect on yields of buying 1 million USD worth of treasuries, they might not care whether the estimate corresponds to any investor's slope of demand. Perhaps the Fed only wants to know the resulting aggregate multiplier, taking into account of all complex equilibrium mechanisms triggering arbitrary tilts and shifts of some investors' demand curves in this particular contest. If this is the case, we can take the measured relationship, akin to $\mathcal{M}(\cdot)$, between quantities and prices as the main object of interest. While, the estimated relationship would not correspond to the quantity response of a given investor group to a change in prices, it might provide sophisticated benchmarks for the literature on what "canonical" models imply on this aggregate multiplier and how sensitive it is to changes of parameters in the economy.

For this route, Appendix E provides some guidance. Taking the index inclusion effect studied in Section 6.1 as an example, we illustrate that the magnitude of $\mathcal{M}(\cdot)$ depends critically on two model ingredients: the calibrated persistence parameter of the demand shift induced by index inclusion or exclusion, λ, and the equilibrium wealth share of index investors, w. The former is closely related to the duration effect emphasized by Davis et al. (2023) and van Binsbergen et al. (2025),[26] and the quantitative effect of λ is shown in Table 2. The latter is closely related to the intermediary asset pricing literature He and Krishnamurthy (2013); note that the calibration of w should also take into account that not all non-index investors trade actively, hence the effective wealth share of investors might be

[26] In contrast to the inelastic financial market hypothesis of Gabaix and Koijen (2022), Berk and Van Binsbergen (2025) argue that the classic CAPM framework is consistent with the observed index inclusion effect documented by Greenwood and Sammon (2024). They emphasize that researchers must properly account for the translation between price and return. To this end, they divide the observed price change upon index inclusion by a duration of 40 years, corresponding to the average dividend yield. While we agree with Berk and Van Binsbergen (2025) on the conceptual point, we contend that the relevant duration should instead reflect the typical horizon over which stocks rotate in and out of benchmark indices—roughly five years, according to Greenwood and Sammon (2024); Pavlova and Sikorskaya (2023). This shorter duration helps explain why the implied price elasticity in our baseline model (399) remains substantially higher than the empirical estimate of 2.7 reported by Greenwood and Sammon (2024).

higher than what our baseline calibration suggests. We also refer interested readers to the calibrations in our companion paper, He et al. (2025), for a more detailed discussion.

7 Concluding Remarks

Measuring how investors were to react to exogenous shocks to returns, or its mirror image, the reaction of returns to changes in aggregate quantities is crucial for a wide range of policy relevant questions. However, according to standard finance theory the slope of the demand curve is determined in equilibrium. In particular, in standard models it depends on the dynamics of future asset prices in general, and in the covariance of asset returns in particular. In this paper, we emphasize that exogenous shocks to the residual *supply* curve, a commonly used instrument in the empirical literature to identify the slope of investors' demand curve, tends to change the future dynamics of asset prices and therefore necessarily changes the investors' demand curve, violating the exclusion restriction. In this paper, we calibrate this effect and find that the dynamic slope tends to be two to seven times larger than its measured counterpart. Importantly, this general equilibrium effect does not vanish when the shock is small.

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Appendix

A Proofs

A.1 Proof of Proposition 1 and Proposition 4

A. Hedgers' Problem

We first consider a hedger's problem. Let $Q_t^\xi \equiv \mathrm{diag}(P_t^\xi)^{-1} X_t^\xi$ denote the hedger's asset holdings in units when the economy is in state $\xi \in \{n, s\}$. From time t to $t + dt$, the hedger's change in wealth is given by

$$dv_t^\xi = rv_t^\xi dt + Q_t^{\xi\top}(dD_t + dP_t^\xi - rP_t^\xi dt) + u^{\xi\top} dD_t. \tag{A.1}$$

Using (4) and (7), we have

$$\begin{aligned} dv_t^\xi =& \Big[rv_t^\xi + Q_t^{\xi\top}(\bar{D} + \mu_{Pt}^\xi - rP_t^\xi) + u^{\xi\top}\bar{D}\Big]dt \\ &+ \Big[(\sigma_{Pt}^\xi + \sigma)Q_t^\xi + \sigma u_t^\xi\Big]^\top dB_t + Q_t^{\xi\top} J_t^\xi dN_t^\xi. \end{aligned} \tag{A.2}$$

Thus,

$$\frac{\mathbb{E}_t[dv_t^\xi]}{dt} = rv_t^\xi + Q_t^{\xi\top}(\bar{D} + \mu_{Pt}^\xi - rP_t^\xi + \lambda^\xi J_t^\xi) + u^{\xi\top}\bar{D}, \tag{A.3}$$

$$\frac{\mathrm{Var}_t(dv_t^\xi)}{dt} = \left[Q_t^{\xi\top}(\sigma_{Pt}^\xi + \sigma)^\top + u_t^{\xi\top}\sigma^\top\right]\left[(\sigma_{Pt}^\xi + \sigma)Q_t^\xi + \sigma u_t^\xi\right] + \lambda^\xi (Q_t^{\xi\top} J_t^\xi)^2. \tag{A.4}$$

Substituting (A.3) and (A.4) into the hedger's mean-variance objective and taking first-order condition yields

$$\bar{D} + \mu_{Pt}^\xi - rP_t^\xi + \lambda^\xi J_t^\xi - \alpha\Big\{(\sigma_{Pt}^\xi + \sigma)^\top\left[(\sigma_{Pt}^\xi + \sigma)Q_t^\xi + \sigma u^\xi\right] + \lambda^\xi J_t^\xi J_t^{\xi\top} Q_t^\xi\Big\} = 0. \tag{A.5}$$

Rearranging yields

$$Q_t^\xi = \Big[(\sigma_{Pt}^\xi + \sigma)^\top(\sigma_{Pt}^\xi + \sigma) + \lambda^\xi J_t^\xi J_t^{\xi\top}\Big]^{-1}\Big[\frac{\bar{D} + \mu_{Pt}^\xi - rP_t^\xi + \lambda^\xi J_t^\xi}{\alpha} - (\sigma_{Pt}^\xi + \sigma)^\top \sigma u^\xi\Big]. \tag{A.6}$$

Note that $\mu_{Rt}^\xi \equiv \mathrm{diag}(P_t^\xi)^{-1}(\bar{D} + \mu_{Pt}^\xi)$, $\sigma_{Rt}^\xi \equiv (\sigma_{Pt}^\xi + \sigma)\,\mathrm{diag}(P_t^\xi)^{-1}$, and $J_{Rt}^\xi \equiv \mathrm{diag}(P_t^\xi)^{-1} J_t^\xi$. Then from (A.6), $X_t^\xi = \mathrm{diag}(P_t^\xi) Q_t^\xi$ can be written as

$$X_t^\xi = \Big[\Omega_t^\xi + \lambda^\xi J_{Rt}^\xi J_{Rt}^{\xi\top}\Big]^{-1}\Big[\frac{\mu_{Rt}^\xi - r\mathbf{1} + \lambda^\xi J_{Rt}^\xi}{\alpha} - (\sigma_{Rt}^\xi)^\top \sigma u^\xi\Big], \tag{A.7}$$

where $\Omega_t^\xi \equiv (\sigma_{Rt}^\xi)^\top \sigma_{Rt}^\xi$.

B. Investors' Problem

Let $\hat{w}_t^\xi$ denote an individual investor's time t wealth when the economy is in state ξ. Let $\hat{Y}_t^\xi \equiv \text{diag}(P_t^\xi)^{-1}\hat{w}_t^\xi \hat{y}_t^\xi$ denote the individual investor's corresponding asset holdings (in units). The investor's wealth $\hat{w}_t^\xi$ evolves according to

$$\begin{aligned} d\hat{w}_t^\xi &= r\hat{w}_t^\xi dt + \hat{Y}_t^{\xi\top}(dD_t + dP_t^\xi - rP_t^\xi dt) - \hat{c}_t^\xi dt \\ &= \left[r\hat{w}_t^\xi - \hat{c}_t^\xi + \hat{Y}_t^{\xi\top}(\bar{D} + \mu_{Pt}^\xi - rP_t^\xi)\right] dt + \hat{Y}_t^{\xi\top}(\sigma_{Pt}^\xi + \sigma)^\top dB_t + \hat{Y}_t^{\xi\top} J_t^\xi dN_t^\xi. \end{aligned} \tag{A.8}$$

Let w_t^ξ denote the aggregate wealth of investors at time t, and let Y_t^ξ denote the aggregate counterpart of $\hat{Y}_t^\xi$. Then w_t^ξ follows

$$dw_t^\xi = \left[rw_t^\xi - c_t^\xi + Y_t^{\xi\top}(\bar{D} + \mu_{Pt}^\xi - rP_t^\xi)\right] dt + Y_t^{\xi\top}(\sigma_{Pt}^\xi + \sigma)^\top dB_t + Y_t^{\xi\top} J_t^\xi dN_t^\xi. \tag{A.9}$$

The individual investor's problem is to choose the optimal consumption and asset holdings $\{\hat{c}_t^\xi, \hat{Y}_t^\xi\}$ to maximize their discounted utility over intertemporal consumption, subject to the wealth dynamic (A.8) while taking prices P_t^ξ as given. Let $V^\xi(\hat{w}_t^\xi, w_t^\xi)$ denote the investor's value function at time t in state ξ. The investor's HJB equation is

$$\begin{aligned} \rho V^\xi = \max_{\hat{c}_t^\xi, \hat{Y}_t^\xi} \Big\{ & u(\hat{c}_t^\xi) + V_{\hat{w}}^\xi \mu_t^{\hat{w},\xi} + \frac{1}{2} V_{\hat{w}\hat{w}}^\xi (\sigma_t^{\hat{w},\xi})^\top \sigma_t^{\hat{w},\xi} + V_w^\xi \mu_t^{w,\xi} + \frac{1}{2} V_{ww}^\xi (\sigma_t^{w,\xi})^\top \sigma_t^{w,\xi} \\ & + V_{\hat{w}w}^\xi (\sigma_t^{\hat{w},\xi})^\top \sigma_t^{w,\xi} + \lambda^\xi (V^{-\xi,+} - V^\xi) \Big\}, \end{aligned} \tag{A.10}$$

where V^ξ stands for the value function $V^\xi(\hat{w}_t^\xi, w_t^\xi)$, while the superscript $+$ on $V^{-\xi,+}$ denotes that the value function is evaluated after the jump, that is

$$V^{-\xi,+} \equiv V^{-\xi}\left(\hat{w}_t^\xi + \hat{Y}_t^{\xi\top} J_t^\xi, w_t^\xi + Y_t^{\xi\top} J_t^\xi\right). \tag{A.11}$$

Moreover, $\mu_t^{\hat{w},\xi}$ and $\sigma_t^{\hat{w},\xi}$ are the drift and the diffusion of the individual investor's wealth $\hat{w}_t^\xi$, while $\mu_t^{w,\xi}$ and $\sigma_t^{w,\xi}$ are the drift and the diffusion of investors' aggregate wealth w_t^ξ. From (A.8) and (A.9),

$$\mu_t^{\hat{w},\xi} \equiv r\hat{w}_t^\xi - \hat{c}_t^\xi + \hat{Y}_t^{\xi\top}(\bar{D} + \mu_{Pt}^\xi - rP_t^\xi), \tag{A.12}$$

$$\sigma_t^{\hat{w},\xi} \equiv (\sigma_{Pt}^\xi + \sigma)\hat{Y}_t^\xi, \tag{A.13}$$

$$\mu_t^{w,\xi} \equiv rw_t^\xi - c_t^\xi + Y_t^{\xi\top}(\bar{D} + \mu_{Pt}^\xi - rP_t^\xi), \tag{A.14}$$

$$\sigma_t^{w,\xi} \equiv (\sigma_{Pt}^\xi + \sigma)Y_t^\xi. \tag{A.15}$$

The first-order condition with respect to consumption implies that

$$u'(\hat{c}_t^\xi) = V_{\hat{w}}^\xi. \tag{A.16}$$

Similarly, the first-order condition with respect to asset holdings implies that $\hat{Y}_t^\xi$ satisfies

$$\begin{aligned} 0 =& V_{\hat{w}}^\xi(\bar{D} + \mu_{Pt}^\xi - rP_t^\xi) + V_{\hat{w}\hat{w}}^\xi(\sigma_{Pt}^\xi + \sigma)^\top(\sigma_{Pt}^\xi + \sigma)\hat{Y}_t^\xi \\ &+ V_{\hat{w}w}^\xi(\sigma_{Pt}^\xi + \sigma)^\top(\sigma_{Pt}^\xi + \sigma)Y_t^\xi + \lambda^\xi V_{\hat{w}}^{-\xi,+}J_t^\xi. \end{aligned} \tag{A.17}$$

Rearranging the first-order condition (A.17), we can write the individual investor's asset demand as

$$\hat{Y}_t^\xi = \frac{V_{\hat{w}}^\xi}{(-V_{\hat{w}\hat{w}}^\xi)}(\Pi_t^\xi)^{-1}(\bar{D} + \mu_{Pt}^\xi - rP_t^\xi) + \frac{V_{\hat{w}w}^\xi}{(-V_{\hat{w}\hat{w}}^\xi)}Y_t^\xi + \lambda^\xi\frac{V_{\hat{w}}^{-\xi,+}}{(-V_{\hat{w}\hat{w}}^\xi)}(\Pi_t^\xi)^{-1}J_t^\xi, \tag{A.18}$$

where we define $\Pi_t^\xi \equiv (\sigma_{Pt}^\xi + \sigma)^\top(\sigma_{Pt}^\xi + \sigma)$. By definition, $\hat{y}_t^\xi = \frac{1}{\hat{w}_t^\xi}\,\mathrm{diag}(P_t^\xi)\hat{Y}_t^\xi$ and $y_t^\xi = \frac{1}{w_t^\xi}\,\mathrm{diag}(P_t^\xi)Y_t^\xi$. Then (A.8) can be rewritten as

$$\begin{aligned} d\hat{w}_t^\xi &= r\hat{w}_t^\xi dt + \hat{w}_t^\xi\hat{y}_t^{\xi\top}\left(dR_t^\xi - r\mathbf{1}dt\right) - \hat{c}_t^\xi dt \\ &= \left[(1 - \hat{y}_t^{\xi\top}\mathbf{1})r\hat{w}_t^\xi - \hat{c}_t^\xi + \hat{w}_t^\xi\hat{y}_t^{\xi\top}\mu_{Rt}^\xi\right] + \hat{w}_t^\xi\hat{y}_t^{\xi\top}(\sigma_{Rt}^\xi)^\top dB_t + \hat{w}_t^\xi\hat{y}_t^{\xi\top}J_{Rt}^\xi dN_t^\xi. \end{aligned} \tag{A.19}$$

Similarly, (A.9) can be rewritten as

$$dw_t^\xi = \left[(1 - y_t^{\xi\top}\mathbf{1})rw_t^\xi - c_t^\xi + w_t^\xi y_t^{\xi\top}\mu_{Rt}^\xi\right] + w_t^\xi y_t^{\xi\top}(\sigma_{Rt}^\xi)^\top dB_t + w_t^\xi y_t^{\xi\top}J_{Rt}^\xi dN_t^\xi. \tag{A.20}$$

And from (A.17), we have

$$\begin{aligned} \hat{y}_t^\xi =& \frac{V_{\hat{w}}^\xi}{\hat{w}_t^\xi(-V_{\hat{w}\hat{w}}^\xi)}(\Omega_t^\xi)^{-1}(\mu_{Rt}^\xi - r\mathbf{1}) + \frac{V_{\hat{w}w}^\xi}{\hat{w}_t^\xi(-V_{\hat{w}\hat{w}}^\xi)}w_t^\xi y_t^\xi + \lambda^\xi\frac{V_{\hat{w}}^{-\xi,+}}{\hat{w}_t^\xi(-V_{\hat{w}\hat{w}}^\xi)}(\Omega_t^\xi)^{-1}J_{Rt}^\xi \\ =& \frac{V_{\hat{w}}^\xi}{\hat{w}_t^\xi(-V_{\hat{w}\hat{w}}^\xi)}(\Omega_t^\xi)^{-1}\left(\mu_{Rt}^\xi - r\mathbf{1} + \lambda^\xi J_{Rt}^\xi\right) + \frac{V_{\hat{w}w}^\xi}{\hat{w}_t^\xi(-V_{\hat{w}\hat{w}}^\xi)}w_t^\xi y_t^\xi \\ &+ \frac{1}{\hat{w}_t^\xi}\frac{V_{\hat{w}}^{-\xi,+} - V_{\hat{w}}^\xi}{(-V_{\hat{w}\hat{w}}^\xi)}(\Omega_t^\xi)^{-1}\lambda^\xi J_{Rt}^\xi. \end{aligned} \tag{A.21}$$

A.1.1 Log Investors

A log investor maximizes

$$\mathbb{E}_t\Big[\int_t^\infty e^{-\rho(\tau-t)}\log\hat{c}_\tau d\tau\Big], \tag{A.22}$$

subject to the wealth dynamic (A.8). To facilitate solution, we let $\hat{z}_t^\xi \equiv \hat{Y}_t^\xi/\hat{w}_t^\xi$ and $z_t^\xi \equiv Y_t^\xi/w_t^\xi$. Then the investor's wealth dynamic (A.8) becomes

$$d\hat{w}_t^\xi = r\hat{w}_t^\xi dt + \hat{w}_t^\xi\hat{z}_t^{\xi\top}(dD_t + dP_t^\xi - rP_t^\xi dt) - \hat{c}_t^\xi dt. \tag{A.23}$$

Substituting (4) and (7) into the individual investor's wealth dynamic (A.23) yields

$$\begin{aligned} d\hat{w}_t^\xi =& \Big[r\hat{w}_t^\xi - \hat{c}_t^\xi + \hat{w}_t^\xi \hat{z}_t^{\xi\top}(\bar{D} + \mu_{Pt}^\xi - rP_t^\xi)\Big]dt + \hat{w}_t^\xi \hat{z}_t^{\xi\top}(\sigma_{Pt}^\xi + \sigma)^\top dB_t \\ &+ \hat{w}_t^\xi \hat{z}_t^{\xi\top} J_t^\xi dN_t^\xi. \end{aligned} \tag{A.24}$$

Similarly, the investors' total wealth dynamic aggregates to

$$\begin{aligned} dw_t^\xi =& \Big[rw_t^\xi - c_t^\xi + w_t^\xi z_t^{\xi\top}(\bar{D} + \mu_{Pt}^\xi - rP_t^\xi)\Big]dt + w_t^\xi z_t^{\xi\top}(\sigma_{Pt}^\xi + \sigma)^\top dB_t \\ &+ w_t^\xi z_t^{\xi\top} J_t^\xi dN_t^\xi. \end{aligned} \tag{A.25}$$

The investor's HJB equation is

$$\begin{aligned} \rho V^\xi = \max_{\hat{c}_t^\xi, \hat{z}_t^\xi} \Big\{ & \log \hat{c}_t^\xi + V_{\hat{w}}^\xi \mu_t^{\hat{w},\xi} + \frac{1}{2} V_{\hat{w}\hat{w}}^\xi (\sigma_t^{\hat{w},\xi})^\top \sigma_t^{\hat{w},\xi} + V_w^\xi \mu_t^{w,\xi} + \frac{1}{2} V_{ww}^\xi (\sigma_t^{w,\xi})^\top \sigma_t^{w,\xi} \\ &+ V_{\hat{w}w}^\xi (\sigma_t^{\hat{w},\xi})^\top \sigma_t^{w,\xi} + \lambda^\xi (V^{-\xi,+} - V^\xi) \Big\}, \end{aligned} \tag{A.26}$$

where

$$\mu_t^{\hat{w},\xi} = r\hat{w}_t^\xi - \hat{c}_t^\xi + \hat{w}_t^\xi \hat{z}_t^{\xi\top}(\bar{D} + \mu_{Pt}^\xi - rP_t^\xi), \tag{A.27}$$

$$\sigma_t^{\hat{w},\xi} = (\sigma_{Pt}^\xi + \sigma)\hat{z}_t^\xi \hat{w}_t^\xi, \tag{A.28}$$

$$\mu_t^{w,\xi} = rw_t^\xi - c_t^\xi + w_t^\xi z_t^{\xi\top}(\bar{D}_t + \mu_{Pt}^\xi - rP_t^\xi), \tag{A.29}$$

$$\sigma_t^{w,\xi} = (\sigma_{Pt}^\xi + \sigma) z_t^\xi w_t^\xi. \tag{A.30}$$

We conjecture (and later verify) that the investor's value function takes the form

$$V^\xi(\hat{w}_t^\xi, w_t^\xi) = \frac{1}{\rho}\log \hat{w}_t^\xi + \bar{q}^\xi(w_t^\xi). \tag{A.31}$$

Substituting (A.31) into (A.26) yields

$$\begin{aligned} &\log \hat{w}_t^\xi + \rho \bar{q}^\xi(w_t^\xi) = \\ &\max_{\hat{c}_t^\xi, \hat{z}_t^\xi} \Big\{ \log \hat{c}_t^\xi + \frac{1}{\rho \hat{w}_t^\xi}\mu_t^{\hat{w},\xi} - \frac{1}{2\rho(\hat{w}_t^\xi)^2}(\sigma_t^{\hat{w},\xi})^\top \sigma^{\hat{w},\xi} + \bar{q}^{\xi\prime}(w_t^\xi)\mu_t^{w,\xi} + \frac{1}{2}\bar{q}^{\xi\prime\prime}(w_t^\xi)(\sigma_t^{w,\xi})^\top \sigma_t^{w,\xi} \\ &+ \lambda^\xi \Big[\frac{1}{\rho}\log\Big(\hat{w}_t^\xi + \hat{w}_t^\xi \hat{z}_t^{\xi\top}(P_t^{-\xi} - P_t^\xi)\Big) + \bar{q}^{-\xi}\Big(w_t^\xi + w^\xi z_t^{\xi\top}(P_t^{-\xi} - P_t^\xi)\Big) - \frac{1}{\rho}\log \hat{w}_t^\xi - \bar{q}^\xi(w_t^\xi)\Big]\Big\}. \end{aligned} \tag{A.32}$$

Taking the first-order condition with respect to consumption yields

$$\frac{1}{\hat{c}_t^\xi} - \frac{1}{\rho \hat{w}_t^\xi} = 0. \tag{A.33}$$

Thus,

$$\hat{c}_t^\xi = \rho \hat{w}_t^\xi. \tag{A.34}$$

Taking the first-order condition with respect to the investor's wealth-scaled asset holdings implies that $\hat{z}_t^\xi$ satisfies

$$0 =(\bar{D} + \mu_{Pt}^\xi - rP_t^\xi) - (\sigma_{Pt}^\xi + \sigma)^\top(\sigma_{Pt}^\xi + \sigma)\hat{z}_t^\xi + \frac{\lambda^\xi(P_t^{-\xi} - P_t^\xi)}{1 + \hat{z}_t^{\xi\top}(P_t^{-\xi} - P_t^\xi)}. \tag{A.35}$$

Rearranging, we get

$$\bar{D} + \mu_{Pt}^\xi - rP_t^\xi = (\sigma_{Pt}^\xi + \sigma)^\top(\sigma_{Pt}^\xi + \sigma)\hat{z}_t^\xi - \frac{\lambda^\xi(P_t^{-\xi} - P_t^\xi)}{1 + \hat{z}_t^{\xi\top}(P_t^{-\xi} - P_t^\xi)}. \tag{A.36}$$

Then

$$\hat{z}_t^\xi = (\Pi_t^\xi)^{-1}(\bar{D} + \mu_{Pt}^\xi - rP_t^\xi) + \lambda^\xi \frac{(\Pi_t^\xi)^{-1}(P_t^{-\xi} - P_t^\xi)}{1 + \hat{z}_t^{\xi\top}(P_t^{-\xi} - P_t^\xi)}. \tag{A.37}$$

From (A.37), since $\hat{y}_t^\xi = \text{diag}(P_t^\xi)\hat{z}_t^\xi$,

$$\begin{aligned} \hat{y}_t^\xi &= (\Omega_t^\xi)^{-1}(\mu_{Rt}^\xi - r\mathbf{1}) + \lambda^\xi \frac{(\Omega_t^\xi)^{-1}J_{Rt}^\xi}{1 + \hat{y}_t^{\xi\top}J_{Rt}^\xi} \\ &= (\Omega_t^\xi)^{-1}\left(\mu_{Rt}^\xi - r\mathbf{1} + \lambda^\xi J_{Rt}^\xi\right) + \lambda^\xi(\Omega_t^\xi)^{-1}\left(\frac{1}{1 + \hat{y}_t^{\xi\top}J_{Rt}^\xi} - 1\right)J_{Rt}^\xi. \end{aligned} \tag{A.38}$$

A. Derivation of ODEs

Note that $\hat{z}_t^\xi = z_t^\xi$, $\hat{c}_t^\xi = c_t^\xi$ and $\hat{w}_t^\xi = w_t^\xi$ in equilibrium, then (A.36) implies

$$\bar{D} + \mu_{Pt}^\xi - rP_t^\xi = (\sigma_{Pt}^\xi + \sigma)^\top(\sigma_{Pt}^\xi + \sigma)z_t^\xi - \frac{\lambda^\xi(P_t^{-\xi} - P_t^\xi)}{1 + z_t^{\xi\top}(P_t^{-\xi} - P_t^\xi)}, \tag{A.39}$$

and from (A.34), $c_t^\xi = \rho w_t^\xi$. Substituting into the HJB equation (A.32) yields

$$\begin{aligned} \rho\bar{q}^\xi(w_t^\xi) &= \log\rho + \frac{1}{\rho}\Big[r - \rho + \hat{z}_t^{\xi\top}(\bar{D} + \mu_{Pt}^\xi - rP_t^\xi)\Big] \\ &+ \frac{1}{2\rho}\hat{z}_t^{\xi\top}(\sigma_{Pt}^\xi + \sigma)^\top(\sigma_{Pt}^\xi + \sigma)\hat{z}_t^\xi \\ &+ \bar{q}^{\xi\prime}(w_t^\xi)w_t^\xi\Big[r - \rho + z_t^{\xi\top}(\bar{D} + \mu_{Pt}^\xi - rP_t^\xi)\Big] \\ &+ \frac{1}{2}\bar{q}^{\xi\prime\prime}(w_t^\xi)(w_t^\xi)^2 z_t^{\xi\top}(\sigma_{Pt}^\xi + \sigma)^\top(\sigma_{Pt}^\xi + \sigma)z_t^\xi \\ &+ \lambda^\xi\Big[\frac{1}{\rho}\log(1 + \hat{z}_t^{\xi\top}(P_t^{-\xi} - P_t^\xi)) + \bar{q}^{-\xi}\Big(w_t^\xi + w_t^\xi z_t^{\xi\top}(P_t^{-\xi} - P_t^\xi)\Big) \; - \bar{q}^\xi(w_t^\xi)\Big]. \end{aligned} \tag{A.40}$$

Note that $\hat{w}_t^\xi$ have all canceled out. Thus, we have verified the conjectured value function

(A.31). Using the fact that $\hat{z}_t^\xi = z_t^\xi$, we get

$$\begin{aligned}\rho\bar{q}^\xi(w_t^\xi) =& \log\rho + \Big[\bar{q}^{\xi\prime}(w_t^\xi)w_t^\xi + \frac{1}{\rho}\Big]\Big[r - \rho + z_t^{\xi\top}(\bar{D} + \mu_{Pt}^\xi - rP_t^\xi)\Big] \\ &+ \frac{1}{2}\Big[\bar{q}^{\xi\prime\prime}(w_t^\xi)(w_t^\xi)^2 - \frac{1}{\rho}\Big]z_t^{\xi\top}(\sigma_{Pt}^\xi + \sigma)^\top(\sigma_{Pt}^\xi + \sigma)z_t^\xi \\ &+ \lambda^\xi\Big[\frac{1}{\rho}\log(1 + z_t^{\xi\top}(P_t^{-\xi} - P_t^\xi)) + \bar{q}^{-\xi}\Big(w_t^\xi + w_t^\xi z_t^{\xi\top}(P_t^{-\xi} - P_t^\xi)\Big) - \bar{q}^\xi(w_t^\xi)\Big].\end{aligned} \tag{A.41}$$

Using (A.39),

$$\begin{aligned}&\rho\bar{q}^\xi(w_t^\xi) \\ &= \log\rho + \Big[\bar{q}^{\xi\prime}(w_t^\xi)w_t^\xi + \frac{1}{\rho}\Big]\Big[r - \rho + z_t^{\xi\top}(\sigma_{Pt}^\xi + \sigma)^\top(\sigma_{Pt}^\xi + \sigma)z_t^\xi\Big] \\ &- \Big[\bar{q}^{\xi\prime}(w_t^\xi)w_t^\xi + \frac{1}{\rho}\Big]\frac{\lambda^\xi z_t^{\xi\top}(P_t^{-\xi} - P_t^\xi)}{1 + z_t^{\xi\top}(P_t^{-\xi} - P_t^\xi)} \\ &+ \frac{1}{2}\Big[\bar{q}^{\xi\prime\prime}(w_t^\xi)(w_t^\xi)^2 - \frac{1}{\rho}\Big]z_t^{\xi\top}(\sigma_{Pt}^\xi + \sigma)^\top(\sigma_{Pt}^\xi + \sigma)z_t^\xi \\ &+ \lambda^\xi\Big[\frac{1}{\rho}\log(1 + z_t^{\xi\top}(P_t^{-\xi} - P_t^\xi)) + \bar{q}^{-\xi}\Big(w_t^\xi + w_t^\xi z_t^{\xi\top}(P_t^{-\xi} - P_t^\xi)\Big) - \bar{q}^\xi(w_t^\xi)\Big].\end{aligned} \tag{A.42}$$

Collecting terms, we get

$$\begin{aligned}\rho\bar{q}^\xi(w_t^\xi) =& \log\rho + \Big[\bar{q}^{\xi\prime}(w_t^\xi)w_t^\xi + \frac{1}{\rho}\Big](r - \rho) \\ &+ \Big[\bar{q}^{\xi\prime}(w_t^\xi)w_t^\xi + \frac{1}{2}\bar{q}^{\xi\prime\prime}(w_t^\xi)(w_t^\xi)^2 + \frac{1}{2\rho}\Big]z_t^{\xi\top}(\sigma_{Pt}^\xi + \sigma)^\top(\sigma_{Pt}^\xi + \sigma)z_t^\xi \\ &+ \lambda^\xi\Big[\frac{1}{\rho}\log(1 + z_t^{\xi\top}(P_t^{-\xi} - P_t^\xi)) + \bar{q}^{-\xi}\Big(w_t^\xi + w_t^\xi z_t^{\xi\top}(P_t^{-\xi} - P_t^\xi)\Big) - \bar{q}^\xi(w_t^\xi) \\ &\qquad - \Big(\bar{q}^{\xi\prime}(w_t^\xi)w_t^\xi + \frac{1}{\rho}\Big)\frac{z_t^{\xi\top}(P_t^{-\xi} - P_t^\xi)}{1 + z_t^{\xi\top}(P_t^{-\xi} - P_t^\xi)}\Big].\end{aligned} \tag{A.43}$$

Substituting the investors' aggregate consumption c_t^ξ into the total wealth dynamic for investors, we get

$$dw_t^\xi = w_t^\xi\Big[r - \rho + z_t^{\xi\top}(\bar{D} + \mu_{Pt}^\xi - rP_t^\xi)\Big]dt + w_t^\xi z_t^{\xi\top}(\sigma_{Pt}^\xi + \sigma)^\top dB_t + w_t^\xi z_t^{\xi\top}J_t^\xi dN_t^\xi. \tag{A.44}$$

The investors' total wealth w_t^ξ and the state of the economy ξ are the two state variables in the model. Thus, the risky assets' prices in state ξ must be functions of the investors' total wealth in the state, that is $P_t^\xi \equiv P^\xi(w_t^\xi)$. Let

$$\tilde{\mu}_t^{w,\xi} \equiv w_t^\xi\big[r - \rho + z_t^{\xi\top}(\bar{D} + \mu_{Pt}^\xi - rP_t^\xi)\big], \tag{A.45}$$

$$\tilde{\sigma}_t^{w,\xi} \equiv (\sigma_{Pt}^\xi + \sigma)z_t^\xi w_t^\xi. \tag{A.46}$$

By Itô's lemma, we have

$$
\begin{aligned}
dP_t^\xi =& \Big[\tilde{\mu}_t^{w,\xi} P^{\xi\prime}(w_t^\xi) + \frac{1}{2}(\tilde{\sigma}_t^{w,\xi})^\top \tilde{\sigma}_t^{w,\xi} P^{\xi\prime\prime}(w_t^\xi)\Big] dt + P^{\xi\prime}(w_t^\xi)(\tilde{\sigma}_t^{w,\xi})^\top dB_t \\
&+ (P_t^{-\xi} - P_t^\xi) dN_t^\xi.
\end{aligned}
\tag{A.47}
$$

From (A.45) and (A.46), the drift of the risky assets' prices become

$$
\begin{aligned}
&\tilde{\mu}_t^{w,\xi} P^{\xi\prime}(w_t^\xi) + \frac{1}{2}(\tilde{\sigma}_t^{w,\xi})^\top \tilde{\sigma}_t^{w,\xi} P^{\xi\prime\prime}(w_t^\xi) \\
&= w_t^\xi \big[r - \rho + z_t^{\xi\top}(\bar{D} + \mu_{Pt}^\xi - rP_t^\xi)\big] P^{\xi\prime}(w_t^\xi) + \frac{1}{2} z_t^{\xi\top}(\sigma_{Pt}^\xi + \sigma)^\top (\sigma_{Pt}^\xi + \sigma) z_t^\xi P^{\xi\prime\prime}(w_t^\xi)(w_t^\xi)^2 \\
&= \Big[r - \rho + z_t^{\xi\top}(\sigma_{Pt}^\xi + \sigma)^\top(\sigma_{Pt}^\xi + \sigma) z_t^\xi - \frac{\lambda^\xi z_t^{\xi\top}(P_t^{-\xi} - P_t^\xi)}{1 + z_t^{\xi\top}(P_t^{-\xi} - P_t^\xi)}\Big] P^{\xi\prime}(w_t^\xi) w_t^\xi \\
&\quad + \frac{1}{2} z_t^{\xi\top}(\sigma_{Pt}^\xi + \sigma)^\top(\sigma_{Pt}^\xi + \sigma) z_t^\xi P^{\xi\prime\prime}(w_t^\xi)(w_t^\xi)^2 \\
&= \Big[r - \rho - \frac{\lambda^\xi z_t^{\xi\top}(P_t^{-\xi} - P_t^\xi)}{1 + z_t^{\xi\top}(P_t^{-\xi} - P_t^\xi)}\Big] P^{\xi\prime}(w_t^\xi) w_t^\xi \\
&\quad + z_t^{\xi\top}(\sigma_{Pt}^\xi + \sigma)^\top(\sigma_{Pt}^\xi + \sigma) z_t^\xi \Big[P^{\xi\prime}(w_t^\xi) w_t^\xi + \frac{1}{2} P^{\xi\prime\prime}(w_t^\xi)(w_t^\xi)^2\Big].
\end{aligned}
\tag{A.48}
$$

The diffusion of the risky assets' prices are

$$
\tilde{\sigma}_t^{w,\xi} P^{\xi\prime}(w_t^\xi)^\top = (\sigma_{Pt}^\xi + \sigma) z_t^\xi w_t^\xi P^{\xi\prime}(w_t^\xi)^\top. \tag{A.49}
$$

The risky assets' prices are conjectured to follow the Itô process (7). By matching coefficients, we get

$$
J_t^\xi = P_t^{-\xi} - P_t^\xi, \tag{A.50}
$$

while μ_{Pt}^ξ and σ_{Pt}^ξ are respectively given by

$$
\begin{aligned}
\mu_{Pt}^\xi =& \Big[r - \rho - \frac{\lambda^\xi z_t^{\xi\top}(P_t^{-\xi} - P_t^\xi)}{1 + z_t^{\xi\top}(P_t^{-\xi} - P_t^\xi)}\Big] P^{\xi\prime}(w_t^\xi) w_t^\xi \\
&+ z_t^{\xi\top}(\sigma_{Pt}^\xi + \sigma)^\top(\sigma_{Pt}^\xi + \sigma) z_t^\xi \Big[P^{\xi\prime}(w_t^\xi) w_t^\xi + \frac{1}{2} P^{\xi\prime\prime}(w_t^\xi)(w_t^\xi)^2\Big],
\end{aligned}
\tag{A.51}
$$

and

$$
\sigma_{Pt}^\xi = (\sigma_{Pt}^\xi + \sigma) z_t^\xi w_t^\xi P^{\xi\prime}(w_t^\xi)^\top. \tag{A.52}
$$

From (A.52),

$$
(\sigma_{Pt}^\xi + \sigma) z_t^\xi = (\sigma_{Pt}^\xi + \sigma) z_t^\xi P^{\xi\prime}(w_t^\xi)^\top z_t^\xi w_t^\xi + \sigma z_t^\xi. \tag{A.53}
$$

Thus,

$$\left[1 - P^{\xi\prime}(w_t^\xi)^\top z_t^\xi w_t^\xi\right](\sigma_{Pt}^\xi + \sigma) z_t^\xi = \sigma z_t^\xi. \tag{A.54}$$

Then

$$(\sigma_{Pt}^\xi + \sigma) z_t^\xi = \frac{\sigma z_t^\xi}{1 - P^{\xi\prime}(w_t^\xi)^\top z_t^\xi w_t^\xi}. \tag{A.55}$$

Substituting (A.55) into (A.52) yields

$$\sigma_{Pt}^\xi = \frac{\sigma z_t^\xi P^{\xi\prime}(w_t^\xi)^\top w_t^\xi}{1 - P^{\xi\prime}(w_t^\xi)^\top z_t^\xi w_t^\xi}. \tag{A.56}$$

Adding σ to both sides yields

$$\sigma_{Pt}^\xi + \sigma = \frac{\sigma z_t^\xi P^{\xi\prime}(w_t^\xi)^\top w_t^\xi + [1 - P^{\xi\prime}(w_t^\xi)^\top z_t^\xi w_t^\xi]\sigma}{1 - P^{\xi\prime}(w_t^\xi)^\top z_t^\xi w_t^\xi}. \tag{A.57}$$

Let

$$\hat{\sigma}_t^\xi \equiv \sigma z_t^\xi P^{\xi\prime}(w_t^\xi)^\top w_t^\xi + [1 - P^{\xi\prime}(w_t^\xi)^\top z_t^\xi w_t^\xi]\sigma. \tag{A.58}$$

Then

$$\sigma_{Pt}^\xi + \sigma = \frac{\hat{\sigma}_t^\xi}{1 - P^{\xi\prime}(w_t^\xi)^\top z_t^\xi w_t^\xi}. \tag{A.59}$$

Substituting (A.55) into (A.51) yields

$$\begin{aligned}\mu_{Pt}^\xi =& \left[r - \rho - \frac{\lambda^\xi z_t^{\xi\top}(P_t^{-\xi} - P_t^\xi)}{1 + z_t^{\xi\top}(P_t^{-\xi} - P_t^\xi)}\right] P^{\xi\prime}(w_t^\xi) w_t^\xi \\ &+ \frac{z_t^{\xi\top}\Sigma z_t^\xi}{[1 - P^{\xi\prime}(w_t^\xi)^\top z_t^\xi w_t^\xi]^2}\left[P^{\xi\prime}(w_t^\xi) w_t^\xi + \frac{1}{2} P^{\xi\prime\prime}(w_t^\xi)(w_t^\xi)^2\right].\end{aligned} \tag{A.60}$$

Thus

$$\begin{aligned}&\bar{D} + \mu_{Pt}^\xi - r P_t^\xi \\ &= \bar{D} - r P^\xi(w_t^\xi) + \left[r - \rho - \frac{\lambda^\xi z_t^{\xi\top}(P_t^{-\xi} - P_t^\xi)}{1 + z_t^{\xi\top}(P_t^{-\xi} - P_t^\xi)}\right] P^{\xi\prime}(w_t^\xi) w_t^\xi \\ &+ \frac{z_t^{\xi\top}\Sigma z_t^\xi}{[1 - P^{\xi\prime}(w_t^\xi)^\top z_t^\xi w_t^\xi]^2}\left[P^{\xi\prime}(w_t^\xi) w_t^\xi + \frac{1}{2} P^{\xi\prime\prime}(w_t^\xi)(w_t^\xi)^2\right].\end{aligned} \tag{A.61}$$

From (A.39), and using (A.55) and (A.59), we have

$$\bar{D} + \mu_{Pt}^{\xi} - rP_t^{\xi} = \frac{\hat{\sigma}_t^{\xi\top}\sigma z_t^{\xi}}{[1 - P^{\xi\prime}(w_t^{\xi})^\top z_t^{\xi} w_t^{\xi}]^2} - \frac{\lambda^{\xi}(P_t^{-\xi} - P_t^{\xi})}{1 + z_t^{\xi\top}(P_t^{-\xi} - P_t^{\xi})}. \tag{A.62}$$

Hence,

$$\begin{aligned} &\bar{D} - rP^{\xi}(w_t^{\xi}) + \Big[r - \rho - \frac{\lambda^{\xi} z_t^{\xi\top}(P_t^{-\xi} - P_t^{\xi})}{1 + z_t^{\xi\top}(P_t^{-\xi} - P_t^{\xi})}\Big] P^{\xi\prime}(w_t^{\xi}) w_t^{\xi} \\ &+ \frac{z_t^{\xi\top}\Sigma z_t^{\xi}}{[1 - P^{\xi\prime}(w_t^{\xi})^\top z_t^{\xi} w_t^{\xi}]^2}\Big[P^{\xi\prime}(w_t^{\xi}) w_t^{\xi} + \frac{1}{2}P^{\xi\prime\prime}(w_t^{\xi})(w_t^{\xi})^2\Big] \\ &= \frac{\hat{\sigma}_t^{\xi\top}\sigma z_t^{\xi}}{[1 - P^{\xi\prime}(w_t^{\xi})^\top z_t^{\xi} w_t^{\xi}]^2} - \frac{\lambda^{\xi}(P_t^{-\xi} - P_t^{\xi})}{1 + z_t^{\xi\top}(P_t^{-\xi} - P_t^{\xi})}. \end{aligned} \tag{A.63}$$

The equation (A.63) follows from the investors' first-order condition and the Itô's lemma. Moreover, from (A.62) we have

$$z_t^{\xi\top}(\bar{D} + \mu_{Pt}^{\xi} - rP_t^{\xi}) = \frac{z_t^{\xi\top}\Sigma z_t^{\xi}}{[1 - P^{\xi\prime}(w_t^{\xi})^\top z_t^{\xi} w_t^{\xi}]^2} - \frac{\lambda^{\xi} z_t^{\xi\top}(P_t^{-\xi} - P_t^{\xi})}{1 + z_t^{\xi\top}(P_t^{-\xi} - P_t^{\xi})}. \tag{A.64}$$

Since there is a measure one of symmetric hedgers, the aggregate risky asset holdings by hedgers at time t in state ξ are given by

$$\begin{aligned} Q_t^{\xi} =& \Big[(\sigma_{Pt}^{\xi} + \sigma)^\top(\sigma_{Pt} + \sigma) + \lambda^{\xi} J_t^{\xi} J_t^{\xi\top}\Big]^{-1}\Big[\frac{\bar{D} + \mu_{Pt}^{\xi} - rP_t^{\xi} + \lambda^{\xi} J_t^{\xi}}{\alpha} - (\sigma_{Pt}^{\xi} + \sigma)^\top \sigma u^{\xi}\Big] \\ =& \frac{1}{\alpha}\Big\{\frac{\hat{\sigma}_t^{\xi\top}\hat{\sigma}_t^{\xi}}{[1 - P^{\xi\prime}(w_t^{\xi})^\top z_t^{\xi} w_t^{\xi}]^2} + \lambda^{\xi}(P_t^{-\xi} - P_t^{\xi})(P_t^{-\xi} - P_t^{\xi})^\top\Big\}^{-1}\Big\{\lambda^{\xi}(P_t^{-\xi} - P_t^{\xi}) \\ &+ \bar{D} - rP^{\xi}(w_t^{\xi}) + \Big[r - \rho - \frac{\lambda^{\xi} z_t^{\xi\top}(P_t^{-\xi} - P_t^{\xi})}{1 + z_t^{\xi\top}(P_t^{-\xi} - P_t^{\xi})}\Big] P^{\xi\prime}(w_t^{\xi}) w_t^{\xi} \\ &+ \frac{z_t^{\xi\top}\Sigma z_t^{\xi}}{[1 - P^{\xi\prime}(w_t^{\xi})^\top z_t^{\xi} w_t^{\xi}]^2}\Big[P^{\xi\prime}(w_t^{\xi}) w_t^{\xi} + \frac{1}{2}P^{\xi\prime\prime}(w_t^{\xi})(w_t^{\xi})^2\Big] - \frac{\alpha\hat{\sigma}_t^{\xi\top}\sigma u^{\xi}}{1 - P^{\xi\prime}(w_t^{\xi})^\top z_t^{\xi} w_t^{\xi}}\Big\}. \end{aligned} \tag{A.65}$$

Market clearing requires that

$$Q_t^{\xi} + Y_t^{\xi} = Q_t^{\xi} + z_t^{\xi} w_t^{\xi} = S. \tag{A.66}$$

That is,

$$
\begin{aligned}
&\frac{1}{\alpha}\Big\{\frac{\hat{\sigma}_t^{\xi\top}\hat{\sigma}_t^{\xi}}{[1-P^{\xi\prime}(w_t^{\xi})^{\top}z_t^{\xi}w_t^{\xi}]^2}+\lambda^{\xi}(P_t^{-\xi}-P_t^{\xi})(P_t^{-\xi}-P_t^{\xi})^{\top}\Big\}^{-1}\Big\{\lambda^{\xi}(P_t^{-\xi}-P_t^{\xi})\\
&\quad+\bar{D}-rP^{\xi}(w_t^{\xi})+\Big[r-\rho-\frac{\lambda^{\xi}z_t^{\xi\top}(P_t^{-\xi}-P_t^{\xi})}{1+z_t^{\xi\top}(P_t^{-\xi}-P_t^{\xi})}\Big]P^{\xi\prime}(w_t^{\xi})w_t^{\xi}\\
&\quad+\frac{z_t^{\xi\top}\Sigma z_t^{\xi}}{[1-P^{\xi\prime}(w_t^{\xi})^{\top}z_t^{\xi}w_t^{\xi}]^2}\Big[P^{\xi\prime}(w_t^{\xi})w_t^{\xi}+\frac{1}{2}P^{\xi\prime\prime}(w_t^{\xi})(w_t^{\xi})^2\Big]-\frac{\alpha\hat{\sigma}_t^{\xi\top}\sigma u^{\xi}}{1-P^{\xi\prime}(w_t^{\xi})^{\top}z_t^{\xi}w_t^{\xi}}\Big\}\\
&\quad=S-z_t^{\xi}w_t^{\xi}.
\end{aligned}
\tag{A.67}
$$

The equation (A.67) follows from the market clearing condition.

When the economy switches from state ξ to $-\xi$ at time t, the investor's wealth jumps from w_t^{ξ} to $w_t^{-\xi}$ according to the following fixed point condition

$$
w_t^{-\xi}=w_t^{\xi}+w_t^{\xi}z_t^{\xi\top}(P_t^{-\xi}-P_t^{\xi}).
\tag{A.68}
$$

Substituting (A.68) into (A.63) yields

$$
\begin{aligned}
&\bar{D}-rP^{\xi}(w_t^{\xi})+\left[r-\rho-\frac{\lambda^{\xi}\frac{w_t^{-\xi}-w_t^{\xi}}{w_t^{\xi}}}{\frac{w_t^{-\xi}}{w_t^{\xi}}}\right]P^{\xi\prime}(w_t^{\xi})w_t^{\xi}\\
&+\frac{z_t^{\xi\top}\Sigma z_t^{\xi}}{[1-P^{\xi\prime}(w_t^{\xi})^{\top}z_t^{\xi}w_t^{\xi}]^2}\Big[P^{\xi\prime}(w_t^{\xi})w_t^{\xi}+\frac{1}{2}P^{\xi\prime\prime}(w_t^{\xi})(w_t^{\xi})^2\Big]\\
&=\frac{\hat{\sigma}_t^{\xi\top}\sigma z_t^{\xi}}{[1-P^{\xi\prime}(w_t^{\xi})^{\top}z_t^{\xi}w_t^{\xi}]^2}-\frac{\lambda^{\xi}(P_t^{-\xi}-P_t^{\xi})}{\frac{w_t^{-\xi}}{w_t^{\xi}}}.
\end{aligned}
\tag{A.69}
$$

Furthermore, substituting (A.68) into (A.67) yields

$$
\begin{aligned}
&\bar{D}-rP^{\xi}(w_t^{\xi})+\left[r-\rho-\frac{\lambda^{\xi}\frac{w_t^{-\xi}-w_t^{\xi}}{w_t^{\xi}}}{\frac{w_t^{-\xi}}{w_t^{\xi}}}\right]P^{\xi\prime}(w_t^{\xi})w_t^{\xi}\\
&+\frac{z_t^{\xi\top}\Sigma z_t^{\xi}}{[1-P^{\xi\prime}(w_t^{\xi})^{\top}z_t^{\xi}w_t^{\xi}]^2}\Big[P^{\xi\prime}(w_t^{\xi})w_t^{\xi}+\frac{1}{2}P^{\xi\prime\prime}(w_t^{\xi})(w_t^{\xi})^2\Big]\\
&=\alpha\Big\{\frac{\hat{\sigma}_t^{\xi\top}\hat{\sigma}_t^{\xi}}{[1-P^{\xi\prime}(w_t^{\xi})^{\top}z_t^{\xi}w_t^{\xi}]^2}+\lambda^{\xi}(P_t^{-\xi}-P_t^{\xi})(P_t^{-\xi}-P_t^{\xi})^{\top}\Big\}(S-z_t^{\xi}w_t^{\xi})\\
&\quad+\frac{\alpha\hat{\sigma}_t^{\xi\top}\sigma u^{\xi}}{1-P^{\xi\prime}(w_t^{\xi})^{\top}z_t^{\xi}w_t^{\xi}}-\lambda^{\xi}(P_t^{-\xi}-P_t^{\xi}).
\end{aligned}
\tag{A.70}
$$

Combining (A.69) and (A.70) yields

$$
\begin{aligned}
&\frac{\hat{\sigma}_t^{\xi\top}\sigma z_t^{\xi}}{[1-P^{\xi\prime}(w_t^{\xi})^{\top}z_t^{\xi}w_t^{\xi}]^2}-\frac{\lambda^{\xi}(P_t^{-\xi}-P_t^{\xi})}{\frac{w_t^{-\xi}}{w_t^{\xi}}}\\
&=\frac{\alpha\hat{\sigma}_t^{\xi\top}\hat{\sigma}_t^{\xi}(S-z_t^{\xi}w_t^{\xi})}{[1-P^{\xi\prime}(w_t^{\xi})^{\top}z_t^{\xi}w_t^{\xi}]^2}+\alpha\lambda^{\xi}(P_t^{-\xi}-P_t^{\xi})(P_t^{-\xi}-P_t^{\xi})^{\top}(S-z_t^{\xi}w_t^{\xi})\\
&\quad+\frac{\alpha\hat{\sigma}_t^{\xi\top}\sigma u^{\xi}}{1-P^{\xi\prime}(w_t^{\xi})^{\top}z_t^{\xi}w_t^{\xi}}-\lambda^{\xi}(P_t^{-\xi}-P_t^{\xi}).
\end{aligned}
\tag{A.71}
$$

Rearranging yields

$$
\begin{aligned}
&\frac{(1+\alpha w_t^{\xi})\hat{\sigma}_t^{\xi\top}\sigma z_t^{\xi}-\alpha\hat{\sigma}_t^{\xi\top}\hat{\sigma}_t^{\xi}S}{[1-P^{\xi\prime}(w_t^{\xi})^{\top}z_t^{\xi}w_t^{\xi}]^2}-\frac{\alpha\hat{\sigma}_t^{\xi\top}\sigma u^{\xi}}{1-P^{\xi\prime}(w_t^{\xi})^{\top}z_t^{\xi}w_t^{\xi}}\\
&=\lambda^{\xi}\left[\left(\frac{1}{\frac{w_t^{-\xi}}{w_t^{\xi}}}-1\right)(P_t^{-\xi}-P_t^{\xi})+\alpha(P_t^{-\xi}-P_t^{\xi})(P_t^{-\xi}-P_t^{\xi})^{\top}(S-z_t^{\xi}w_t^{\xi})\right].
\end{aligned}
\tag{A.72}
$$

Note that

$$
\hat{\sigma}_t^{\xi\top}\sigma z_t^{\xi}=w_t^{\xi}P^{\xi\prime}(w_t^{\xi})z_t^{\xi\top}\Sigma z_t^{\xi}+[1-w_t^{\xi}P^{\xi\prime}(w_t^{\xi})^{\top}z_t^{\xi}]\Sigma z_t^{\xi}. \tag{A.73}
$$

Hence, (A.69) simplifies to

$$
\begin{aligned}
&\bar{D}-rP^{\xi}(w_t^{\xi})+\left[r-\rho-\frac{\lambda^{\xi}\frac{w_t^{-\xi}-w_t^{\xi}}{w_t^{\xi}}}{\frac{w_t^{-\xi}}{w_t^{\xi}}}\right]P^{\xi\prime}(w_t^{\xi})w_t^{\xi}+\frac{z_t^{\xi\top}\Sigma z_t^{\xi}}{2[1-P^{\xi\prime}(w_t^{\xi})^{\top}z_t^{\xi}w_t^{\xi}]^2}P^{\xi\prime\prime}(w_t^{\xi})(w_t^{\xi})^2\\
&=\frac{\Sigma z_t^{\xi}}{1-P^{\xi\prime}(w_t^{\xi})^{\top}z_t^{\xi}w_t^{\xi}}-\frac{\lambda^{\xi}(P_t^{-\xi}-P_t^{\xi})}{\frac{w_t^{-\xi}}{w_t^{\xi}}}.
\end{aligned}
\tag{A.74}
$$

Moreover, $\bar{q}^{\xi}(w_t^{\xi})$ is solution to the ODE (A.43), which can be rewritten as

$$
\begin{aligned}
&\log\rho+\left[\bar{q}^{\xi\prime}(w_t^{\xi})w_t^{\xi}+\frac{1}{\rho}\right](r-\rho)-\rho\bar{q}^{\xi}(w_t^{\xi})\\
&+\left[\bar{q}^{\xi\prime}(w_t^{\xi})w_t^{\xi}+\frac{1}{2}\bar{q}^{\xi\prime\prime}(w_t^{\xi})(w_t^{\xi})^2+\frac{1}{2\rho}\right]\frac{z_t^{\xi\top}\Sigma z_t^{\xi}}{[1-P^{\xi\prime}(w_t^{\xi})^{\top}z_t^{\xi}w_t^{\xi}]^2}\\
&+\lambda^{\xi}\left[\frac{1}{\rho}\log\left(\frac{w_t^{-\xi}}{w_t^{\xi}}\right)+\bar{q}^{-\xi}(w_t^{-\xi})-\bar{q}^{\xi}(w_t^{\xi})-\left(\bar{q}^{\xi\prime}(w_t^{\xi})w_t^{\xi}+\frac{1}{\rho}\right)\frac{\frac{w_t^{-\xi}-w_t^{\xi}}{w_t^{\xi}}}{\frac{w_t^{-\xi}}{w_t^{\xi}}}\right]=0.
\end{aligned}
\tag{A.75}
$$

The investors' wealth-scaled asset holdings z_t^{ξ} are determined by (A.72), while prices

$P^\xi(w_t^\xi)$ and hedging demand $\bar{q}^\xi(w_t^\xi)$ are solutions to the ODEs (A.74) and (A.75).

B. Summary of ODE System

Define

$$\pi^\xi(w_t^\xi) \equiv \frac{\bar{D}}{r} - P^\xi(w_t^\xi). \tag{A.76}$$

Then the investors' wealth-scaled asset holdings z_t^ξ satisfies

$$\begin{aligned} &\frac{(1+\alpha w_t^\xi)\hat{\sigma}_t^{\xi\top}\sigma z_t^\xi - \alpha\hat{\sigma}_t^{\xi\top}\hat{\sigma}_t^\xi S}{[1+w_t^\xi\pi^{\xi\prime}(w_t^\xi)^\top z_t^\xi]^2} - \frac{\alpha\hat{\sigma}_t^{\xi\top}\sigma u^\xi}{1+w_t^\xi\pi^{\xi\prime}(w_t^\xi)^\top z_t^\xi} \\ &= \lambda^\xi\left[\Big(\frac{w_t^\xi}{w_t^{-\xi}} - 1\Big)(P_t^{-\xi} - P_t^\xi) + \alpha(P_t^{-\xi} - P_t^\xi)(P_t^{-\xi} - P_t^\xi)^\top(S - z_t^\xi w_t^\xi)\right], \end{aligned} \tag{A.77}$$

where

$$\hat{\sigma}_t^\xi = -w_t^\xi\sigma z_t^\xi\pi^{\xi\prime}(w_t^\xi)^\top + \big[1 + w_t^\xi\pi^{\xi\prime}(w_t^\xi)^\top z_t^\xi\big]\sigma, \tag{A.78}$$

$$P_t^{-\xi} - P_t^\xi = \pi^\xi(w_t^\xi) - \pi^{-\xi}(w_t^{-\xi}). \tag{A.79}$$

The functions $\pi^\xi(w_t^\xi)$ and $\bar{q}^\xi(w_t^\xi)$ are solutions to the following ODEs

$$\begin{aligned} &r\pi^\xi(w_t^\xi) - \Big[r - \rho - \frac{\lambda^\xi(w_t^{-\xi} - w_t^\xi)}{w_t^{-\xi}}\Big]\pi^{\xi\prime}(w_t^\xi)w_t^\xi - \frac{z_t^{\xi\top}\Sigma z_t^\xi}{2[1+w_t^\xi\pi^{\xi\prime}(w_t^\xi)^\top z_t^\xi]}\pi^{\xi\prime\prime}(w_t^\xi)(w_t^\xi)^2 \\ &= \frac{\Sigma z_t^\xi}{1+w_t^\xi\pi^{\xi\prime}(w_t^\xi)^\top z_t^\xi} - \frac{\lambda^\xi[\pi^\xi(w_t^\xi) - \pi^{-\xi}(w_t^{-\xi})]}{\frac{w_t^{-\xi}}{w_t^\xi}}, \end{aligned} \tag{A.80}$$

and

$$\begin{aligned} &\log\rho + \Big[\bar{q}^{\xi\prime}(w_t^\xi)w_t^\xi + \frac{1}{\rho}\Big](r-\rho) - \rho\bar{q}^\xi(w_t^\xi) \\ &+ \Big[\bar{q}^{\xi\prime}(w_t^\xi)w_t^\xi + \frac{1}{2}\bar{q}^{\xi\prime\prime}(w_t^\xi)(w_t^\xi)^2 + \frac{1}{2\rho}\Big]\frac{z_t^{\xi\top}\Sigma z_t^\xi}{[1+w_t^\xi\pi^{\xi\prime}(w_t^\xi)^\top z_t^\xi]^2} \\ &+ \lambda^\xi\left[\frac{1}{\rho}\log\Big(\frac{w_t^{-\xi}}{w_t^\xi}\Big) + \bar{q}^{-\xi}(w_t^{-\xi}) - \bar{q}^\xi(w_t^\xi) - \Big(\bar{q}^{\xi\prime}(w_t^\xi)w_t^\xi + \frac{1}{\rho}\Big)\frac{w_t^{-\xi} - w_t^\xi}{w_t^{-\xi}}\right] = 0. \end{aligned} \tag{A.81}$$

Moreover, the following fixed point condition must be satisfied

$$w_t^{-\xi} - w_t^\xi = w_t^\xi z_t^{\xi\top}\big[\pi^\xi(w_t^\xi) - \pi^{-\xi}(w_t^{-\xi})\big]. \tag{A.82}$$

The endogenous percentage return variance-covariance matrix is given by $\Omega_t^\xi \equiv (\sigma_{Rt}^\xi)^\top\sigma_{Rt}^\xi$, where $\sigma^{\xi}_{Rt} \equiv (\sigma^{\xi}_{Pt} + \sigma)\,\mathrm{diag}(P^{\xi}_t)^{-1}$. Then from (A.57),

$$\begin{aligned}\Omega^{\xi}_t &= \mathrm{diag}(P^{\xi}_t)^{-1}(\sigma^{\xi}_{Pt}+\sigma)^{\top}(\sigma^{\xi}_{Pt}+\sigma)\,\mathrm{diag}(P^{\xi}_t)^{-1}\\ &= \mathrm{diag}(P^{\xi}_t)^{-1}\left[\sigma + \frac{\sigma z^{\xi}_t P^{\xi\prime}(w^{\xi}_t)^{\top} w^{\xi}_t}{1-P^{\xi\prime}(w^{\xi}_t)^{\top} z^{\xi}_t w^{\xi}_t}\right]^{\top}\left[\sigma + \frac{\sigma z^{\xi}_t P^{\xi\prime}(w^{\xi}_t)^{\top} w^{\xi}_t}{1-P^{\xi\prime}(w^{\xi}_t)^{\top} z^{\xi}_t w^{\xi}_t}\right]\mathrm{diag}(P^{\xi}_t)^{-1}.\end{aligned} \tag{A.83}$$

Thus, the variance-covariance matrix exhibits a factor structure and depends on the endogenous state variable, the investors' total wealth w^{ξ}_t.

C. Limit Cases with $\lambda \to 0$ and $\lambda \to \infty$

The log investor's optimal portfolio policy $\hat{y}^{\xi}_t$ is given by (A.38). Our setup focuses on $\lambda^n = 0$ and $\lambda^s = \lambda$. Since we define $\mathbb{E}_t(R^{\xi}) \equiv \mathbb{E}_t(dR^{\xi}_t)/dt = \mu^{\xi}_{Rt} + \lambda^{\xi} J^{\xi}_{Rt}$, we can then rewrite $\hat{y}^{\xi}_t$ as

$$\hat{y}^{\xi}_t = (\Omega^{\xi}_t)^{-1}\Big(\mathbb{E}_t(R^{\xi}) - r\mathbf{1}\Big) + \lambda^{\xi}(\Omega^{\xi}_t)^{-1}\left(\frac{1}{1+\hat{y}^{\xi\top}_t J^{\xi}_{Rt}} - 1\right) J^{\xi}_{Rt}. \tag{A.84}$$

It is immediate that in state n,

$$\hat{y}^{n}_t = (\Omega^{n}_t)^{-1}\left(\mathbb{E}_t(R^{n}) - r\mathbf{1}\right). \tag{A.85}$$

Similarly, when $\lambda \to 0$, the investor's demand in state s is

$$\lim_{\lambda\to 0} \hat{y}^{s}_t = (\Omega^{s}_t)^{-1}\left(\mathbb{E}_t(R^{s}) - r\mathbf{1}\right). \tag{A.86}$$

On the other hand, when $\lambda \to \infty$, $J^{s}_{Rt} \to 0$. Thus, the second term on the right-hand side of (A.84) converges to zero, and

$$\lim_{\lambda\to \infty} \hat{y}^{s}_t = (\Omega^{s}_t)^{-1}\left(\mathbb{E}_t(R^{s}) - r\mathbf{1}\right). \tag{A.87}$$

Consequently, both in the limits $\lambda \to 0$ and $\lambda \to \infty$, we have

$$\hat{y}^{\xi}_t = (\Omega^{\xi}_t)^{-1}\big(\mathbb{E}_t(R^{\xi}) - r\mathbf{1}\big). \tag{A.88}$$

A.1.2 CRRA Investors

We now consider the case with CRRA investors. An CRRA investor maximizes

$$\mathbb{E}_t\Big[\int_t^{\infty} e^{-\rho(\tau-t)}\frac{\hat{c}_{\tau}^{1-\gamma}}{1-\gamma}d\tau\Big], \tag{A.89}$$

subject to the wealth dynamic (A.8). The investor's HJB equation can be written as

$$\begin{aligned}\rho V^{\xi} = \max_{\hat{c}_t^{\xi}, \hat{z}_t^{\xi}} \Big\{ &\frac{\hat{c}_t^{1-\gamma}}{1-\gamma} + V_{\hat{w}}^{\xi}\mu_t^{\hat{w},\xi} + \frac{1}{2}V_{\hat{w}\hat{w}}^{\xi}(\sigma_t^{\hat{w},\xi})^{\top}\sigma_t^{\hat{w},\xi} + V_{w}^{\xi}\mu_t^{w,\xi} + \frac{1}{2}V_{ww}^{\xi}(\sigma_t^{w,\xi})^{\top}\sigma_t^{w,\xi} \\ &+ V_{\hat{w}w}^{\xi}(\sigma_t^{\hat{w},\xi})^{\top}\sigma_t^{w,\xi} + \lambda^{\xi}(V^{-\xi,+} - V^{\xi})\Big\},\end{aligned} \tag{A.90}$$

where $\mu_t^{\hat{w},\xi}$, $\sigma_t^{\hat{w},\xi}$, $\mu_t^{w,\xi}$ and $\sigma_t^{w,\xi}$ are given by (A.27)-(A.30).

We conjecture (and later verify) that the investor's value function takes the form

$$V^{\xi}(\hat{w}_t^{\xi}, w_t^{\xi}) = q^{\xi}(w_t^{\xi})\frac{(\hat{w}_t^{\xi})^{1-\gamma}}{1-\gamma}. \tag{A.91}$$

Substituting the conjectured value function into the investor's HJB equation (A.90) yields

$$\begin{aligned}\rho q^{\xi}(w_t^{\xi})\frac{(\hat{w}_t^{\xi})^{1-\gamma}}{1-\gamma} = \max_{\hat{c}_t^{\xi}, \hat{z}_t^{\xi}} \Big\{ &\frac{(\hat{c}_t^{\xi})^{1-\gamma}}{1-\gamma} + q^{\xi}(w_t^{\xi})(\hat{w}_t^{\xi})^{-\gamma}\mu_t^{\hat{w},\xi} - \frac{\gamma}{2}q^{\xi}(w_t^{\xi})(\hat{w}_t^{\xi})^{-\gamma-1}(\sigma_t^{\hat{w},\xi})^{\top}\sigma_t^{\hat{w},\xi} \\ &+ q^{\xi\prime}(w_t^{\xi})\frac{(\hat{w}_t^{\xi})^{1-\gamma}}{1-\gamma}\mu_t^{w,\xi} + \frac{1}{2}q^{\xi\prime\prime}(w_t^{\xi})\frac{(\hat{w}_t^{\xi})^{1-\gamma}}{1-\gamma}(\sigma_t^{w,\xi})^{\top}\sigma_t^{w,\xi} + q^{\xi\prime}(w_t^{\xi})(\hat{w}_t^{\xi})^{-\gamma}(\sigma_t^{\hat{w},t})^{\top}\sigma_t^{w,\xi} \\ &+ \lambda^{\xi}\Big[q^{-\xi}\big(w_t^{\xi} + w_t^{\xi}z_t^{\xi\top}(P_t^{-\xi} - P_t^{\xi})\big)\frac{[\hat{w}_t^{\xi} + \hat{w}_t^{\xi}\hat{z}_t^{\xi\top}(P_t^{-\xi} - P_t^{\xi})]^{1-\gamma}}{1-\gamma} - q^{\xi}(w_t^{\xi})\frac{(\hat{w}_t^{\xi})^{1-\gamma}}{1-\gamma}\Big]\Big\}.\end{aligned} \tag{A.92}$$

Taking the first-order condition with respect to consumption yields

$$(\hat{c}_t^{\xi})^{-\gamma} - q^{\xi}(w_t^{\xi})(\hat{w}_t^{\xi})^{-\gamma} = 0. \tag{A.93}$$

Thus,

$$\hat{c}_t^{\xi} = q^{\xi}(w_t^{\xi})^{-\frac{1}{\gamma}}\hat{w}_t^{\xi}. \tag{A.94}$$

Let $\hat{z}_t^{\xi} \equiv \hat{Y}_t^{\xi}/\hat{w}_t^{\xi}$ denote the investor's optimal wealth-scaled asset holdings at time t if the economy is in state ξ. Let $z_t^{\xi} \equiv Y_t^{\xi}/w_t^{\xi}$ denote the investors' aggregate wealth-scaled asset holdings. Taking the first-order condition, $\hat{z}_t^{\xi}$ satisfies

$$\begin{aligned}0 =\,&q^{\xi}(w_t^{\xi})(\hat{w}_t^{\xi})^{1-\gamma}(\bar{D} + \mu_{Pt}^{\xi} - rP_t^{\xi}) - \gamma q^{\xi}(w_t^{\xi})(\hat{w}_t^{\xi})^{1-\gamma}(\sigma_{Pt}^{\xi} + \sigma)^{\top}(\sigma_{Pt}^{\xi} + \sigma)\hat{z}_t^{\xi} \\ &+ q^{\xi\prime}(w_t^{\xi})(\hat{w}_t^{\xi})^{1-\gamma}(\sigma_{Pt}^{\xi} + \sigma)^{\top}(\sigma_{Pt}^{\xi} + \sigma)z_t^{\xi}w_t^{\xi} \\ &+ \lambda^{\xi}q^{-\xi}\big(w_t^{\xi} + w_t^{\xi}z_t^{\xi\top}(P_t^{-\xi} - P_t^{\xi})\big)[1 + \hat{z}_t^{\xi\top}(P_t^{-\xi} - P_t^{\xi})]^{-\gamma}(\hat{w}_t^{\xi})^{1-\gamma}(P_t^{-\xi} - P_t^{\xi}).\end{aligned} \tag{A.95}$$

Rearranging, we get

$$
\begin{aligned}
\bar{D}+\mu_{Pt}^{\xi}-rP_t^{\xi}=&\gamma(\sigma_{Pt}^{\xi}+\sigma)^{\top}(\sigma_{Pt}^{\xi}+\sigma)\hat{z}_t^{\xi}-\frac{q^{\xi\prime}(w_t^{\xi})}{q^{\xi}(w_t^{\xi})}w_t^{\xi}(\sigma_{Pt}^{\xi}+\sigma)^{\top}(\sigma_{Pt}^{\xi}+\sigma)z_t^{\xi}\\
&-\frac{\lambda^{\xi}q^{-\xi}\big(w_t^{\xi}[1+z_t^{\xi\top}(P_t^{-\xi}-P_t^{\xi})]\big)(P_t^{-\xi}-P_t^{\xi})}{q^{\xi}(w_t^{\xi})[1+\hat{z}_t^{\xi\top}(P_t^{-\xi}-P_t^{\xi})]^{\gamma}},
\end{aligned}
\tag{A.96}
$$

or equivalently

$$
\begin{aligned}
\hat{z}_t^{\xi}=&\frac{1}{\gamma}(\Pi_t^{\xi})^{-1}(\bar{D}+\mu_{Pt}^{\xi}-rP_t^{\xi})+\frac{1}{\gamma}\frac{q^{\xi\prime}(w_t^{\xi})}{q^{\xi}(w_t^{\xi})}w_t^{\xi}z_t^{\xi}\\
&+\frac{\lambda^{\xi}}{\gamma}\frac{q^{-\xi}(w_t^{\xi}+w_t^{\xi}z_t^{\xi\top}(P_t^{-\xi}-P_t^{\xi}))}{q^{\xi}(w_t^{\xi})[1+\hat{z}_t^{\xi\top}(P_t^{-\xi}-P_t^{\xi})]^{\gamma}}(\Pi_t^{\xi})^{-1}(P_t^{-\xi}-P_t^{\xi}),
\end{aligned}
\tag{A.97}
$$

where $\Pi_t^{\xi}\equiv(\sigma_{Pt}^{\xi}+\sigma)^{\top}(\sigma_{Pt}^{\xi}+\sigma)$. Since $\hat{y}_t^{\xi}=\mathrm{diag}(P_t^{\xi})\hat{z}_t^{\xi}$, then

$$
\begin{aligned}
\hat{y}_t^{\xi}=&\frac{1}{\gamma}(\Omega_t^{\xi})^{-1}(\mu_{Rt}^{\xi}-r\mathbf{1}+\lambda^{\xi}J_{Rt}^{\xi})+\frac{1}{\gamma}\frac{q^{\xi\prime}(w_t^{\xi})}{q^{\xi}(w_t^{\xi})}w_t^{\xi}y_t^{\xi}\\
&+\frac{\lambda^{\xi}}{\gamma}(\Omega_t^{\xi})^{-1}\left(\frac{q^{-\xi}(w_t^{\xi}(1+y_t^{\xi\top}J_{Rt}^{\xi}))}{q^{\xi}(w_t^{\xi})[1+\hat{y}_t^{\xi\top}J_{Rt}^{\xi}]^{\gamma}}-1\right)J_{Rt}^{\xi}.
\end{aligned}
\tag{A.98}
$$

Note that $\hat{z}_t^{\xi}=z_t^{\xi}$, $\hat{c}_t^{\xi}=c_t^{\xi}$ and $\hat{w}_t^{\xi}=w_t^{\xi}$ in equilibrium, then (A.96) implies

$$
\begin{aligned}
\bar{D}+\mu_{Pt}^{\xi}-rP_t^{\xi}=&\Big[\gamma-\frac{q^{\xi\prime}(w_t^{\xi})}{q^{\xi}(w_t^{\xi})}w_t^{\xi}\Big](\sigma_{Pt}^{\xi}+\sigma)^{\top}(\sigma_{Pt}^{\xi}+\sigma)z_t^{\xi}\\
&-\frac{\lambda^{\xi}q^{-\xi}\big(w_t^{\xi}[1+z_t^{\xi\top}(P_t^{-\xi}-P_t^{\xi})]\big)(P_t^{-\xi}-P_t^{\xi})}{q^{\xi}(w_t^{\xi})[1+z_t^{\xi\top}(P_t^{-\xi}-P_t^{\xi})]^{\gamma}}.
\end{aligned}
\tag{A.99}
$$

Let

$$
\Gamma^{\xi}(w_t^{\xi})\equiv\gamma-\frac{q^{\xi\prime}(w_t^{\xi})}{q^{\xi}(w_t^{\xi})}w_t^{\xi},
\tag{A.100}
$$

Then, the above equation can be rewritten as

$$
\begin{aligned}
\bar{D}+\mu_{Pt}^{\xi}-rP_t^{\xi}=&\Gamma^{\xi}(w_t^{\xi})(\sigma_{Pt}^{\xi}+\sigma)^{\top}(\sigma_{Pt}^{\xi}+\sigma)z_t^{\xi}\\
&-\frac{\lambda^{\xi}q^{-\xi}\big(w_t^{\xi}[1+z_t^{\xi\top}(P_t^{-\xi}-P_t^{\xi})]\big)(P_t^{-\xi}-P_t^{\xi})}{q^{\xi}(w_t^{\xi})[1+z_t^{\xi\top}(P_t^{-\xi}-P_t^{\xi})]^{\gamma}}.
\end{aligned}
\tag{A.101}
$$

Moreover, from (A.94), the investors' total consumption aggregates to $c_t^{\xi}=q^{\xi}(w_t^{\xi})^{-\frac{1}{\gamma}}w_t^{\xi}$.

Substituting it into the HJB equation (A.92) yields

$$
\begin{aligned}
&\rho q^\xi(w_t^\xi)\frac{(\hat{w}_t^\xi)^{1-\gamma}}{1-\gamma} = \frac{q^\xi(w_t^\xi)^{-\frac{1-\gamma}{\gamma}}(\hat{w}_t^\xi)^{1-\gamma}}{1-\gamma} \\
&+ q^\xi(w_t^\xi)(\hat{w}_t^\xi)^{1-\gamma}\Big[r - q^\xi(w_t^\xi)^{-\frac{1}{\gamma}} + \hat{z}_t^{\xi\top}(\bar{D} + \mu_{Pt}^\xi - rP_t^\xi)\Big] \\
&- \frac{\gamma}{2}q^\xi(w_t^\xi)(\hat{w}_t^\xi)^{1-\gamma}\hat{z}_t^{\xi\top}(\sigma_{Pt}^\xi + \sigma)^\top(\sigma_{Pt}^\xi + \sigma)\hat{z}_t^\xi \\
&+ q^{\xi\prime}(w_t^\xi)\frac{(\hat{w}_t^\xi)^{1-\gamma}}{1-\gamma}w_t^\xi\Big[r - q^\xi(w_t^\xi)^{-\frac{1}{\gamma}} + z_t^{\xi\top}(\bar{D} + \mu_{Pt}^\xi - rP_t^\xi)\Big] \\
&+ \frac{1}{2}q^{\xi\prime\prime}(w_t^\xi)\frac{(\hat{w}_t^\xi)^{1-\gamma}}{1-\gamma}(w_t^\xi)^2 z_t^{\xi\top}(\sigma_{Pt}^\xi + \sigma)^\top(\sigma_{Pt}^\xi + \sigma)z_t^\xi \\
&+ q^{\xi\prime}(w_t^\xi)(\hat{w}_t^\xi)^{1-\gamma}w_t^\xi\hat{z}_t^{\xi\top}(\sigma_{Pt}^\xi + \sigma)^\top(\sigma_{Pt}^\xi + \sigma)z_t^\xi \\
&+ \lambda^\xi\Big[q^{-\xi}\big(w_t^\xi[1 + z_t^{\xi\top}(P_t^{-\xi} - P_t^\xi)]\big)\frac{[1 + \hat{z}_t^{\xi\top}(P_t^{-\xi} - P_t^\xi)]^{1-\gamma}(\hat{w}_t^\xi)^{1-\gamma}}{1-\gamma} - q^\xi(w_t^\xi)\frac{(\hat{w}_t^\xi)^{1-\gamma}}{1-\gamma}\Big].
\end{aligned}
\tag{A.102}
$$

Note that $\hat{w}_t^{1-\gamma}$ cancel out on both sides. Thus, we have verified the conjectured value function (A.91). Using $\hat{z}_t^\xi = z_t^\xi$, we get

$$
\begin{aligned}
&\rho q^\xi(w_t^\xi) \\
&= q^\xi(w_t^\xi)^{-\frac{1-\gamma}{\gamma}} + \Big[q^{\xi\prime}(w_t^\xi)w_t^\xi + (1-\gamma)q^\xi(w_t^\xi)\Big]\Big[r - q^\xi(w_t^\xi)^{-\frac{1}{\gamma}} + z_t^{\xi\top}(\bar{D} + \mu_{Pt}^\xi - rP_t^\xi)\Big] \\
&+ \frac{1}{2}\Big[q^{\xi\prime\prime}(w_t^\xi)(w_t^\xi)^2 - \gamma(1-\gamma)q^\xi(w_t^\xi) + 2(1-\gamma)q^{\xi\prime}(w_t^\xi)w_t^\xi\Big]z_t^{\xi\top}(\sigma_{Pt}^\xi + \sigma)^\top(\sigma_{Pt}^\xi + \sigma)z_t^\xi \\
&+ \lambda^\xi\Big[q^{-\xi}\big(w_t^\xi[1 + z_t^{\xi\top}(P_t^{-\xi} - P_t^\xi)]\big)[1 + z_t^{\xi\top}(P_t^{-\xi} - P_t^\xi)]^{1-\gamma} - q^\xi(w_t^\xi)\Big].
\end{aligned}
\tag{A.103}
$$

Using (A.101) and dividing both sides by $q^\xi(w_t^\xi)$ and rearranging, we find

$$
\begin{aligned}
&q^\xi(w_t^\xi)^{-\frac{1}{\gamma}} + \Big[\frac{q^{\xi\prime}(w_t^\xi)}{q^\xi(w_t^\xi)}w_t^\xi + 1 - \gamma\Big]\Big[r - q^\xi(w_t^\xi)^{-\frac{1}{\gamma}} + \Gamma^\xi(w_t^\xi)z_t^{\xi\top}(\sigma_{Pt}^\xi + \sigma)^\top(\sigma_{Pt}^\xi + \sigma)z_t^\xi\Big] \\
&- \Big[\frac{q^{\xi\prime}(w_t^\xi)}{q^\xi(w_t^\xi)}w_t^\xi + 1 - \gamma\Big]\frac{\lambda^\xi q^{-\xi}(w_t^\xi[1 + z_t^{\xi\top}(P_t^{-\xi} - P_t^\xi)])z_t^{\xi\top}(P_t^{-\xi} - P_t^\xi)}{q^\xi(w_t^\xi)[1 + z_t^{\xi\top}(P_t^{-\xi} - P_t^\xi)]^\gamma} \\
&+ \frac{1}{2}\Big[\frac{q^{\xi\prime\prime}(w_t^\xi)}{q^\xi(w_t^\xi)}(w_t^\xi)^2 - \gamma(1-\gamma) + 2(1-\gamma)\frac{q^{\xi\prime}(w_t^\xi)}{q^\xi(w_t^\xi)}w_t^\xi\Big]z_t^{\xi\top}(\sigma_{Pt}^\xi + \sigma)^\top(\sigma_{Pt}^\xi + \sigma)z_t^\xi \\
&+ \lambda^\xi\Big[\frac{q^{-\xi}(w_t^\xi[1 + z_t^{\xi\top}(P_t^{-\xi} - P_t^\xi)])}{q^\xi(w_t^\xi)}[1 + z_t^{\xi\top}(P_t^{-\xi} - P_t^\xi)]^{1-\gamma} - 1\Big] = \rho.
\end{aligned}
\tag{A.104}
$$

From (A.100),

$$
\begin{aligned}
\Gamma^{\xi\prime}(w_t^\xi) &= -\frac{q^{\xi\prime\prime}(w_t^\xi)q^\xi(w_t^\xi) - q^{\xi\prime}(w_t^\xi)^2}{q^\xi(w_t^\xi)^2}w_t^\xi - \frac{q^{\xi\prime}(w_t^\xi)}{q^\xi(w_t^\xi)} \\
&= -\frac{q^{\xi\prime\prime}(w_t^\xi)}{q^\xi(w_t^\xi)}w_t^\xi + \frac{q^{\xi\prime}(w_t^\xi)^2}{q^\xi(w_t^\xi)^2}w_t^\xi - \frac{q^{\xi\prime}(w_t^\xi)}{q^\xi(w_t^\xi)}.
\end{aligned}
\tag{A.105}
$$

Thus,

$$
\begin{aligned}
&\frac{q^{\xi\prime\prime}(w_t^\xi)}{q^\xi(w_t^\xi)}(w_t^\xi)^2 - \gamma(1-\gamma) + 2(1-\gamma)\frac{q^{\xi\prime}(w_t^\xi)}{q^\xi(w_t^\xi)}w_t^\xi \\
=&\Big[\frac{q^{\xi\prime\prime}(w_t^\xi)}{q^\xi(w_t^\xi)}w_t^\xi - \frac{q^{\xi\prime}(w_t^\xi)^2}{q^\xi(w_t^\xi)^2}w_t^\xi + \frac{q^{\xi\prime}(w_t^\xi)}{q^\xi(w_t^\xi)}\Big]w_t^\xi + \Big[\frac{q^{\xi\prime}(w_t^\xi)}{q^\xi(w_t^\xi)}w_t^\xi - \gamma\Big]^2 + \frac{q^{\xi\prime}(w_t^\xi)}{q^\xi(w_t^\xi)}w_t^\xi - \gamma \\
=&-\Gamma^{\xi\prime}(w_t^\xi)w_t^\xi + \Gamma^\xi(w_t^\xi)^2 - \Gamma^\xi(w_t^\xi).
\end{aligned}
\tag{A.106}
$$

Substituting into (A.104) and subtracting r from both sides, we get

$$
\begin{aligned}
&\rho - r = q^\xi(w_t^\xi)^{-\frac{1}{\gamma}} - r + \Big[\frac{q^{\xi\prime}(w_t^\xi)}{q^\xi(w_t^\xi)}w_t^\xi + 1 - \gamma\Big]\Big[r - q^\xi(w_t^\xi)^{-\frac{1}{\gamma}}\Big] \\
&- \lambda^\xi\Big[\frac{q^{\xi\prime}(w_t^\xi)}{q^\xi(w_t^\xi)}w_t^\xi + 1 - \gamma\Big]\frac{q^{-\xi}(w_t^\xi[1 + z_t^{\xi\top}(P_t^{-\xi} - P_t^\xi)])z_t^{\xi\top}(P_t^{-\xi} - P_t^\xi)}{q^\xi(w_t^\xi)[1 + z_t^{\xi\top}(P_t^{-\xi} - P_t^\xi)]^\gamma} \\
&+ \frac{1}{2}\Big[-\Gamma^{\xi\prime}(w_t^\xi)w_t^\xi + \Gamma^\xi(w_t^\xi)^2 - \Gamma^\xi(w_t^\xi) + 2[1 - \Gamma^\xi(w_t^\xi)]\Gamma^\xi(w_t^\xi)\Big]z_t^{\xi\top}(\sigma_{Pt}^\xi + \sigma)^\top(\sigma_{Pt} + \sigma)z_t^\xi \\
&+ \lambda^\xi\Big[\frac{q^{-\xi}(w_t^\xi[1 + z_t^{\xi\top}(P_t^{-\xi} - P_t^\xi)])}{q^\xi(w_t^\xi)}[1 + z_t^{\xi\top}(P_t^{-\xi} - P_t^\xi)]^{1-\gamma} - 1\Big] \\
&= \Big[q^\xi(w_t^\xi)^{-\frac{1}{\gamma}} - r\Big]\Gamma^\xi(w_t^\xi) - \frac{1}{2}\Big[\Gamma^{\xi\prime}(w_t^\xi)w_t^\xi - \Gamma^\xi(w_t^\xi) + \Gamma^\xi(w_t^\xi)^2\Big]z_t^{\xi\top}(\sigma_{Pt}^\xi + \sigma)^\top(\sigma_{Pt}^\xi + \sigma)z_t^\xi \\
&+ \lambda^\xi\Big\{\frac{[\Gamma^\xi(w_t^\xi)z_t^{\xi\top}(P_t^{-\xi} - P_t^\xi) + 1]q^{-\xi}(w_t^\xi[1 + z_t^{\xi\top}(P_t^{-\xi} - P_t^\xi)])}{q^\xi(w_t^\xi)[1 + z_t^{\xi\top}(P_t^{-\xi} - P_t^\xi)]^\gamma} - 1\Big\}.
\end{aligned}
\tag{A.107}
$$

Substituting the investors' aggregate consumption c_t^ξ into the total wealth dynamic for investors, we get

$$
\begin{aligned}
dw_t^\xi =& w_t^\xi\big[r - q^\xi(w_t^\xi)^{-\frac{1}{\gamma}} + z_t^{\xi\top}(\bar{D} + \mu_{Pt}^\xi - rP_t^\xi)\big]dt + w_t^\xi z_t^{\xi\top}(\sigma_{Pt}^\xi + \sigma)^\top dB_t \\
&+ w_t^\xi z_t^{\xi\top}J_t^\xi dN_t^\xi.
\end{aligned}
\tag{A.108}
$$

The investors' total wealth w_t^ξ and the state of the economy ξ are the two state variables in the model. Thus, the risky assets' prices in state ξ must be functions of the investors' total wealth in the state, that is, $P_t^\xi = P^\xi(w_t^\xi)$. Let

$$\tilde{\mu}_t^{w,\xi} \equiv w_t^\xi\big[r - q^\xi(w_t^\xi)^{-\frac{1}{\gamma}} + z_t^{\xi\top}(\bar{D} + \mu_{Pt}^\xi - rP_t^\xi)\big], \tag{A.109}$$

$$\tilde{\sigma}_t^{w,\xi} \equiv (\sigma_{Pt}^\xi + \sigma) z_t^\xi w_t^\xi, \tag{A.110}$$

which are the drift and the diffusion of the investors' total wealth w_t^ξ after substituting in the investors' optimal consumption and wealth-scaled asset holdings. By Itô's lemma, we have

$$\begin{aligned} dP_t^\xi =& \Big[\tilde{\mu}_t^{w,\xi} P^{\xi\prime}(w_t^\xi) + \frac{1}{2}(\tilde{\sigma}_t^{w,\xi})^\top \tilde{\sigma}_t^{w,\xi} P^{\xi\prime\prime}(w_t^\xi)\Big] dt + P^{\xi\prime}(w_t^\xi)(\tilde{\sigma}_t^{w,\xi})^\top dB_t \\ &+ (P_t^{-\xi} - P_t^\xi) dN_t^\xi. \end{aligned} \tag{A.111}$$

From (A.109) and (A.110), the drift of the risky assets' prices become

$$\begin{aligned} &\tilde{\mu}_t^{w,\xi} P^{\xi\prime}(w_t^\xi) + \frac{1}{2}(\tilde{\sigma}_t^{w,\xi})^\top \tilde{\sigma}_t^{w,\xi} P^{\xi\prime\prime}(w_t^\xi) \\ &= w_t^\xi\big[r - q^\xi(w_t^\xi)^{-\frac{1}{\gamma}} + z_t^{\xi\top}(\bar{D} + \mu_{Pt}^\xi - rP_t^\xi)\big] P^{\xi\prime}(w_t^\xi) \\ &\quad + \frac{1}{2} z_t^{\xi\top}(\sigma_{Pt}^\xi + \sigma)^\top(\sigma_{Pt}^\xi + \sigma) z_t^\xi P^{\xi\prime\prime}(w_t^\xi)(w_t^\xi)^2 \\ &= w_t^\xi\Big[r - q^\xi(w_t^\xi)^{-\frac{1}{\gamma}} + \Gamma^\xi(w_t^\xi) z_t^{\xi\top}(\sigma_{Pt}^\xi + \sigma)^\top(\sigma_{Pt}^\xi + \sigma) z_t^\xi \\ &\qquad - \frac{\lambda^\xi q^{-\xi}\big(w_t^\xi[1 + z_t^{\xi\top}(P_t^{-\xi} - P_t^\xi)]\big) z_t^{\xi\top}(P_t^{-\xi} - P_t^\xi)}{q^\xi(w_t^\xi)[1 + z_t^{\xi\top}(P_t^{-\xi} - P_t^\xi)]^\gamma}\Big] P^{\xi\prime}(w_t^\xi) \\ &\quad + \frac{1}{2} z_t^{\xi\top}(\sigma_{Pt}^\xi + \sigma)^\top(\sigma_{Pt}^\xi + \sigma) z_t^\xi P^{\xi\prime\prime}(w_t^\xi)(w_t^\xi)^2 \\ &= \Big[r - q^\xi(w_t^\xi)^{-\frac{1}{\gamma}} - \frac{\lambda^\xi q^{-\xi}\big(w_t^\xi[1 + z_t^{\xi\top}(P_t^{-\xi} - P_t^\xi)]\big) z_t^{\xi\top}(P_t^{-\xi} - P_t^\xi)}{q^\xi(w_t^\xi)[1 + z_t^{\xi\top}(P_t^{-\xi} - P_t^\xi)]^\gamma}\Big] P^{\xi\prime}(w_t^\xi) w_t^\xi \\ &\quad + z_t^{\xi\top}(\sigma_{Pt}^\xi + \sigma)^\top(\sigma_{Pt}^\xi + \sigma) z_t^\xi \Big[\Gamma^\xi(w_t^\xi) P^{\xi\prime}(w_t^\xi) w_t^\xi + \frac{1}{2} P^{\xi\prime\prime}(w_t^\xi)(w_t^\xi)^2\Big], \end{aligned} \tag{A.112}$$

where the second equality follows from (A.101). The diffusion of the risky assets' prices are

$$\tilde{\sigma}_t^{w,\xi} P^{\xi\prime}(w_t^\xi)^\top = (\sigma_{Pt}^\xi + \sigma) z_t^\xi w_t^\xi P^{\xi\prime}(w_t^\xi)^\top. \tag{A.113}$$

The risky assets' prices are conjectured to follow the Itô process (7). Thus, by matching coefficients, we get

$$J_t^\xi = P_t^{-\xi} - P_t^\xi, \tag{A.114}$$

while μ_{Pt}^ξ and σ_{Pt}^ξ are given by

$$\begin{aligned} \mu_{Pt}^\xi =& \Big[r - q^\xi(w_t^\xi)^{-\frac{1}{\gamma}} - \frac{\lambda^\xi q^{-\xi}\big(w_t^\xi[1 + z_t^{\xi\top}(P_t^{-\xi} - P_t^\xi)]\big) z_t^{\xi\top}(P_t^{-\xi} - P_t^\xi)}{q^\xi(w_t^\xi)[1 + z_t^{\xi\top}(P_t^{-\xi} - P_t^\xi)]^\gamma}\Big] P^{\xi\prime}(w_t^\xi) w_t^\xi \\ &+ z_t^{\xi\top}(\sigma_{Pt}^\xi + \sigma)^\top(\sigma_{Pt}^\xi + \sigma) z_t^\xi \Big[\Gamma^\xi(w_t^\xi) P^{\xi\prime}(w_t^\xi) w_t^\xi + \frac{1}{2} P^{\xi\prime\prime}(w_t^\xi)(w_t^\xi)^2\Big], \end{aligned} \tag{A.115}$$

and

$$\sigma^{\xi}_{Pt} = (\sigma^{\xi}_{Pt} + \sigma) z^{\xi}_t w^{\xi}_t P^{\xi\prime}(w^{\xi}_t)^{\top}. \tag{A.116}$$

From (A.116),

$$(\sigma^{\xi}_{Pt} + \sigma) z^{\xi}_t = (\sigma^{\xi}_{Pt} + \sigma) z^{\xi}_t P^{\xi\prime}(w^{\xi}_t)^{\top} z^{\xi}_t w^{\xi}_t + \sigma z^{\xi}_t. \tag{A.117}$$

Note that $P^{\xi\prime}(w^{\xi}_t)^{\top} z^{\xi}_t w^{\xi}_t$ is a scalar. Thus,

$$\big[1 - P^{\xi\prime}(w^{\xi}_t)^{\top} z^{\xi}_t w^{\xi}_t\big](\sigma^{\xi}_{Pt} + \sigma) z^{\xi}_t = \sigma z^{\xi}_t. \tag{A.118}$$

Then

$$(\sigma^{\xi}_{Pt} + \sigma) z^{\xi}_t = \frac{\sigma z^{\xi}_t}{1 - P^{\xi\prime}(w^{\xi}_t)^{\top} z^{\xi}_t w^{\xi}_t}. \tag{A.119}$$

Substituting (A.119) into (A.116) yields

$$\sigma^{\xi}_{Pt} = \frac{\sigma z^{\xi}_t P^{\xi\prime}(w^{\xi}_t)^{\top} w^{\xi}_t}{1 - P^{\xi\prime}(w^{\xi}_t)^{\top} z^{\xi}_t w^{\xi}_t}. \tag{A.120}$$

Adding σ to both sides yields

$$\sigma^{\xi}_{Pt} + \sigma = \frac{\sigma z^{\xi}_t P^{\xi\prime}(w^{\xi}_t)^{\top} w^{\xi}_t + [1 - P^{\xi\prime}(w^{\xi}_t)^{\top} z^{\xi}_t w^{\xi}_t]\sigma}{1 - P^{\xi\prime}(w^{\xi}_t)^{\top} z^{\xi}_t w^{\xi}_t}. \tag{A.121}$$

Let

$$\hat{\sigma}^{\xi}_t \equiv \sigma z^{\xi}_t P^{\xi\prime}(w^{\xi}_t)^{\top} w^{\xi}_t + [1 - P^{\xi\prime}(w^{\xi}_t)^{\top} z^{\xi}_t w^{\xi}_t]\sigma. \tag{A.122}$$

Then

$$\sigma^{\xi}_{Pt} + \sigma = \frac{\hat{\sigma}^{\xi}_t}{1 - P^{\xi\prime}(w^{\xi}_t)^{\top} z^{\xi}_t w^{\xi}_t}. \tag{A.123}$$

Substituting (A.119) into (A.115) yields

$$\begin{aligned} \mu^{\xi}_{Pt} = {} & \Big[r - q^{\xi}(w^{\xi}_t)^{-\frac{1}{\gamma}} - \frac{\lambda^{\xi} q^{-\xi}\big(w^{\xi}_t[1 + z^{\xi\top}_t(P^{-\xi}_t - P^{\xi}_t)]\big) z^{\xi\top}_t(P^{-\xi}_t - P^{\xi}_t)}{q^{\xi}(w^{\xi}_t)[1 + z^{\xi\top}_t(P^{-\xi}_t - P^{\xi}_t)]^{\gamma}}\Big] P^{\xi\prime}(w^{\xi}_t) w^{\xi}_t \\ & + \frac{z^{\xi\top}_t \Sigma z^{\xi}_t}{[1 - P^{\xi\prime}(w^{\xi}_t)^{\top} z^{\xi}_t w^{\xi}_t]^2}\Big[\Gamma^{\xi}(w^{\xi}_t) P^{\xi\prime}(w^{\xi}_t) w^{\xi}_t + \frac{1}{2} P^{\xi\prime\prime}(w^{\xi}_t)(w^{\xi}_t)^2\Big]. \end{aligned} \tag{A.124}$$

It is thus immediate that

$$
\begin{aligned}
&\bar{D} + \mu^\xi_{Pt} - rP^\xi_t \\
&= \Big[r - q^\xi(w^\xi_t)^{-\frac{1}{\gamma}} - \frac{\lambda^\xi q^{-\xi}\big(w^\xi_t[1 + z^{\xi\top}_t(P^{-\xi}_t - P^\xi_t)]\big)z^{\xi\top}_t(P^{-\xi}_t - P^\xi_t)}{q^\xi(w^\xi_t)[1 + z^{\xi\top}_t(P^{-\xi}_t - P^\xi_t)]^\gamma}\Big]P^{\xi\prime}(w^\xi_t)w^\xi_t \\
&\quad + \frac{z^{\xi\top}_t\Sigma z^\xi_t}{[1 - P^{\xi\prime}(w^\xi_t)^\top z^\xi_t w^\xi_t]^2}\Big[\Gamma^\xi(w^\xi_t)P^{\xi\prime}(w^\xi_t)w^\xi_t + \frac{1}{2}P^{\xi\prime\prime}(w^\xi_t)(w^\xi_t)^2\Big] + \bar{D} - rP^\xi(w^\xi_t).
\end{aligned} \tag{A.125}
$$

From (A.101) and using (A.119) and (A.123), we have

$$
\bar{D} + \mu^\xi_{Pt} - rP^\xi_t = \frac{\Gamma^\xi(w^\xi_t)\hat{\sigma}^{\xi\top}_t\sigma z^\xi_t}{[1 - P^{\xi\prime}(w^\xi_t)^\top z^\xi_t w^\xi_t]^2} - \frac{\lambda^\xi q^{-\xi}\big(w^\xi_t[1 + z^{\xi\top}_t(P^{-\xi}_t - P^\xi_t)]\big)(P^{-\xi}_t - P^\xi_t)}{q^\xi(w^\xi_t)[1 + z^{\xi\top}_t(P^{-\xi}_t - P^\xi_t)]^\gamma}. \tag{A.126}
$$

From (A.125) and (A.126),

$$
\begin{aligned}
&\Big[r - q^\xi(w^\xi_t)^{-\frac{1}{\gamma}} - \frac{\lambda^\xi q^{-\xi}\big(w^\xi_t[1 + z^{\xi\top}_t(P^{-\xi}_t - P^\xi_t)]\big)z^{\xi\top}_t(P^{-\xi}_t - P^\xi_t)}{q^\xi(w^\xi_t)[1 + z^{\xi\top}_t(P^{-\xi}_t - P^\xi_t)]^\gamma}\Big]P^{\xi\prime}(w^\xi_t)w^\xi_t \\
&\quad + \frac{z^{\xi\top}_t\Sigma z^\xi_t}{[1 - P^{\xi\prime}(w^\xi_t)^\top z^\xi_t w^\xi_t]^2}\Big[\Gamma^\xi(w^\xi_t)P^{\xi\prime}(w^\xi_t)w^\xi_t + \frac{1}{2}P^{\xi\prime\prime}(w^\xi_t)(w^\xi_t)^2\Big] + \bar{D} - rP^\xi(w^\xi_t) \\
&= \frac{\Gamma^\xi(w^\xi_t)\hat{\sigma}^{\xi\top}\sigma z^\xi_t}{[1 - P^{\xi\prime}(w^\xi_t)^\top z^\xi_t w^\xi_t]^2} - \frac{\lambda^\xi q^{-\xi}\big(w^\xi_t[1 + z^{\xi\top}_t(P^{-\xi}_t - P^\xi_t)]\big)(P^{-\xi}_t - P^\xi_t)}{q^\xi(w^\xi_t)[1 + z^{\xi\top}_t(P^{-\xi}_t - P^\xi_t)]^\gamma}.
\end{aligned} \tag{A.127}
$$

The equation (A.127) follows from the investors' first-order condition and the Itô's lemma. Moreover, (A.126) implies that

$$
\begin{aligned}
z^{\xi\top}_t(\bar{D} + \mu^\xi_{Pt} - rP^\xi_t) = \Gamma^\xi(w^\xi_t)&\frac{z^{\xi\top}_t\Sigma z^\xi_t}{[1 - z^{\xi\top}_t P^{\xi\prime}(w^\xi_t)w^\xi_t]^2} \\
&- \frac{\lambda^\xi q^{-\xi}\big(w^\xi_t[1 + z^{\xi\top}_t(P^{-\xi}_t - P^\xi_t)]\big)z^{\xi\top}_t(P^{-\xi}_t - P^\xi_t)}{q^\xi(w^\xi_t)[1 + z^{\xi\top}_t(P^{-\xi}_t - P^\xi_t)]^\gamma}.
\end{aligned} \tag{A.128}
$$

Since there is a measure one of symmetric hedgers, the aggregate risky asset holdings by hedgers at time t in state ξ are given by

$$
Q^\xi_t = \Big[(\sigma^\xi_{Pt} + \sigma)^\top(\sigma^\xi_{Pt} + \sigma) + \lambda^\xi J^\xi_t J^{\xi\top}_t\Big]^{-1}\Big[\frac{\bar{D} + \mu^\xi_{Pt} - rP^\xi_t + \lambda^\xi J^\xi_t}{\alpha} - (\sigma^\xi_{Pt} + \sigma)^\top\sigma u^\xi\Big]. \tag{A.129}
$$

From (A.119), (A.123) and (A.125) and using the fact that $J_t^\xi = P_t^{-\xi} - P_t^\xi$,

$$
\begin{aligned}
Q_t^\xi = \frac{1}{\alpha}\Big\{ & \frac{\hat{\sigma}_t^{\xi\top}\hat{\sigma}_t^\xi}{[1 - P^{\xi\prime}(w_t^\xi)^\top z_t^\xi w_t^\xi]^2} + \lambda^\xi (P_t^{-\xi} - P_t^\xi)(P_t^{-\xi} - P_t^\xi)^\top \Big\}^{-1} \Big\{ \lambda^\xi (P_t^{-\xi} - P_t^\xi) \\
& + \Big[r - q^\xi (w_t^\xi)^{-\frac{1}{\gamma}} - \frac{\lambda^\xi q^{-\xi}\big(w_t^\xi [1 + z_t^{\xi\top}(P_t^{-\xi} - P_t^\xi)]\big) z_t^{\xi\top}(P_t^{-\xi} - P_t^\xi)}{q^\xi(w_t^\xi)[1 + z_t^{\xi\top}(P_t^{-\xi} - P_t^\xi)]^\gamma} \Big] P^{\xi\prime}(w_t^\xi) w_t^\xi \\
& + \frac{z_t^{\xi\top}\Sigma z_t^\xi}{[1 - P^{\xi\prime}(w_t^\xi)^\top z_t^\xi w_t^\xi]^2} \Big[\Gamma^\xi(w_t^\xi) P^{\xi\prime}(w_t^\xi) w_t^\xi + \frac{1}{2} P^{\xi\prime\prime}(w_t^\xi)(w_t^\xi)^2 \Big] + \bar{D} - r P^\xi(w_t^\xi) \\
& - \frac{\alpha \hat{\sigma}_t^{\xi\top} \sigma u^\xi}{1 - P^{\xi\prime}(w_t^\xi)^\top z_t^\xi w_t^\xi} \Big\}.
\end{aligned}
\tag{A.130}
$$

Market clearing requires that

$$
Q_t^\xi + Y_t^\xi = Q_t^\xi + z_t^\xi w_t^\xi = S. \tag{A.131}
$$

Combining (A.130) and (A.131) yields

$$
\begin{aligned}
\frac{1}{\alpha}\Big\{ & \frac{\hat{\sigma}_t^{\xi\top}\hat{\sigma}_t^\xi}{[1 - P^{\xi\prime}(w_t^\xi)^\top z_t^\xi w_t^\xi]^2} + \lambda^\xi (P_t^{-\xi} - P_t^\xi)(P_t^{-\xi} - P_t^\xi)^\top \Big\}^{-1} \Big\{ \lambda^\xi (P_t^{-\xi} - P_t^\xi) \\
& + \Big[r - q^\xi (w_t^\xi)^{-\frac{1}{\gamma}} - \frac{\lambda^\xi q^{-\xi}\big(w_t^\xi [1 + z_t^{\xi\top}(P_t^{-\xi} - P_t^\xi)]\big) z_t^{\xi\top}(P_t^{-\xi} - P_t^\xi)}{q^\xi(w_t^\xi)[1 + z_t^{\xi\top}(P_t^{-\xi} - P_t^\xi)]^\gamma} \Big] P^{\xi\prime}(w_t^\xi) w_t^\xi \\
& + \frac{z_t^{\xi\top}\Sigma z_t^\xi}{[1 - P^{\xi\prime}(w_t^\xi)^\top z_t^\xi w_t^\xi]^2} \Big[\Gamma^\xi(w_t^\xi) P^{\xi\prime}(w_t^\xi) w_t^\xi + \frac{1}{2} P^{\xi\prime\prime}(w_t^\xi)(w_t^\xi)^2 \Big] + \bar{D} - r P^\xi(w_t^\xi) \\
& - \frac{\alpha \hat{\sigma}_t^{\xi\top} \sigma u^\xi}{1 - P^{\xi\prime}(w_t^\xi)^\top z_t^\xi w_t^\xi} \Big\} = S - z_t^\xi w_t^\xi.
\end{aligned}
\tag{A.132}
$$

The equation (A.132) follows from the market clearing condition.

When the economy switches from state ξ to state $-\xi$ at time t, the investor's wealth jumps from w_t^ξ to $w_t^{-\xi}$ according to the following fixed point condition

$$
w_t^{-\xi} = w_t^\xi + w_t^\xi z_t^{\xi\top}(P_t^{-\xi} - P_t^\xi). \tag{A.133}
$$

Substituting (A.133) into (A.127) yields

$$\begin{aligned}
&\bar{D} - rP^{\xi}(w_t^{\xi}) + \Big[r - q^{\xi}(w_t^{\xi})^{-\frac{1}{\gamma}} - \frac{\lambda^{\xi} q^{-\xi}(w_t^{-\xi})\frac{w_t^{-\xi} - w_t^{\xi}}{w_t^{\xi}}}{q^{\xi}(w_t^{\xi})(\frac{w_t^{-\xi}}{w_t^{\xi}})^{\gamma}}\Big] P^{\xi\prime}(w_t^{\xi}) w_t^{\xi} \\
&+ \frac{z_t^{\xi\top}\Sigma z_t^{\xi}}{[1 - P^{\xi\prime}(w_t^{\xi})^{\top} z_t^{\xi} w_t^{\xi}]^2}\Big[\Gamma^{\xi}(w_t^{\xi}) P^{\xi\prime}(w_t^{\xi}) w_t^{\xi} + \frac{1}{2} P^{\xi\prime\prime}(w_t^{\xi})(w_t^{\xi})^2\Big] \\
&= \frac{\Gamma^{\xi}(w_t^{\xi})\hat{\sigma}_t^{\xi\top}\sigma z_t^{\xi}}{[1 - P^{\xi\prime}(w_t^{\xi})^{\top} z_t^{\xi} w_t^{\xi}]^2} - \frac{\lambda^{\xi} q^{-\xi}(w_t^{-\xi})(P_t^{-\xi} - P_t^{\xi})}{q^{\xi}(w_t^{\xi})(\frac{w_t^{-\xi}}{w_t^{\xi}})^{\gamma}}.
\end{aligned} \tag{A.134}$$

Furthermore, substituting (A.133) into (A.132) yields

$$\begin{aligned}
&\bar{D} - rP_t^{\xi} + \Big[r - q^{\xi}(w_t^{\xi})^{-\frac{1}{\gamma}} - \frac{\lambda^{\xi} q^{-\xi}(w_t^{-\xi})\frac{w_t^{-\xi} - w_t^{\xi}}{w_t^{\xi}}}{q^{\xi}(w_t^{\xi})(\frac{w_t^{-\xi}}{w_t^{\xi}})^{\gamma}}\Big] P^{\xi\prime}(w_t^{\xi}) w_t^{\xi} \\
&+ \frac{z_t^{\xi\top}\Sigma z_t^{\xi}}{[1 - P^{\xi\prime}(w_t^{\xi})^{\top} z_t^{\xi} w_t^{\xi}]^2}\Big[\Gamma^{\xi}(w_t^{\xi}) P^{\xi\prime}(w_t^{\xi}) w_t^{\xi} + \frac{1}{2} P^{\xi\prime\prime}(w_t^{\xi})(w_t^{\xi})^2\Big] \\
&= \alpha\Big\{\frac{\hat{\sigma}_t^{\xi\top}\hat{\sigma}_t^{\xi}}{[1 - P^{\xi\prime}(w_t^{\xi})^{\top} z_t^{\xi} w_t^{\xi}]^2} + \lambda^{\xi}(P^{-\xi} - P^{\xi})(P^{-\xi} - P^{\xi})^{\top}\Big\}(S - z_t^{\xi} w_t^{\xi}) \\
&\quad + \frac{\alpha\hat{\sigma}_t^{\xi\top}\sigma u^{\xi}}{1 - P^{\xi\prime}(w_t^{\xi})^{\top} z_t^{\xi} w_t^{\xi}} - \lambda^{\xi}(P_t^{-\xi} - P_t^{\xi}).
\end{aligned} \tag{A.135}$$

Combining (A.134) and (A.135) yields

$$\begin{aligned}
&\frac{\Gamma^{\xi}(w_t^{\xi})\hat{\sigma}_t^{\xi\top}\sigma z_t^{\xi}}{[1 - w_t^{\xi} P^{\xi\prime}(w_t^{\xi})^{\top} z_t^{\xi}]^2} - \frac{\lambda^{\xi} q^{-\xi}(w_t^{-\xi})(P_t^{-\xi} - P_t^{\xi})}{q^{\xi}(w_t^{\xi})(\frac{w_t^{-\xi}}{w_t^{\xi}})^{\gamma}} \\
&= \frac{\alpha\hat{\sigma}_t^{\xi\top}\hat{\sigma}_t^{\xi}(S - z_t^{\xi} w_t^{\xi})}{[1 - w_t^{\xi} P^{\xi\prime}(w_t^{\xi})^{\top} z_t^{\xi}]^2} + \alpha\lambda^{\xi}(P^{-\xi} - P^{\xi})(P^{-\xi} - P^{\xi})^{\top}(S - z_t^{\xi} w_t^{\xi}) \\
&\quad + \frac{\alpha\hat{\sigma}^{\xi\top}\sigma u^{\xi}}{1 - w_t^{\xi} P^{\xi\prime}(w_t^{\xi})^{\top} z_t^{\xi}} - \lambda^{\xi}(P^{-\xi} - P_t^{\xi}).
\end{aligned} \tag{A.136}$$

Rearranging yields

$$\begin{aligned}
&\frac{[\Gamma^{\xi}(w_t^{\xi}) + \alpha w_t^{\xi}]\hat{\sigma}_t^{\xi\top}\sigma z_t^{\xi} - \alpha\hat{\sigma}_t^{\xi\top}\sigma(\sigma^{-1}\hat{\sigma}_t^{\xi})S}{[1 - w_t^{\xi} P^{\xi\prime}(w_t^{\xi})^{\top} z_t^{\xi}]^2} - \frac{\alpha\hat{\sigma}_t^{\xi\top}\sigma u^{\xi}}{1 - w_t^{\xi} P^{\xi\prime}(w_t^{\xi})^{\top} z_t^{\xi}} \\
&= \lambda^{\xi}\Big\{\Big[\frac{q^{-\xi}(w_t^{-\xi})}{q^{\xi}(w_t^{\xi})(\frac{w_t^{-\xi}}{w_t^{\xi}})^{\gamma}} - 1\Big](P_t^{-\xi} - P_t^{\xi}) + \alpha(P_t^{-\xi} - P_t^{\xi})(P_t^{-\xi} - P_t^{\xi})^{\top}(S - z_t^{\xi} w_t^{\xi})\Big\}.
\end{aligned} \tag{A.137}$$

Note that

$$\hat{\sigma}_t^{\xi} = w_t^{\xi} \sigma z_t^{\xi} P^{\xi\prime}(w_t^{\xi})^{\top} + [1 - w_t^{\xi} P^{\xi\prime}(w_t^{\xi})^{\top} z_t^{\xi}]\sigma. \tag{A.138}$$

Thus,

$$\hat{\sigma}_t^{\xi\top} \sigma z_t^{\xi} = w_t^{\xi} P^{\xi\prime}(w_t^{\xi}) z_t^{\xi\top} \Sigma z_t^{\xi} + [1 - w_t^{\xi} P^{\xi\prime}(w_t^{\xi})^{\top} z_t^{\xi}]\Sigma z_t^{\xi}. \tag{A.139}$$

Then, (A.134) simplifies to

$$\begin{aligned} &\bar{D} - rP^{\xi}(w_t^{\xi}) + \Big[r - q^{\xi}(w_t^{\xi})^{-\frac{1}{\gamma}} - \frac{\lambda^{\xi} q^{-\xi}(w_t^{-\xi}) \frac{w_t^{-\xi} - w_t^{\xi}}{w_t^{\xi}}}{q^{\xi}(w_t^{\xi})(\frac{w_t^{-\xi}}{w_t^{\xi}})^{\gamma}}\Big] P^{\xi\prime}(w_t^{\xi}) w_t^{\xi} \\ &+ \frac{z_t^{\xi\top} \Sigma z_t^{\xi}}{2[1 - w_t^{\xi} P^{\xi\prime}(w_t^{\xi})^{\top} z_t^{\xi}]^2} P^{\xi\prime\prime}(w_t^{\xi})(w_t^{\xi})^2 = \frac{\Gamma^{\xi}(w_t^{\xi}) \Sigma z_t^{\xi}}{1 - w_t^{\xi} P^{\xi\prime}(w_t^{\xi})^{\top} z_t^{\xi}} - \frac{\lambda^{\xi} q^{-\xi}(w_t^{-\xi})(P_t^{-\xi} - P_t^{\xi})}{q^{\xi}(w_t^{\xi})(\frac{w_t^{-\xi}}{w_t^{\xi}})^{\gamma}}. \end{aligned} \tag{A.140}$$

Moreover, $q^{\xi}(w_t^{\xi})$ is solution to the ODE (A.107). Using (A.119), we can rewrite (A.107) as

$$\begin{aligned} &\frac{q^{\xi}(w_t^{\xi})^{-\frac{1}{\gamma}} - r}{\rho - r} \Gamma^{\xi}(w_t^{\xi}) - \frac{\Gamma^{\xi\prime}(w_t^{\xi}) w_t^{\xi} - \Gamma^{\xi}(w_t^{\xi}) + \Gamma^{\xi}(w_t^{\xi})^2}{2(\rho - r)} \frac{z_t^{\xi\top} \Sigma z_t^{\xi}}{[1 - w_t^{\xi} P^{\xi\prime}(w_t^{\xi})^{\top} z_t^{\xi}]^2} \\ &+ \frac{\lambda^{\xi}}{\rho - r} \Big\{ \frac{[\Gamma^{\xi}(w_t^{\xi}) \frac{w_t^{-\xi} - w_t^{\xi}}{w_t^{\xi}} + 1] q^{-\xi}(w_t^{-\xi})}{q^{\xi}(w_t^{\xi})(\frac{w_t^{-\xi}}{w_t^{\xi}})^{\gamma}} - 1 \Big\} = 1. \end{aligned} \tag{A.141}$$

The investors' wealth-scaled asset holdings z_t^{ξ} are determined by (A.137), while prices $P^{\xi}(w_t^{\xi})$ and hedging demand $q^{\xi}(w_t^{\xi})$ are solutions to the ODEs (A.140) and (A.141).

A. Summary of ODE System

We can define $\pi^{\xi}(w_t^{\xi}) \equiv \frac{\bar{D}}{r} - P^{\xi}(w_t^{\xi})$. Then, $P^{\xi\prime}(w_t^{\xi}) = -\pi^{\xi\prime}(w_t^{\xi})$ and $P^{\xi\prime\prime}(w_t^{\xi}) = -\pi^{\xi\prime\prime}(w_t^{\xi})$. The ODE (A.140) becomes

$$\begin{aligned} &r\pi^{\xi}(w_t^{\xi}) - \Big[r - q^{\xi}(w_t^{\xi})^{-\frac{1}{\gamma}} - \frac{\lambda^{\xi} q^{-\xi}(w_t^{-\xi}) \frac{w_t^{-\xi} - w_t^{\xi}}{w_t^{\xi}}}{q^{\xi}(w_t^{\xi})(\frac{w_t^{-\xi}}{w_t^{\xi}})^{\gamma}}\Big] \pi^{\xi\prime}(w_t^{\xi}) w_t^{\xi} - \frac{z_t^{\xi\top} \Sigma z_t^{\xi} \pi^{\xi\prime\prime}(w_t^{\xi})(w_t^{\xi})^2}{2[1 + w_t^{\xi} \pi^{\xi\prime}(w_t^{\xi})^{\top} z_t^{\xi}]^2} \\ &= \frac{\Gamma^{\xi}(w_t^{\xi}) \Sigma z_t^{\xi}}{1 + w_t^{\xi} \pi^{\xi\prime}(w_t^{\xi})^{\top} z_t^{\xi}} - \frac{\lambda^{\xi} q^{-\xi}(w_t^{-\xi})[\pi^{\xi}(w_t^{\xi}) - \pi^{-\xi}(w_t^{-\xi})]}{q^{\xi}(w_t^{\xi})(\frac{w_t^{-\xi}}{w_t^{\xi}})^{\gamma}}. \end{aligned} \tag{A.142}$$

The ODE (A.141) becomes

$$
\begin{aligned}
&\frac{q^\xi(w_t^\xi)^{-\frac{1}{\gamma}}-r}{\rho-r}\Gamma^\xi(w_t^\xi)-\frac{\Gamma^{\xi\prime}(w_t^\xi)w_t^\xi-\Gamma^\xi(w_t^\xi)+\Gamma^\xi(w_t^\xi)^2}{2(\rho-r)}\frac{z_t^{\xi\top}\Sigma z_t^\xi}{[1+w_t^\xi\pi^{\xi\prime}(w_t^\xi)^\top z_t^\xi]^2}\\
&+\frac{\lambda^\xi}{\rho-r}\Big\{\frac{[\Gamma^\xi(w_t^\xi)\frac{w_t^{-\xi}-w_t^\xi}{w_t^\xi}+1]q^{-\xi}(w_t^{-\xi})}{q^\xi(w_t^\xi)(\frac{w_t^{-\xi}}{w_t^\xi})^\gamma}-1\Big\}=1.
\end{aligned}
\tag{A.143}
$$

Moreover, (A.137) can be written as

$$
\begin{aligned}
&\frac{[\Gamma^\xi(w_t^\xi)+\alpha w_t^\xi]\hat{\sigma}_t^{\xi\top}\sigma z_t^\xi-\alpha\hat{\sigma}_t^{\xi\top}\sigma(\sigma^{-1}\hat{\sigma}_t^\xi)S}{[1+w_t^\xi\pi^{\xi\prime}(w_t^\xi)^\top z_t^\xi]^2}-\frac{\alpha\hat{\sigma}_t^{\xi\top}\sigma u^\xi}{1+w_t^\xi\pi^{\xi\prime}(w_t^\xi)^\top z_t^\xi}\\
&=\lambda^\xi\Big\{\Big[\frac{q^{-\xi}(w_t^{-\xi})}{q^\xi(w_t^\xi)(\frac{w_t^{-\xi}}{w_t^\xi})^\gamma}-1\Big](P_t^{-\xi}-P_t^\xi)+\alpha(P_t^{-\xi}-P_t^\xi)(P_t^{-\xi}-P_t^\xi)^\top(S-z_t^\xi w_t^\xi)\Big\}.
\end{aligned}
\tag{A.144}
$$

Here,

$$
\hat{\sigma}_t^\xi=-w_t^\xi\sigma z_t^\xi\pi^{\xi\prime}(w_t^\xi)^\top+[1+w_t^\xi\pi^{\xi\prime}(w_t^\xi)^\top z_t^\xi]\sigma, \tag{A.145}
$$

$$
\Gamma^\xi(w_t^\xi)=\gamma-\frac{q^{\xi\prime}(w_t^\xi)}{q^\xi(w_t^\xi)}w_t^\xi. \tag{A.146}
$$

Moreover, the following fixed point condition must be satisfied

$$
w_t^{-\xi}-w_t^\xi=w_t^\xi y_t^{\xi\top}\big[\pi^\xi(w_t^\xi)-\pi^{-\xi}(w_t^{-\xi})\big]. \tag{A.147}
$$

B. Variance-Covariance Matrix of Percentage Returns

As in the case with log investors, the endogenous percentage return variance-covariance matrix is given by $\Omega_t^\xi\equiv(\sigma_{Rt}^\xi)^\top\sigma_{Rt}^\xi$, where $\sigma_{Rt}^\xi\equiv(\sigma_{Pt}^\xi+\sigma)\,\mathrm{diag}(P_t^\xi)^{-1}$. Then

$$
\begin{aligned}
\Omega_t^\xi&=\mathrm{diag}(P_t^\xi)^{-1}(\sigma_{Pt}^\xi+\sigma)^\top(\sigma_{Pt}^\xi+\sigma)\,\mathrm{diag}(P_t^\xi)^{-1}\\
&=\mathrm{diag}(P_t^\xi)^{-1}\left[\sigma+\frac{\sigma z_t^\xi P^{\xi\prime}(w_t^\xi)^\top w_t^\xi}{1-P^{\xi\prime}(w_t^\xi)^\top z_t^\xi w_t^\xi}\right]^\top\left[\sigma+\frac{\sigma z_t^\xi P^{\xi\prime}(w_t^\xi)^\top w_t^\xi}{1-P^{\xi\prime}(w_t^\xi)^\top z_t^\xi w_t^\xi}\right]\mathrm{diag}(P_t^\xi)^{-1}.
\end{aligned}
\tag{A.148}
$$

The variance-covariance matrix exhibits a factor structure and depends on the endogenous state variable, the investors' total wealth w_t^ξ.

A.2 Proof of Proposition 2

Based on the derivations for Proposition 1 and Proposition 4 and specializing the ODEs to the one-asset case, we find that investors' aggregate wealth-scaled asset holding z_t^ξ satisfies

$$\begin{aligned}&\frac{(1+\alpha w_t^\xi)\sigma^2 z_t^\xi - \alpha\sigma^2 S}{[1+\pi^{\xi\prime}(w_t^\xi)w_t^\xi z_t^\xi]^2} - \frac{\alpha\sigma^2 u^\xi}{1+\pi^{\xi\prime}(w_t^\xi)w_t^\xi z_t^\xi}\\&= \lambda^\xi\Big[\Big(\frac{w_t^\xi}{w_t^{-\xi}}-1\Big)\frac{w_t^{-\xi}-w_t^\xi}{w_t^\xi z_t^\xi} + \alpha(S-w_t^\xi z_t^\xi)\Big(\frac{w_t^{-\xi}-w_t^\xi}{w_t^\xi z_t^\xi}\Big)^2\Big].\end{aligned} \tag{A.149}$$

The functions $\pi^\xi(w_t^\xi)$ and $\bar{q}^\xi(w_t^\xi)$ are solutions to the following ODEs

$$\begin{aligned}&r\pi^\xi(w_t^\xi) - \Big[r-\rho-\lambda^\xi\frac{w_t^{-\xi}-w_t^\xi}{w_t^{-\xi}}\Big]\pi^{\xi\prime}(w_t^\xi)w_t^\xi - \frac{1}{2}\Big[\frac{\sigma z_t^\xi}{1+\pi^{\xi\prime}(w_t^\xi)w_t^\xi z_t^\xi}\Big]^2\pi^{\xi\prime\prime}(w_t^\xi)(w_t^\xi)^2\\&= \frac{\sigma^2 z_t^\xi}{1+\pi^{\xi\prime}(w_t^\xi)w_t^\xi z_t^\xi} - \lambda^\xi\frac{\pi^\xi(w_t^\xi)-\pi^{-\xi}(w_t^{-\xi})}{\frac{w_t^{-\xi}}{w_t^\xi}},\end{aligned} \tag{A.150}$$

and

$$\begin{aligned}&\log\rho + \Big[\bar{q}^{\xi\prime}(w_t^\xi)w_t^\xi + \frac{1}{\rho}\Big](r-\rho) - \rho\bar{q}^\xi(w_t^\xi)\\&+\Big[\bar{q}^{\xi\prime}(w_t^\xi)w_t^\xi + \frac{1}{2}\bar{q}^{\xi\prime\prime}(w_t^\xi)(w_t^\xi)^2 + \frac{1}{2\rho}\Big]\Big[\frac{\sigma z_t^\xi}{1+\pi^{\xi\prime}(w_t^\xi)w_t^\xi z_t^\xi}\Big]^2\\&+\lambda^\xi\Big[\frac{1}{\rho}\log\Big(\frac{w_t^{-\xi}}{w_t^\xi}\Big) + \bar{q}^{-\xi}(w_t^{-\xi}) - \bar{q}^\xi(w_t^\xi) - \Big(\bar{q}^{\xi\prime}(w_t^\xi)w_t^\xi + \frac{1}{\rho}\Big)\frac{w_t^{-\xi}-w_t^\xi}{w_t^{-\xi}}\Big] = 0.\end{aligned} \tag{A.151}$$

Moreover, the following fixed point condition must be satisfied

$$w_t^{-\xi} - w_t^\xi = w_t^\xi z_t^\xi[\pi^\xi(w_t^\xi) - \pi^{-\xi}(w_t^{-\xi})]. \tag{A.152}$$

Suppose that we are standing at $\xi = s$ and the state prior to shock is denoted by $n-$. We want to analyze the asymptotic behavior at $\lambda^s = \lambda \to \infty$. Let $\epsilon \equiv 1/\lambda$. Expand the function $\pi^s(w)$ according to

$$\pi^s(w) = \pi^n(w) + \epsilon\pi_1(w) + \epsilon^2\pi_2(w) + \dots \tag{A.153}$$

where higher-order terms have been omitted. If $w_t^s = w$, then from the fixed point condition (A.152), we have

$$\begin{aligned}w_t^{n-} &= w - (S - Q_t^{n-})\big[\pi^n(w_t^{n-}) - \pi^s(w)\big]\\&= w - (S - Q_t^{n-})\big[\pi^n(w_t^{n-}) - \pi^n(w) - \epsilon\pi_1(w) - \epsilon^2\pi_2(w)\big].\end{aligned} \tag{A.154}$$

For small ϵ, we have

$$w_t^{n-} = w - (S - Q_t^{n-})\big[\pi^{n\prime}(w)(w_t^{n-} - w) - \epsilon\pi_1(w) - \epsilon^2\pi_2(w)\big], \tag{A.155}$$

which implies

$$w_t^{n-} = w + \frac{S - Q_t^{n-}}{1 + (S - Q_t^{n-})\pi^{n\prime}(w)} \big[\pi_1(w)\epsilon + \pi_2(w)\epsilon^2\big]. \tag{A.156}$$

When $\lambda \to \infty$, $w_t^{n-} \to w$. Moreover, $\pi^n(w_t^{n-}) \to \pi^s(w)$.

From (A.58) and (A.59) and specializing to the one-asset case, we get

$$\sigma_{Pt}^{\xi} + \sigma = \frac{\sigma}{1 + \pi^{\xi\prime}(w_t^{\xi}) w_t^{\xi} z_t^{\xi}} = \frac{\sigma}{1 + \pi^{\xi\prime}(w_t^{\xi})(S - Q_t^{\xi})}. \tag{A.157}$$

The second equality follows from the market clearing condition $w_t z_t^{\xi} = S - Q_t^{\xi}$. Then

$$\sigma_{Rt}^{\xi} \equiv \frac{\sigma_{Pt}^{\xi} + \sigma}{P_t^{\xi}} = \frac{\sigma}{[1 + \pi^{\xi\prime}(w_t^{\xi})(S - Q_t^{\xi})][\frac{\bar{D}}{r} - \pi^{\xi}(w_t^{\xi})]}. \tag{A.158}$$

Note that in the one-asset case,

$$Q_t^{\xi} = \Big[(\sigma_{Pt}^{\xi} + \sigma)^2 + \lambda^{\xi}(J_t^{\xi})^2\Big]^{-1} \Big[\frac{\bar{D} + \mu_{Pt}^{\xi} - rP_t^{\xi} + \lambda^{\xi} J_t^{\xi}}{\alpha} - (\sigma_{Pt}^{\xi} + \sigma)\sigma u^{\xi}\Big]. \tag{A.159}$$

In the limit with $\lambda \to \infty$, (A.37) implies

$$(\sigma_{Pt}^{s} + \sigma)^2 \hat{z}_t^{s} = \bar{D} + \mu_{Pt}^{s} - rP_t^{s} + \lambda J_t^{s}, \tag{A.160}$$

where λJ_t^s is bounded away from zero, and (A.159) becomes

$$Q_t^{s} = \frac{1}{\alpha \hat{w}_t^{s}}(S - Q_t^{s}) - \frac{\sigma}{\sigma_{Pt}^{s} + \sigma} u^{\xi} = \frac{1}{\alpha \hat{w}_t^{s}}(S - Q_t^{s}) - [1 + \pi^{s\prime}(w_t^{s})(S - Q_t^{s})]u^{s}. \tag{A.161}$$

Rearranging yields

$$Q_t^{s} = \frac{\frac{S}{\alpha \hat{w}_t^{s}} - [1 + \pi^{s\prime}(w_t^{s})S]u^{s}}{1 + \frac{1}{\alpha \hat{w}_t^{s}} - \pi^{s\prime}(w_t^{s})u^{s}}. \tag{A.162}$$

Similarly, in the normal state,

$$Q_t^{n} = \frac{\frac{S}{\alpha \hat{w}_t^{n}} - [1 + \pi^{n\prime}(w_t^{n})S]u^{n}}{1 + \frac{1}{\alpha \hat{w}_t^{n}} - \pi^{n\prime}(w_t^{n})u^{n}}. \tag{A.163}$$

Since $\pi^{n\prime}(w) < 0$, we can show that $Q_t^s < Q_t^n$ for $u^s > u^n$. Substituting into (A.158), we see that $\sigma_{Rt}^s > \sigma_{Rt}^n$. Note that $\Delta\mathbb{E}_t(R) = \mu_{Rt}^s + \lambda J_{Rt}^s - \mu_{Rt}^n$, that is

$$\Delta\mathbb{E}_t(R) = \frac{(\sigma_{Pt}^{s} + \sigma)^2(S - Q_t^{s})}{\hat{w}_t^{s}(\frac{\bar{D}}{r} - \pi^{s}(w_t^{s}))} - \frac{(\sigma_{Pt}^{n} + \sigma)^2(S - Q_t^{n})}{\hat{w}_t^{n}(\frac{\bar{D}}{r} - \pi^{n}(w_t^{n}))}. \tag{A.164}$$

In the limit, $\Delta\mathbb{E}_t(R) > 0$. From (23), $\mathcal{M}^{\log}(w_t^n; u^n, u^s) < \mathcal{C}^{\log}(w_t^n; \delta)|_{\delta = \Delta\mathbb{E}_t(R)}$.

(The proof for

$$\lim_{u^n \to u^s} \left(\mathcal{M}^{log}(w_t^n; u^n, u^s) - \mathcal{C}^{log}(w_t^n; \delta)|_{\delta = \Delta \mathbb{E}_t(R)} \right) < 0.$$

is TBC)

A.3 Proof of Proposition 3

In this proof we show that

$$\lim_{\Delta \upsilon \to 0} \frac{1}{\gamma} \frac{\left[\left((\Omega_t^s)^{-1} - (\Omega_t^n)^{-1} \right) \left(\mathbb{E}_t \left(R^s \right) - r\mathbf{1} \right) \right]}{\Delta E \left(R \right)} =$$

$$= -\frac{P_t^2}{\gamma} \left(I \left(s + \upsilon \right) F \left(w_t \right) + \frac{r}{\bar{D}} 2\pi \left(w_t \right) \left(1 - \sigma^2 I \left(s + \upsilon \right)^2 F \left(w_t \right) \right) \right) \left(s + \upsilon \right) < 0$$

where $\frac{1}{\sigma^2 I(s+\upsilon)^2} > F \left(w_t \right) > 0$ and $\bar{F} \left(w_t \right) > 0$ are scalar functions defined by ODEs and P_t, w_t and $\pi \left(\cdot \right)$ are the limit of P_t^ξ, w_t^ξ and $\pi^\xi \left(\cdot \right)$ for both states $\xi = s, n$.

We start with the following Lemma which is based on results in Kondor-Vayanos.

Lemma A.1. *When $\lambda^s, \lambda^n \to 0$, price is*

$$P_t^\xi = \frac{\bar{D}}{r} - \pi^\xi \left(\omega_t \right) \Sigma \left(s + u^\xi \right)$$

expected return has the form of

$$\mathbb{E}_t(R^\xi) - r\mathbf{1} = diag \left(P_t^\xi \right)^{-1} \left(\bar{D} + \mu_{Pt}^\xi - rP_t^\xi \right) = diag \left(P_t^\xi \right)^{-1} B^\xi \left(w_t^\xi \right) \Sigma \left(s + u^\xi \right)$$

investors equilibrium position has the form of

$$Y_t = G^\xi \left(w_t^\xi \right) \left(s + u^\xi \right)$$

and, the inverse of the return covariance matrix has the form of

$$\left(\Omega_t^\xi \right)^{-1} = diag \left(P_t^\xi \right) \Sigma^{-1} diag \left(P_t^\xi \right) - F^\xi \left(w_t^\xi \right) diag \left(P_t^\xi \right) \left(s + u^\xi \right) \left(s + u^\xi \right)^\intercal diag \left(P_t^\xi \right)$$

where $\pi^\xi \left(\cdot \right), F^\xi \left(\cdot \right)$ and $B^\xi \left(\cdot \right)$ and $G^\xi \left(\cdot \right)$ are scalar functions depending on the vectors of u^ξ and s only through the terms $\left(s + u^\xi \right)^\intercal \Sigma \left(s + u^\xi \right)$ and $\left(s + u^\xi \right)^\intercal \Sigma s$.

Proof. With $\lambda^s, \lambda^n \to 0$, our model is equivalent with the positive supply, short-term hedgers' version of KV discussed in Section 6 . Expected return is given by (6.1) in Proposition 6.2 in KV which is, with our notation,

$$\bar{D} + \mu_{Pt}^\xi - rP_t^\xi = B^\xi \left(w_t^\xi \right) \Sigma \left(s + u^\xi \right)$$

where

$$B^{\xi}\left(w_t^{\xi}\right)=\frac{\alpha\left(\frac{1}{w_t^{\xi}}\right)}{\alpha+\frac{1}{w_t^{\xi}}+\alpha\pi^{\xi\prime}\left(w_t^{\xi}\right)\left(s+u^{\xi}\right)^{\intercal}\Sigma s}\left[1-\left(s+u^{\xi}\right)^{\intercal}\Sigma\left(s+u^{\xi}\right)\frac{\alpha\pi^{\xi\prime}\left(w_t\right)}{\alpha+\frac{1}{w_t^{\xi}}+\alpha\pi^{\xi\prime}\left(w_t^{\xi}\right)\left(s+u^{\xi}\right)^{\intercal}\Sigma s}\right]$$

where $\pi^{\xi}\left(w_t^{\xi}\right)$ solves

$$\begin{aligned}&\frac{\alpha^2\left(s+u^{\xi}\right)^{\intercal}\Sigma\left(s+u^{\xi}\right)}{2\left[\alpha+\frac{1}{w_t^{\xi}}+\alpha\pi^{\xi\prime}(w_t)\left(s+u^{\xi}\right)^{\intercal}\Sigma s\right]^2}\left(-\pi^{\prime\prime\xi}\left(w_t^{\xi}\right)\right)-\left(r-q(w_t)^{-\frac{1}{\gamma}}\right)\pi^{\xi\prime}(w_t)w_t+r\pi^{\xi}\left(w_t^{\xi}\right)\\&=\frac{\alpha\left(\frac{\gamma}{w_t^{\xi}}-\frac{q^{\xi\prime}(w_t)}{q^{\xi}(w_t)}\right)}{\alpha+\frac{1}{w_t^{\xi}}+\alpha\pi^{\xi\prime}(w_t)\left(s+u^{\xi}\right)^{\intercal}\Sigma s},\end{aligned}\tag{A.165}$$

with boundary conditions

$$\pi^{\xi}(0)=\frac{\alpha}{r},\ \lim_{w_t\to\infty}\pi^{\xi}(w_t)=0.$$

Theorem 3 in Kondor and Vayanos (2019) implies that $\pi^{\xi}(w_t)$ is monotonically decreasing. For Π_t^{ξ}, as the proof of Proposition 6.1 shows, we have the following expressions (using our notation).

$$\sigma_{P_t}^{\xi}=\frac{\alpha}{\alpha+\frac{1}{w_t^{\xi}}}\pi^{\xi\prime}\left(w_t^{\xi}\right)b_t\left(s+u^{\xi}\right)^{\intercal}\Sigma$$

where

$$b_t=\frac{\sigma\left(s+u^{\xi}\right)}{1-\frac{\alpha}{\alpha+\frac{1}{w_t^{\xi}}}\pi^{\xi\prime}\left(w_t^{\xi}\right)\left(s+u^{\xi}\right)^{\intercal}\Sigma s}$$

Hence, we can write

$$\sigma_{P_t}^{\xi}=\frac{\alpha}{\alpha+\frac{1}{w_t^{\xi}}}\pi^{\xi\prime}\left(w_t^{\xi}\right)\frac{1}{1-\frac{\alpha}{\alpha+\frac{1}{w_t^{\xi}}}\pi^{\xi\prime}\left(w_t^{\xi}\right)\left(s+u^{\xi}\right)^{\intercal}\Sigma s}\sigma\left(s+u^{\xi}\right)\left(s+u^{\xi}\right)^{\intercal}\Sigma$$

or

$$\sigma_{P_t}^{\xi}=\hat{F}^{\xi}\left(w_t^{\xi}\right)\sigma\left(s+u^{\xi}\right)\left(s+u^{\xi}\right)^{\intercal}\Sigma$$

with

$$\hat{F}^{\xi}\left(w_t^{\xi}\right)\equiv\frac{\alpha\pi^{\xi\prime}\left(w_t^{\xi}\right)}{\alpha+\frac{1}{w_t^{\xi}}-\alpha\pi^{\xi\prime}\left(w_t^{\xi}\right)\left(s+u^{\xi}\right)^{\intercal}\Sigma s}.$$

Then

$$\sigma_{P_t}^{\xi\intercal} = \hat{F}^{\xi}\left(w_t^{\xi}\right)\left(\sigma\left(s+u^{\xi}\right)\left(s+u^{\xi}\right)^{\intercal}\Sigma\right)^{\intercal} = \hat{F}^{\xi}\left(w_t^{\xi}\right)\left(\left(s+u^{\xi}\right)^{\intercal}\Sigma\right)^{\intercal}\left(\sigma\left(s+u^{\xi}\right)\right)^{\intercal} =$$
$$= \hat{F}^{\xi}\left(w_t^{\xi}\right)\Sigma\left(s+u^{\xi}\right)\left(s+u^{\xi}\right)^{\intercal}\sigma^{\intercal}$$

implying

$$\sigma_{P_t}^{\xi\intercal}\sigma_{P_t}^{\xi} = \left(s+u^{\xi}\right)^{\intercal}\Sigma\left(s+u^{\xi}\right)\left(\hat{F}^{\xi}\left(w_t^{\xi}\right)\right)^2\Sigma\left(s+u^{\xi}\right)\left(s+u^{\xi}\right)^{\intercal}\Sigma.$$

Then

$$\begin{aligned}
\Pi_t^{\xi} &= \left(\sigma_{P_t}^{\xi}+\sigma\right)^{\intercal}\left(\sigma_{P_t}^{\xi}+\sigma\right) = \sigma_{P_t}^{\xi\intercal}\sigma_{P_t}^{\xi} + \sigma^{\intercal}\sigma_{P_t}^{\xi} + \Sigma + \sigma_{P_t}^{\xi\intercal}\sigma = \\
&= \left(\hat{F}^{\xi}\left(w_t^{\xi}\right)\right)^2\Sigma\left(s+u^{\xi}\right)\left(s+u^{\xi}\right)^{\intercal}\Sigma\left(s+u^{\xi}\right)\left(s+u^{\xi}\right)^{\intercal}\Sigma + 2\hat{F}^{\xi}\left(w_t^{\xi}\right)\Sigma\left(s+u^{\xi}\right)\left(s+u^{\xi}\right)^{\intercal}\Sigma + \Sigma \\
&= \left(s+u^{\xi}\right)^{\intercal}\Sigma\left(s+u^{\xi}\right)\left(\hat{F}^{\xi}\left(w_t^{\xi}\right)\right)^2\Sigma\left(s+u^{\xi}\right)\left(s+u^{\xi}\right)^{\intercal}\Sigma + 2\hat{F}^{\xi}\left(w_t^{\xi}\right)\Sigma\left(s+u^{\xi}\right)\left(s+u^{\xi}\right)^{\intercal}\Sigma + \Sigma \\
&= \hat{F}^{\xi}\left(w_t^{\xi}\right)\left(\hat{F}^{\xi}\left(w_t^{\xi}\right)\left(s+u^{\xi}\right)^{\intercal}\Sigma\left(s+u^{\xi}\right)+2\right)\Sigma\left(s+u^{\xi}\right)\left(s+u^{\xi}\right)^{\intercal}\Sigma + \Sigma
\end{aligned}$$

Or, defining

$$\begin{aligned}
\bar{F}^{\xi}\left(w_t^{\xi}\right) &= \hat{F}^{\xi}\left(w_t^{\xi}\right)\left(\hat{F}^{\xi}\left(w_t^{\xi}\right)\left(s+u^{\xi}\right)^{\intercal}\Sigma\left(s+u^{\xi}\right)+2\right) = \\
&= \frac{-\alpha\pi^{\xi\prime}\left(w_t^{\xi}\right)}{\frac{1}{w_t^{\xi}}+\alpha\left(1+\pi^{\xi\prime}\left(w_t^{\xi}\right)\left(s+u^{\xi}\right)^{\intercal}\Sigma s\right)}\left(\frac{-\alpha\pi^{\xi\prime}\left(w_t^{\xi}\right)}{\frac{1}{w_t^{\xi}}+\alpha\left(1+\pi^{\xi\prime}\left(w_t^{\xi}\right)\left(s+u^{\xi}\right)^{\intercal}\Sigma s\right)}\left(s+u^{\xi}\right)^{\intercal}\Sigma\left(s+u^{\xi}\right)+2\right)
\end{aligned}$$

we have simply

$$\Pi_t^{\xi} = \bar{F}^{\xi}\left(w_t^{\xi}\right)\Sigma\left(s+u^{\xi}\right)\left(s+u^{\xi}\right)^{\intercal}\Sigma + \Sigma$$

where $\bar{F}^{\xi}\left(w_t^{\xi}\right)$ is a scalar. For the inverse, note that by the Sherman Morrison formula

$$\left(\Sigma + vv^{\intercal}\right)^{-1} = \Sigma^{-1} - \frac{\Sigma^{-1}vv^{\intercal}\Sigma^{-1}}{1+v^{\intercal}\Sigma^{-1}v}.$$

Picking

$$v = \sqrt{\bar{F}^{\xi}\left(w_t\right)}\Sigma\left(s+u^{\xi}\right)$$

gives $\left(\Pi_t^{\xi}\right)^{-1} = \left(\Sigma + vv^{\intercal}\right)^{-1}$ and

$$\begin{aligned}
\left(\Pi_t^{\xi}\right)^{-1} &= \Sigma^{-1} - \frac{\Sigma^{-1}vv^{\intercal}\Sigma^{-1}}{1+v^{\intercal}\Sigma^{-1}v} \\
&= \Sigma^{-1} - \frac{\bar{F}^{\xi}\left(w_t^{\xi}\right)}{1+\bar{F}^{\xi}\left(w_t^{\xi}\right)\left(s+u^{\xi}\right)^{\intercal}\Sigma\left(s+u^{\xi}\right)}\left(s+u^{\xi}\right)\left(s+u^{\xi}\right)^{\intercal}.
\end{aligned}$$

Introducing the short-cut

$$F^{\xi}\left(w_t^{\xi}\right) \equiv \frac{\bar{F}^{\xi}\left(w_t^{\xi}\right)}{1+\bar{F}^{\xi}\left(w_t^{\xi}\right)\left(s+u^{\xi}\right)^{\intercal}\Sigma\left(s+u^{\xi}\right)}$$

and the definition of Ω_t^{ξ} givs that the inverse of the return covariance matrix has the form of

$$\left(\Omega_t^{\xi}\right)^{-1} = diag\left(P_t^{\xi}\right)\Sigma^{-1}diag\left(P_t^{\xi}\right) - F^{\xi}\left(w_t^{\xi}\right)diag\left(P_t^{\xi}\right)\left(s+u^{\xi}\right)\left(s+u^{\xi}\right)^{\intercal}diag\left(P_t^{\xi}\right)$$

Finally, Equation (IA148) gives equilibirum positions with the choice of $F'\left(w_t\right)=0$ as

$$Y_t = \frac{\alpha}{\alpha+\frac{1}{w_t^{\xi}}+\alpha\pi'\left(w_t^{\xi}\right)\left(s+u^{\xi}\right)^{\intercal}\Sigma s}\left(\sigma_{P_t}+\sigma\right)^{-1}\sigma\left(s+u\right)$$

then

$$\left(\hat{F}^{\xi}\left(w_t^{\xi}\right)\Sigma\left(s+u^{\xi}\right)\left(s+u^{\xi}\right)^{\intercal}\Sigma+\Sigma\right)Y_t = \frac{\alpha}{\alpha+\frac{1}{w_t^{\xi}}+\alpha\pi'\left(w_t^{\xi}\right)\left(s+u^{\xi}\right)^{\intercal}\Sigma s}\Sigma\left(s+u\right)$$

or

$$Y_t = \frac{\alpha}{\alpha+\frac{1}{w_t^{\xi}}+\alpha\pi'\left(w_t^{\xi}\right)\left(s+u^{\xi}\right)^{\intercal}\Sigma s}\left(\hat{F}^{\xi}\left(w_t^{\xi}\right)\Sigma\left(s+u^{\xi}\right)\left(s+u^{\xi}\right)^{\intercal}\Sigma+\Sigma\right)^{-1}\Sigma\left(s+u\right)$$

which gives the statement with the choice of

$$G^{\xi}\left(w_t^{\xi}\right) = \frac{\alpha}{\alpha+\frac{1}{w_t^{\xi}}+\alpha\pi'\left(w_t^{\xi}\right)\left(s+u^{\xi}\right)^{\intercal}\Sigma s}\frac{1}{1+\hat{F}^{\xi}\left(w_t^{\xi}\right)\left(s+u^{\xi}\right)^{\intercal}\Sigma\left(s+u^{\xi}\right)}.$$

□

First note, that in our example, $(s+u^s)^{\intercal}\Sigma(s+u^s) = (s+u^n)^{\intercal}\Sigma(s+u^n)$ and $(s+u^s)^{\intercal}\Sigma s = (s+u^n)^{\intercal}\Sigma s$ hence the Lemma implies that $\pi^{\xi}(\cdot), F^{\xi}(\cdot)$ and $B^{\xi}(\cdot)$ and $G^{\xi}(\cdot)$ are the same functions in the states $\xi = n, s$. Therefore, in the rest of the proof we omit the subscript ξ.

We are interested in
$$\left[\left((\Omega_t^s)^{-1}-(\Omega_t^n)^{-1}\right)\mathbb{E}\left(R_t^s\right)\right]_1$$
which we can write as a weighted combination of the terms

$$\left[\left((\Omega_t^s)^{-1}-(\Omega_t^n)^{-1}\right)\right]_{1j}.$$

So consider first

$$\left[\left(\left(\Omega_t^s\right)^{-1}-\left(\Omega_t^n\right)^{-1}\right)\right]_{11}=$$
$$=\left(\frac{\bar{D}}{r}-\pi\left(w_t^s\right)\sigma^2\left(s+\upsilon+\Delta\upsilon\right)\right)^2\left(\frac{1}{\sigma^2}-F\left(w_t^s\right)\left(s+\upsilon+\Delta\upsilon\right)^2\right)$$
$$-\left(\frac{\bar{D}}{r}-\pi\left(w_t^s\right)\sigma^2\left(s+\upsilon\right)\right)^2\left(\frac{1}{\sigma^2}-F\left(w_t^s\right)\left(s+\upsilon\right)^2\right)$$

Hence, to a first order we have

$$\left[\left(\left(\Omega_t^s\right)^{-1}-\left(\Omega_t^n\right)^{-1}\right)\right]_{11}\approx\frac{\partial\frac{\left(\frac{\bar{D}}{r}-\pi(w_t)\sigma^2(s+\upsilon)\right)^2}{\sigma^2}-F\left(w_t\right)\left(\frac{\bar{D}}{r}-\pi\left(w_t\right)\sigma^2\left(s+\upsilon\right)\right)^2\left(s+\upsilon\right)^2}{\partial w_t}\left(w_t^s-w_t^n\right)+$$
$$+\frac{\partial\frac{\left(\frac{\bar{D}}{r}-\pi(w_t)\sigma^2(s+\upsilon)\right)^2}{\sigma^2}-F\left(w_t\right)\left(\frac{\bar{D}}{r}-\pi\left(w_t\right)\sigma^2\left(s+\upsilon\right)\right)^2\left(s+\upsilon\right)^2}{\partial \upsilon}\Delta\upsilon$$

First, we show that the first element is of order $\left(\Delta\upsilon\right)^2$. Indeed

$$w_t^s-w_t^n=\hat{Y}_t^{n\intercal}\left(\left(-\pi\left(w_t^s\right)\right)\left(s+u^s\right)-\left(-\pi\left(w_t^n\right)\right)\left(s+u^n\right)\right)=$$
$$=G\left(w_t^n\right)\left(-\pi\left(w_t^s\right)\right)\left(s+u^n\right)^\intercal\Sigma\left(s+u^s\right)-G\left(w_t^n\right)\left(-\pi\left(w_t^n\right)\right)\left(s+u^n\right)^\intercal\Sigma\left(s+u^n\right)=$$
$$=G\left(w_t^n\right)\left(-\pi\left(w_t^s\right)\right)\sigma^2\left(\Delta\upsilon\right)^2+G\left(w_t^n\right)\left(\left(-\pi\left(w_t^s\right)\right)-\left(-\pi\left(w_t^n\right)\right)\right)\left(s+u^n\right)^\intercal\Sigma\left(s+u^n\right)$$

using

$$\left(\left(-\pi\left(w_t^s\right)\right)-\left(-\pi\left(w_t^n\right)\right)\right)\approx-\pi'\left(w_t^n\right)\left(w_t^s-w_t^n\right).$$

this implies

$$w_t^s-w_t^n\approx\frac{G\left(w_t^n\right)\left(-\pi\left(w_t^s\right)\right)}{1+G\left(w_t^n\right)\left(s+u^n\right)^\intercal\Sigma\left(s+u^n\right)\pi'\left(w_t^n\right)}\left(\left(s+u^n\right)^\intercal\Sigma\left(s+u^s\right)-\left(s+u^n\right)^\intercal\Sigma\left(s+u^n\right)\right)=$$
$$=-\frac{G\left(w_t^n\right)\left(-\pi\left(w_t^s\right)\right)}{1+G\left(w_t^n\right)\left(s+u^n\right)^\intercal\Sigma\left(s+u^n\right)\pi'\left(w_t^n\right)}\sigma^2\left(\Delta\upsilon\right)^2.$$

Therefore, we can replace w^n, w^s with its limit w_t. Hence,

$$\left[\left(\left(\Omega_t^s\right)^{-1}-\left(\Omega_t^n\right)^{-1}\right)\right]_{11}\approx\frac{\partial\frac{\left(\frac{\bar{D}}{r}-\pi(w_t)\sigma^2(s+\upsilon)\right)^2}{\sigma^2}-F\left(w_t\right)\left(\frac{\bar{D}}{r}-\pi\left(w_t\right)\sigma^2\left(s+\upsilon\right)\right)^2\left(s+\upsilon\right)^2}{\partial \upsilon}\Delta\upsilon=$$
$$=-2P_t\left(F\left(w_t\right)\left(s+\upsilon\right)P_t-\pi\left(w_t\right)\left(F\left(w_t\right)\left(s+\upsilon\right)\sigma^2\left(s+\upsilon\right)-1\right)\right)\Delta\upsilon$$

Note that

$$F\left(w_t\right)\left(s+\upsilon\right)\sigma^2\left(s+\upsilon\right)=\frac{\bar{F}\left(w_t\right)\left(s+\upsilon\right)\sigma^2\left(s+\upsilon\right)}{1+\bar{F}\left(w_t\right)\sigma^2\left(s+u^\xi\right)^\intercal\left(s+u^\xi\right)}<1$$

and that $\pi\left(w_t\right), P_t>0$ hence $\left[\left(\left(\Omega_t^s\right)^{-1}-\left(\Omega_t^n\right)^{-1}\right)\right]_{11}$ is always negative.

Now consider

$$\left[\left((\Omega_t^s)^{-1} - (\Omega_t^n)^{-1}\right)\right]_{1j}$$

First we consider the case when $j \leq I_1$, then

$$\left[\left((\Omega_t^s)^{-1} - (\Omega_t^n)^{-1}\right)\right]_{1j} =$$
$$= \left(-F(w_t)\left(\frac{\bar{D}}{r} - \pi(w_t)\sigma^2(s+\upsilon)\right)\left(\frac{\bar{D}}{r} - \pi(w_t)\sigma^2(s+\upsilon+\Delta\upsilon)\right)(s+\upsilon)(s+\upsilon+\Delta\upsilon)\right)$$
$$- \left(-F(w_t)\left(\frac{\bar{D}}{r} - \pi(w_t)\sigma^2(s+\upsilon)\right)\left(\frac{\bar{D}}{r} - \pi(w_t)\sigma^2(s+\upsilon)\right)(s+\upsilon)(s+\upsilon)\right)$$

Just as before, the effect of w_t is lower order, so we can write

$$\left[\left((\Omega_t^s)^{-1} - (\Omega_t^n)^{-1}\right)\right]_{1j} \approx \frac{\partial\left(-F(w_t)\left(\frac{\bar{D}}{r} - \pi(w_t)\sigma^2(s+\upsilon)\right)\left(\frac{\bar{D}}{r} - \pi(w_t)\sigma^2(s+\upsilon+\Delta\upsilon)\right)(s+\upsilon)(s+\upsilon+\Delta\upsilon)\right)}{\partial\Delta\upsilon}|_{\Delta\upsilon=0}\Delta\upsilon =$$
$$= -F(w_t)[P_t^s]_1(s+\upsilon)\left([P_t^s]_1 - \pi(w_t)\sigma^2(s+\upsilon)\right)\Delta\upsilon$$

Finally, we have the $I > j > I_1$assets, for those we can write

$$\left[\left((\Omega_t^s)^{-1} - (\Omega_t^n)^{-1}\right)\right]_{1k} \approx -\frac{\partial\left(-F(w_t)\left(\frac{\bar{D}}{r} - \pi(w_t)\sigma^2(s+\upsilon)\right)\left(\frac{\bar{D}}{r} - \pi(w_t)\sigma^2(s+\upsilon+\Delta\upsilon)\right)(s+\upsilon)(s+\upsilon+\Delta\upsilon)\right)}{\partial\Delta\upsilon}|_{\Delta\upsilon=0}(-\Delta\upsilon)$$
$$= -F(w_t)[P_t^s]_1(s+\upsilon)\left([P_t^s]_1 - \pi(w_t)\sigma^2(s+\upsilon)\right)\Delta\upsilon.$$

(It is easy to see that $\left[\left((\Omega_t^s)^{-1} - (\Omega_t^n)^{-1}\right)\right]_{1I} = 0$, as the role of the first and the last asset before and after the shock are symmetric.)

These terms are the same at the limit with uncertain sign. Then, using $\mathbb{E}_t(R_t^s) - r\mathbf{1} = \operatorname{diag}(P_t^s)^{-1} B(w_t^s)\Sigma(s+u^s)$ gives

$$\left[\left((\Omega_t^s)^{-1} - (\Omega_t^n)^{-1}\right)\right]_1 (\mathbb{E}_t(R_t^s) - r\mathbf{1}) =$$
$$= -2\left(F(w_t)(s+\upsilon)P_t - \pi(w_t)\left(F(w_t)(s+\upsilon)\sigma^2(s+\upsilon) - 1\right)\right)B(w_t)\sigma^2(s+\upsilon)\Delta\upsilon -$$
$$(I-2)F(w_t)(s+\upsilon)\left([P_t^s]_1 - \pi(w_t)\sigma^2(s+\upsilon)\right)B(w_t)\sigma^2(s+\upsilon)\Delta\upsilon =$$
$$= \left(-I(s+\upsilon)F(w_t)\frac{\bar{D}}{r} - 2\pi(w_t)\left(1 - \sigma^2 I(s+\upsilon)^2 F(w_t)\right)\right)B(w_t^s)\sigma^2(s+\upsilon)\Delta\upsilon$$

which we scale by $[\Delta\mathbb{E}_t(R)]_1 = B(w_t)\frac{\bar{D}}{r}\frac{\sigma^2}{P_t^2}\Delta\upsilon$ giving

$$-\frac{\left(I(s+\upsilon)\frac{\bar{D}}{r}F(w_t) + 2\pi(w_t)\left(1 - \sigma^2 I(s+\upsilon)^2 F(w_t)\right)\right)}{\frac{\bar{D}}{r}\frac{1}{P_t^2}}(s+\upsilon)$$

which is negative as

$$1-\sigma^{2}I\left(s+\upsilon\right)^{2}F\left(w_{t}\right)=1-\sigma^{2}I\left(s+\upsilon\right)^{2}\frac{\bar{F}\left(w_{t}\right)}{1+\bar{F}\left(w_{t}\right)\left(s+u^{\xi}\right)^{\intercal}\Sigma\left(s+u^{\xi}\right)}=$$
$$=1-\frac{\sigma^{2}I\left(s+\upsilon\right)^{2}\bar{F}\left(w_{t}\right)}{1+\bar{F}\left(w_{t}\right)\sigma^{2}I\left(s+\upsilon\right)^{2}}>0.$$

Finally, we turn to the results on the covariances. Note from Lemma A.1 that

$$[\Omega_{t}^{s}]_{1i}=\bar{F}^{s}\left(w_{t}^{s}\right)\sigma^{2}\left\{\begin{array}{ccc}\frac{(s+\upsilon+\Delta\upsilon)^{2}}{\left(\frac{\bar{D}}{r}+g(\omega_{t})\sigma^{2}(s+\upsilon+\Delta\upsilon)\right)^{2}} & for & i=1\\ \frac{(s+\upsilon+\Delta\upsilon)(s+\upsilon)}{\left(\frac{\bar{D}}{r}+g(\omega_{t})\sigma^{2}(s+\upsilon+\Delta\upsilon)\right)\left(\frac{\bar{D}}{r}+g(\omega_{t})\sigma^{2}(s+\upsilon)\right)} & for & I_{1}>i>1\\ \frac{(s+\upsilon+\Delta\upsilon)^{2}}{\left(\frac{\bar{D}}{r}+g(\omega_{t})\sigma^{2}(s+\upsilon+\Delta\upsilon)\right)^{2}} & for & I>i>I_{1}\\ \frac{(s+\upsilon+\Delta\upsilon)(s+\upsilon)}{\left(\frac{\bar{D}}{r}+g(\omega_{t})\sigma^{2}(s+\upsilon+\Delta\upsilon)\right)\left(\frac{\bar{D}}{r}+g(\omega_{t})\sigma^{2}(s+\upsilon)\right)} & for & i=I\end{array}\right\}$$

while

$$[\Omega_{t}^{n}]_{1i}=\bar{F}^{n}\left(w_{t}^{n}\right)\sigma^{2}\left\{\begin{array}{ccc}\frac{(s+\upsilon)^{2}}{\left(\frac{\bar{D}}{r}+g(\omega_{t})\sigma^{2}(s+\upsilon)\right)^{2}} & for & i=1\\ \frac{(s+\upsilon)(s+\upsilon)}{\left(\frac{\bar{D}}{r}+g(\omega_{t})\sigma^{2}(s+\upsilon)\right)\left(\frac{\bar{D}}{r}+g(\omega_{t})\sigma^{2}(s+\upsilon)\right)} & for & I_{1}>i>1\\ \frac{(s+\upsilon+\Delta\upsilon)(s+\upsilon)}{\left(\frac{\bar{D}}{r}+g(\omega_{t})\sigma^{2}(s+\upsilon+\Delta\upsilon)\right)\left(\frac{\bar{D}}{r}+g(\omega_{t})\sigma^{2}(s+\upsilon)\right)} & for & I>i>I_{1}\\ \frac{(s+\upsilon+\Delta\upsilon)(s+\upsilon)}{\left(\frac{\bar{D}}{r}+g(\omega_{t})\sigma^{2}(s+\upsilon+\Delta\upsilon)\right)\left(\frac{\bar{D}}{r}+g(\omega_{t})\sigma^{2}(s+\upsilon)\right)} & for & i=I\end{array}\right\}$$

Therefore, following the argument above, using the first order Taylor expension around $\Delta\upsilon=0$ and recognizing that $w_{t}^{n}-w_{t}^{s}$ is of order $\left(\Delta\upsilon\right)^{2}$, we have

$$\lim_{\Delta\upsilon\to0}\frac{[\Omega_{t}^{s}-\Omega_{t}^{n}]_{1i}}{\Delta\upsilon}=\bar{F}\left(w_{t}\right)\sigma^{2}\left\{\begin{array}{ccc}2\frac{\bar{D}}{r}\frac{(s+\upsilon)^{2}}{(P_{t}^{n})^{3}} & for & i=1\\ \frac{\bar{D}}{r}\frac{s+\upsilon}{(P_{t}^{n})^{3}} & for & I>i>1\\ 0 & for & i=I\end{array}\right\}$$

A.4 Proof of Proposition 5

Suppose that an investor is offered to buy any asset with a δ additional instantaneous expected return. The hypothetical dividend process is

$$dD_{t}^{h}=\bar{D}dt+\sigma^{\top}dB_{t}+(1-\chi)\operatorname{diag}(P_{t}^{n})\delta dt+\chi\operatorname{diag}(P_{t}^{n})\frac{\delta}{\lambda}dN_{t}^{h}. \tag{A.166}$$

Let $\epsilon\equiv\operatorname{diag}(P_{t}^{n})\delta$. Then

$$dD_{t}^{h}=\bar{D}dt+\sigma^{\top}dB_{t}+(1-\chi)\epsilon dt+\chi\frac{\epsilon}{\lambda}dN_{t}^{h}. \tag{A.167}$$

Given the investor's hypothetical dividend process, let $\hat{z}_t^h \equiv \hat{Y}_t^h / \hat{w}_t^h$ denote the individual investor's wealth-scaled asset holdings and let $\hat{c}_t^h$ denote the investor's consumption. The individual investor's wealth dynamic is

$$\begin{aligned} d\hat{w}_t^h &= r\hat{w}_t^h dt + \hat{w}_t^h \hat{z}_t^{h\top}(dD_t^h + dP_t^n - rP_t^n dt) - \hat{c}_t^h dt \\ &= \Big[r\hat{w}_t^h - \hat{c}_t^h + \hat{w}_t^h \hat{z}_t^{h\top}\big(\bar{D} + \mu_{Pt}^n - rP_t^n + (1-\chi)\epsilon\big)\Big]dt + \hat{w}_t^h \hat{z}_t^{h\top}(\sigma_{Pt}^n + \sigma)^\top dB_t \\ &\quad + \frac{\chi}{\lambda}\hat{w}_t^h \hat{z}_t^{h\top}\epsilon dN_t^h. \end{aligned} \tag{A.168}$$

The corresponding aggregate wealth of investors

$$dw_t^n = \Big[rw_t^n - c_t^n + w_t^n z_t^{n\top}(\bar{D} + \mu_{Pt}^n - rP_t^n)\Big]dt + w_t^n z_t^{n\top}(\sigma_{Pt}^n + \sigma)^\top dB_t. \tag{A.169}$$

A.4.1 Log Investors

The individual investor's HJB equation is

$$\begin{aligned} \rho V^h = \max_{\hat{c}_t^h, \hat{z}_t^h} \Big\{ &\frac{(\hat{c}_t^h)^{1-\gamma}}{1-\gamma} + V_{\hat{w}^h}^h \mu_t^{\hat{w}^h} + \frac{1}{2}V_{\hat{w}^h\hat{w}^h}^h (\sigma_t^{\hat{w}^h})^\top \sigma_t^{\hat{w}^h} + V_w^h \mu_t^{w,n} \\ &+ \frac{1}{2}V_{ww}^h (\sigma_t^{w,n})^\top \sigma_t^{w,n} + V_{\hat{w}^h w}^h (\sigma_t^{\hat{w}^h})^\top \sigma_t^{w,n} + \lambda(V^{n,+} - V^h)\Big\}, \end{aligned} \tag{A.170}$$

where

$$\mu_t^{\hat{w}^h} = r\hat{w}_t^h - \hat{c}_t^h + \hat{w}_t^h \hat{z}_t^{h\top}\big[\bar{D} + \mu_{Pt}^n - rP_t^n + (1-\chi)\epsilon\big], \tag{A.171}$$

$$\sigma_t^{\hat{w}^h} = (\sigma_{Pt}^n + \sigma)\hat{z}_t^h \hat{w}_t^h, \tag{A.172}$$

$$\mu_t^{w,n} = rw_t^n - c_t^n + w_t^n z_t^{n\top}(\bar{D} + \mu_{Pt}^n - rP_t^n), \tag{A.173}$$

$$\sigma_t^{w,n} = (\sigma_{Pt}^n + \sigma)z_t^n w_t^n, \tag{A.174}$$

and

$$V^{n,+} = V^n\left(\hat{w}_t^h + \frac{\chi}{\lambda}\hat{w}_t^h \hat{z}_t^{h\top}\epsilon, w_t^n\right) = \frac{1}{\rho}\log\left(\hat{w}_t^h + \frac{\chi}{\lambda}\hat{w}_t^h \hat{z}_t^{h\top}\epsilon\right) + \bar{q}^n(w_t^n). \tag{A.175}$$

Here, $\bar{q}^n(w_t^n)$ is solution to the ODEs given by Proposition 4.

We conjecture (and verify) that the investor's value function under the hypothetical scenario takes the form

$$V^h(\hat{w}_t^h, w_t^n) = \frac{1}{\rho}\log\hat{w}_t^h + \bar{q}^h(w_t^n). \tag{A.176}$$

Substituting into the HJB equation yields

$$
\begin{aligned}
&\log \hat{w}_t^h + \rho \bar{q}^h(w_t^n) = \\
&\max_{\hat{c}_t^h, \hat{z}_t^h} \Big\{ \log \hat{c}_t^h + \frac{1}{\rho \hat{w}_t^h} \mu_t^{\hat{w}^h} - \frac{1}{2\rho(\hat{w}_t^h)^2} (\sigma_t^{\hat{w}^h})^\top \sigma_t^{\hat{w}^h} + \bar{q}^{h\prime}(w_t^n)\mu_t^{w,n} + \frac{1}{2}\bar{q}^{h\prime\prime}(w_t^n)(\sigma_t^{w,n})^\top \sigma_t^{w,n} \\
&\qquad + \lambda \Big[\frac{1}{\rho} \log \Big(\hat{w}_t^h + \frac{\chi}{\lambda} \hat{w}_t^h \hat{z}_t^{h\top} \epsilon \Big) + \bar{q}^n(w_t^n) - \frac{1}{\rho} \log \hat{w}_t^h - \bar{q}^h(w_t^n) \Big] \Big\}.
\end{aligned}
\tag{A.177}
$$

Taking the first-order condition with respect to consumption yields the optimal consumption by the investor under the hypothetical scenario

$$
\hat{c}_t^h = \rho \hat{w}_t^h. \tag{A.178}
$$

Taking the first-order condition with respect to wealth-scaled asset holdings,

$$
0 = \bar{D} + \mu_{Pt}^n - rP_t^n + (1-\chi)\epsilon - (\sigma_{Pt}^n + \sigma)^\top (\sigma_{Pt}^n + \sigma)\hat{z}_t^h + \frac{\chi \epsilon}{1 + \frac{\chi}{\lambda} \hat{z}_t^{h\top} \epsilon}. \tag{A.179}
$$

Hence,

$$
\hat{z}_t^h = (\Pi_t^n)^{-1} \big[\bar{D} + \mu_{Pt}^n - rP_t^n + (1-\chi)\epsilon \big] + \frac{\chi}{1 + \frac{\chi}{\lambda} \hat{z}^{h\top} \epsilon} (\Pi_t^n)^{-1} \epsilon. \tag{A.180}
$$

From (A.178), the investors' aggregate consumption is given by $c_t^n = \rho w_t^n$. Then we can rewrite the HJB equation as

$$
\begin{aligned}
&\rho \bar{q}^h(w_t^n) = \log \rho + \frac{1}{\rho} \Big[r - \rho + \hat{z}_t^{h\top} \big(\bar{D} + \mu_{Pt}^n - rP_t^n + (1-\chi)\epsilon \big) \Big] \\
&- \frac{1}{2\rho} z_t^{h\top} (\sigma_{Pt}^n + \sigma)^\top (\sigma_{Pt}^n + \sigma) \hat{z}_t^h + \bar{q}^{h\prime}(w_t^n) w_t^n \Big[r - \rho + z_t^{n\top} (\bar{D} + \mu_{Pt}^n - rP_t^n) \Big] \\
&+ \frac{1}{2} \bar{q}^{h\prime\prime}(w_t^n)(w_t^n)^2 z_t^{n\top} (\sigma_{Pt}^n + \sigma)^\top (\sigma_{Pt}^n + \sigma) z_t^n \\
&+ \lambda \Big[\frac{1}{\rho} \log \Big(1 + \frac{\chi}{\lambda} z_t^{h\top} \epsilon \Big) + \bar{q}^n(w_t^n) - \bar{q}^h(w_t^n) \Big].
\end{aligned}
\tag{A.181}
$$

Note that $\hat{w}_t^h$ have all canceled out.

Since $\hat{y}_t^h = \mathrm{diag}(P_t^n)\hat{z}_t^h$, then

$$
\begin{aligned}
\hat{y}_t^h &= (\Omega_t^n)^{-1} \left[\mu_{Rt}^n - r\mathbf{1} + (1-\chi)\delta \right] + \frac{\lambda}{\lambda + \hat{y}^{h\top} \chi \delta} (\Omega_t^n)^{-1} \chi \delta \\
&= (\Omega_t^n)^{-1} (\mu_{Rt}^n - r\mathbf{1} + \delta) + \Big[\frac{\lambda}{\lambda + \hat{y}_t^{h\top} \chi \delta} - 1 \Big] (\Omega_t^n)^{-1} \chi \delta.
\end{aligned}
\tag{A.182}
$$

From (A.38), the investor's demand in the normal state is given by

$$
\hat{y}_t^n = (\Omega_t^n)^{-1} (\mu_{Rt}^n - r\mathbf{1}). \tag{A.183}
$$

The dynamic slope is thus

$$\begin{aligned}\mathcal{C}(w_t^n;\theta) &= \frac{[\hat{y}_t^h - \hat{y}_t^n]_i}{[\delta]_i} \\ &= \frac{[(\Omega_t^n)^{-1}\delta]_i}{[\delta]_i} + \Big[\frac{\lambda}{\lambda + \hat{y}_t^{h\top}\chi\delta} - 1\Big]\frac{[(\Omega_t^n)^{-1}\chi\delta]_i}{[\delta]_i} \\ &= [(\Omega_t^n)^{-1}]_{ii} + \frac{\sum_{j\neq i}[(\Omega_t^n)^{-1}]_{ij}[\delta]_j}{[\delta]_i} + \Big[\frac{\lambda}{\lambda + \hat{y}_t^{h\top}\chi\delta} - 1\Big]\frac{[(\Omega_t^n)^{-1}\chi\delta]_i}{[\delta]_i}.\end{aligned} \tag{A.184}$$

When $\lambda \to \infty$, the last component goes to zero. When $\lambda \to 0$ and $\chi = 0$, the last component is also zero in the limit.

A.4.2 CRRA Investors

The individual investor's HJB equation is

$$\begin{aligned}\rho V^h = \max_{\hat{c}_t^h, \hat{z}_t^h} \Big\{ &\frac{(\hat{c}_t^h)^{1-\gamma}}{1-\gamma} + V_{\hat{w}^h}^h \mu_t^{\hat{w}^h} + \frac{1}{2} V_{\hat{w}^h\hat{w}^h}^h (\sigma_t^{\hat{w}^h})^\top \sigma_t^{\hat{w}^h} + V_w^h \mu_t^{w,n} \\ &+ \frac{1}{2} V_{ww}^h (\sigma_t^{w,n})^\top \sigma_t^{w,n} + V_{\hat{w}^h w}^h (\sigma_t^{\hat{w}^h})^\top \sigma_t^{w,n} + \lambda(V^{n,+} - V^h)\Big\},\end{aligned} \tag{A.185}$$

where

$$\mu_t^{\hat{w}^h} = r\hat{w}_t^h - \hat{c}_t^h + \hat{w}_t^h \hat{z}_t^{h\top}\big[\bar{D} + \mu_{Pt}^n - rP_t^n + (1-\chi)\epsilon\big], \tag{A.186}$$

$$\sigma_t^{\hat{w}^h} = (\sigma_{Pt}^n + \sigma)\hat{z}_t^h \hat{w}_t^h, \tag{A.187}$$

$$\mu_t^{w,n} = rw_t^n - c_t^n + w_t^n z_t^{n\top}(\bar{D} + \mu_{Pt}^n - rP_t^n), \tag{A.188}$$

$$\sigma_t^{w,n} = (\sigma_{Pt}^n + \sigma) z_t^n w_t^n, \tag{A.189}$$

and

$$V^{n,+} = V^n\left(\hat{w}_t^h + \frac{\chi}{\lambda}\hat{w}_t^h \hat{z}_t^{h\top}\epsilon, w_t^n\right) = q^n(w_t^n)\frac{(\hat{w}_t^h + \frac{\chi}{\lambda}\hat{w}_t^h\hat{z}_t^{h\top}\epsilon)^{1-\gamma}}{1-\gamma}. \tag{A.190}$$

Here, $q^n(w_t^n)$ is solution to the ODEs given by Proposition 4.

We conjecture (and verify) that the investor's value function takes the form

$$V^h(\hat{w}_t^h, w_t^n) = q^h(w_t^n;\theta)\frac{(\hat{w}_t^h)^{1-\gamma}}{1-\gamma}. \tag{A.191}$$

Substituting into the HJB equation

$$
\begin{aligned}
&\rho q^h(w_t^n;\lambda,\delta,\chi)\frac{(\hat{w}_t^h)^{1-\gamma}}{1-\gamma} \\
&= \max_{\hat{c}_t^h,\hat{z}_t^h}\Big\{\frac{(\hat{c}_t^h)^{1-\gamma}}{1-\gamma} + q^h(w_t^n;\theta,)(\hat{w}_t^h)^{-\gamma}\mu_t^{\hat{w}^h} - \frac{\gamma}{2}q^h(w_t^n;\theta)(\hat{w}_t^h)^{-\gamma-1}(\sigma_t^{\hat{w}^h})^\top\sigma_t^{\hat{w}^h} \\
&\qquad + q^{h\prime}(w_t^n;\theta,)\frac{(\hat{w}_t^h)^{1-\gamma}}{1-\gamma}\mu_t^{w,n} + \frac{1}{2}q^{h\prime\prime}(w_t^n;\lambda,\delta\chi)\frac{(\hat{w}_t^h)^{1-\gamma}}{1-\gamma}(\sigma_t^{w,n})^\top\sigma_t^{w,n} \\
&\qquad + q^{h\prime}(w_t^n;\lambda,\delta,\chi)(\hat{w}_t^h)^{-\gamma}(\sigma_t^{\hat{w}^h})^\top\sigma_t^{w,n} \\
&\qquad + \lambda\Big[q^n(w_t^n)\frac{(\hat{w}_t^h + \frac{\chi}{\lambda}\hat{w}_t^h\hat{z}_t^{h\top}\epsilon)^{1-\gamma}}{1-\gamma} - q^h(w_t^n;\theta)\frac{(\hat{w}_t^h)^{1-\gamma}}{1-\gamma}\Big]\Big\}.
\end{aligned}
\tag{A.192}
$$

Taking the first-order condition with respect to consumption yields the optimal consumption by the investor as

$$
\hat{c}_t^h = q^h(w_t^n;\theta)^{-\frac{1}{\gamma}}\hat{w}_t^h \tag{A.193}
$$

Taking the first-order condition with respect to wealth-scaled asset holdings,

$$
\begin{aligned}
0 =& q^h(w_t^n;\lambda,\delta,\chi)\big[\bar{D} + \mu_{Pt}^n - rP_t^n + (1-\chi)\epsilon\big] \\
&- \gamma q^h(w_t^n;\lambda,\delta,\chi)(\sigma_{Pt}^n + \sigma)^\top(\sigma_{Pt}^n + \sigma)\hat{z}_t^h \\
&+ q^{h\prime}(w_t^n;\theta)(\sigma_{Pt}^n + \sigma)^\top(\sigma_{Pt}^n + \sigma)Y_t^n + \chi q^n(w_t^n)\Big(1 + \frac{\chi}{\lambda}\hat{z}_t^{h\top}\epsilon\Big)^{-\gamma}\epsilon.
\end{aligned}
\tag{A.194}
$$

Then

$$
\hat{z}_t^h = \frac{1}{\gamma}(\Pi_t^n)^{-1}\big[\bar{D} + \mu_{Pt}^n - rP_t^n + (1-\chi)\epsilon\big] + \frac{1}{\gamma}\frac{q^{h\prime}(w_t^n;\theta)}{q^h(w_t^n;\lambda,\delta,\chi)}Y_t^n \tag{A.195}
$$

$$
+ \frac{\chi}{\gamma}\frac{q^n(w_t^n)}{q^h(w_t^n;\theta)}(\Pi_t^n)^{-1}\Big(1 + \frac{\chi}{\lambda}\hat{z}_t^{h\top}\epsilon\Big)^{-\gamma}\epsilon. \tag{A.196}
$$

Moreover, investors' aggregate consumption is given by

$$
c_t^n = q^n(w_t^n)^{-\frac{1}{\gamma}}w_t^n. \tag{A.197}
$$

Then we can rewrite the HJB equation as

$$
\begin{aligned}
&\rho q^h(w_t^n;\theta)\frac{(\hat{w}_t^h)^{1-\gamma}}{1-\gamma} = q^h(w,t^n;\lambda,\delta\chi)^{1-\frac{1}{\gamma}}\frac{(\hat{w}_t^h)^{1-\gamma}}{1-\gamma} \\
&\quad + q^h(w_t^n;\theta)(\hat{w}_t^h)^{1-\gamma}\Big[r - q^h(w_t^n;\theta)^{-\frac{1}{\gamma}} + \hat{z}_t^{h\top}\big(\bar{D} + \mu_{Pt}^n - rP_t^n + (1-\chi)\epsilon\big)\Big] \\
&\quad - \frac{\gamma}{2}q^h(w_t^n;\theta)(\hat{w}_t^h)^{1-\gamma}\hat{z}_t^{h\top}(\sigma_{Pt}^n+\sigma)^\top(\sigma_{Pt}^n+\sigma)\hat{z}_t^h \\
&\quad + w_t^n q^{h\prime}(w_t^n;\theta)\frac{(\hat{w}_t^h)^{1-\gamma}}{1-\gamma}\Big[r - q^n(w_t^n)^{-\frac{1}{\gamma}} + z_t^{n\top}(\bar{D} + \mu_{Pt}^n - rP_t^n)\Big] \\
&\quad + \frac{1}{2}(w_t^n)^2 q^{h\prime\prime}(w_t^n;\lambda,\delta,\chi)\frac{(\hat{w}_t^h)^{1-\gamma}}{1-\gamma}z_t^{n\top}(\sigma_{Pt}^n+\sigma)^\top(\sigma_{Pt}^n+\sigma)z_t^n \\
&\quad + w_t^n q^{h\prime}(w_t^n;\lambda,\delta,\chi)(\hat{w}_t^h)^{1-\gamma}\hat{z}_t^{h\top}(\sigma_{Pt}^n+\sigma)^\top(\sigma_{Pt}^n+\sigma)z_t^n \\
&\quad + \lambda\Big[q^n(w_t^n)\frac{(\hat{w}_t^h)^{1-\gamma}(1+\frac{\chi}{\lambda}\hat{z}_t^{h\top}\epsilon)^{1-\gamma}}{1-\gamma} - q^h(w_t^n;\theta)\frac{(\hat{w}_t^h)^{1-\gamma}}{1-\gamma}\Big].
\end{aligned}
\tag{A.198}
$$

Dividing both sides by $q^h(w_t^n;\theta)\frac{(\hat{w}_t^h)^{1-\gamma}}{1-\gamma}$ yields

$$
\begin{aligned}
&q^h(w_t^n;\theta)^{-\frac{1}{\gamma}} + (1-\gamma)\Big[r - q^h(w_t^n;\lambda,\delta,\chi)^{-\frac{1}{\gamma}} + \hat{z}_t^{h\top}\big(\bar{D} + \mu_{Pt}^n - rP_t^n + (1-\chi)\epsilon\big)\Big] \\
&+ w_t^n\frac{q^{h\prime}(w_t^n;\theta)}{q^h(w_t^n;\theta)}\Big[r - q^n(w_t^n)^{-\frac{1}{\gamma}} + z_t^{n\top}(\bar{D} + \mu_{Pt}^n - rP_t^n)\Big] \\
&- \frac{\gamma(1-\gamma)}{2}\hat{z}^{h\top}(\sigma_{Pt}^n+\sigma)^\top(\sigma_{Pt}^n+\sigma)\hat{z}_t^h \\
&+ \frac{1}{2}(w_t^n)^2\frac{q^{h\prime\prime}(w_t^n;\lambda,\delta,\chi)}{q^h(w_t^n;\theta)}z_t^{n\top}(\sigma_{Pt}^n+\sigma)^\top(\sigma_{Pt}^n+\sigma)z_t^n \\
&+ (1-\gamma)w_t^n\frac{q^{h\prime}(w_t^n;\theta)}{q^h(w_t^n;\theta)}\hat{z}^{h\top}(\sigma_{Pt}^n+\sigma)^\top(\sigma_{Pt}^n+\sigma)z_t^n \\
&+ \lambda\Big[\frac{q^n(w_t^n)}{q^h(w_t^n;\lambda,\delta,\chi)}\Big(1+\frac{\chi}{\lambda}\hat{z}_t^{h\top}\epsilon\Big)^{1-\gamma} - 1\Big] = \rho
\end{aligned}
\tag{A.199}
$$

Rearranging yields

$$
\begin{aligned}
&q^h(w_t^n;\theta)^{-\frac{1}{\gamma}} \\
&\quad + (1-\gamma)\Big[r - q^h(w_t^n;\lambda,\delta,\chi)^{-\frac{1}{\gamma}} + \hat{z}_t^{h\top}\big(\bar{D} + \mu_{Pt}^n - rP_t^n + (1-\chi)\epsilon\big)\Big] \\
&\quad + w_t^n\frac{q^{h\prime}(w_t^n;\theta)}{q^h(w_t^n;\theta)}\Big[r - q^n(w_t^n)^{-\frac{1}{\gamma}} + z_t^{n\top}(\bar{D} + \mu_{Pt}^n - rP_t^n) + (1-\gamma)\hat{z}_t^{h\top}\Pi_t^n z_t^n\Big] \\
&\quad - \frac{\gamma(1-\gamma)}{2}\hat{z}_t^{h\top}\Pi_t^n\hat{z}_t^h + \frac{1}{2}(w_t^n)^2\frac{q^{h\prime\prime}(w_t^n;\theta)}{q^h(w_t^n;\theta)}z_t^{n\top}\Pi_t^n z_t^n \\
&\quad + \lambda\Big[\frac{q^n(w_t^n)}{q^h(w_t^n;\theta)}\Big(1+\frac{\chi}{\lambda}\hat{z}_t^{h\top}\epsilon\Big)^{1-\gamma} - 1\Big] = \rho.
\end{aligned}
\tag{A.200}
$$

Since

$$\bar{D} + \mu_{Pt}^n - rP_t^n = \frac{\Gamma^n(w_t^n)\hat{\sigma}_t^{n\top}\sigma z_t^n}{[1+\pi^{n\prime}(w_t^n)^\top z_t^n w_t^n]^2}, \tag{A.201}$$

$$z_t^{n\top}(\bar{D} + \mu_{Pt}^n - rP_t^n) = \frac{\Gamma^n(w_t^n) z_t^{n\top}\Sigma z_t^n}{[1+\pi^{n\prime}(w_t^n)^\top z_t^n w_t^n]^2}, \tag{A.202}$$

$$\sigma_{Pt}^n + \sigma = \frac{\hat{\sigma}_t^n}{1+\pi^{n\prime}(w_t^n)^\top z_t^n w_t^n}, \tag{A.203}$$

where

$$\hat{\sigma}_t^n \equiv -w_t^n \sigma z_t^n \pi^{n\prime}(w_t^n)^\top + [1 + w_t^n \pi^{n\prime}(w_t^n)^\top z_t^n]\sigma. \tag{A.204}$$

Hence, the ODE becomes

$$\begin{aligned}
&q^h(w_t^n;\theta)^{-\frac{1}{\gamma}} + (1-\gamma)\Big[r - q^h(w_t^n;\theta)^{-\frac{1}{\gamma}} + \hat{z}^{h\top}\Big(\frac{\Gamma^n(w_t^n)\hat{\sigma}_t^{n\top}\sigma z_t^n}{[1+\pi^{n\prime}(w_t^n)^\top z_t^n w_t^n]^2} + (1-\chi)\epsilon\Big)\Big] \\
&+ w_t^n \frac{q^{h\prime}(w_t^n;\theta)}{q^h(w_t^n;\theta)}\Big[r - q^n(w_t^n)^{-\frac{1}{\gamma}} + \frac{\Gamma^n(w_t^n) z_t^{n\top}\Sigma z_t^n}{[1+\pi^{n\prime}(w_t^n)^\top z_t^n w_t^n]^2} + (1-\gamma)\frac{\hat{z}_t^{h\top}\hat{\sigma}_t^{n\top}\sigma z_t^n}{[1+\pi^{n\prime}(w_t^n)^\top z_t^n w_t^n]^2}\Big] \\
&- \frac{\gamma(1-\gamma)}{2}\frac{\hat{z}_t^{h\top}\hat{\sigma}_t^{n\top}\hat{\sigma}_t^n \hat{z}_t^h}{[1+\pi^{n\prime}(w_t^n)^\top z_t^n w_t^n]^2} + \frac{1}{2}(w_t^n)^2 \frac{q^{h\prime\prime}(w_t^n;\theta)}{q^h(w_t^n;\theta)}\frac{z_t^{n\top}\Sigma z_t^n}{[1+\pi^{n\prime}(w_t^n)^\top z_t^n w_t^n]^2} \\
&+ \lambda\Big[\frac{q^n(w_t^n)}{q^h(w_t^n;\theta)}\Big(1 + \frac{\chi}{\lambda}\hat{z}^{h\top}\epsilon\Big)^{1-\gamma} - 1\Big] = \rho.
\end{aligned} \tag{A.205}$$

Moreover,

$$\begin{aligned}
\hat{z}_t^h =& \frac{1}{\gamma}(\hat{\sigma}_t^{n\top}\hat{\sigma}_t^n)^{-1}\Big\{\Gamma^n(w_t^n)\hat{\sigma}_t^{n\top}\sigma z_t^n + [1+\pi^{n\prime}(w_t^n)^\top z_t^n w_t^n]^2(1-\chi)\epsilon\Big\} \\
&+ \frac{q^{h\prime}(w_t^n;\theta,)}{\gamma q^h(w_t^n;\lambda,\delta\chi)} z_t^n w_t^n \\
&+ \frac{\chi}{\gamma}\frac{q^n(w_t^n)}{q^h(w_t^n;\theta)}[1+\pi^{n\prime}(w_t^n)^\top z_t^n w_t^n]^2(\hat{\sigma}_t^{n\top}\hat{\sigma}_t^n)^{-1}\Big(1 + \frac{\chi}{\lambda}\hat{z}_t^{h\top}\epsilon\Big)^{-\gamma}\epsilon.
\end{aligned} \tag{A.206}$$

That is,

$$\begin{aligned}
\hat{z}_t^h =& \Big[\frac{1}{\gamma}\Gamma^n(w_t^n) + \frac{q^{h\prime}(w_t^n;\theta)}{\gamma q^h(w_t^n;\theta)} w_t^n\Big] z_t^n + \frac{1-\chi}{\gamma}[1+\pi^{n\prime}(w_t^n)^\top z_t^n w_t^n]^2(\hat{\sigma}_t^{n\top}\hat{\sigma}_t^n)^{-1}\epsilon \\
&+ \frac{\chi}{\gamma}\frac{q^n(w_t^n)}{q^h(w_t^n;\theta)}[1+\pi^{n\prime}(w_t^n)^\top z_t^n w_t^n]^2(\hat{\sigma}_t^{n\top}\hat{\sigma}_t^n)^{-1}\Big(1 + \frac{\chi}{\lambda}\hat{z}_t^{h\top}\epsilon\Big)^{-\gamma}\epsilon.
\end{aligned} \tag{A.207}$$

A.5 Proof of Proposition 6

With (5.2), we can restate(42) as

$$\begin{aligned}\mathcal{C}^{\text{dyn}}\left(w_t^n;\theta\right) =& \frac{[\hat{y}_t^h-\hat{y}_t^n]_i}{\delta_i} = \mathcal{C}^{\text{myo}}(w_t^n;\delta) \\ &+\frac{1}{\gamma}\Big[\frac{q^{h\prime}(w_t^n;\theta)}{q^h(w_t^n;\theta)}-\frac{q^{n\prime}(w_t^n)}{q^n(w_t^n)}\Big]\frac{[y_t^n w_t^n]_i}{[\delta]_i} \\ &+\frac{1}{\gamma}\Big[\frac{q^n(w_t^n)}{q^h(w_t^n;\theta)}\Big(1+\hat{y}_t^{h\top}\frac{\chi\delta}{\lambda}\Big)^{-\gamma}-1\Big]\frac{[(\Omega_t^n)^{-1}\chi\delta]_i}{[\delta]_i}.\end{aligned} \tag{A.208}$$

First, we show that the second component (Brownian hedging) of (A.208) goes to zero as $\lambda\to\infty$. Equivalently, we want to show that

$$\lim_{\lambda\to\infty}\frac{q^{h\prime}(w_t^n;\theta)}{q^h(w_t^n;\theta)}-\frac{q^{n\prime}(w_t^n)}{q^n(w_t^n)}=0. \tag{A.209}$$

We only need to show that

$$\left(\frac{q^h(w_t^n;\theta)}{q^n(w_t^n)}\right)'_{w^n}=0, \tag{A.210}$$

that is the limiting ratio does not depend on w_t^n.

Restating the ODE (A.205) as

$$\begin{aligned}&q^h(w_t^n;\theta)^{-\frac{1}{\gamma}}+(1-\gamma)\Big[r-q^h(w_t^n;\theta)^{-\frac{1}{\gamma}}+\hat{z}^{h\top}\Big(\frac{\Gamma^n(w_t^n)\hat{\sigma}_t^{n\top}\sigma z_t^n}{[1+\pi^{n\prime}(w_t^n)^\top z_t^n w_t^n]^2}+(1-\chi)\epsilon\Big)\Big] \\ &+w_t^n\frac{q^{h\prime}(w_t^n;\theta)}{q^h(w_t^n;\theta)}\Big[r-q^n(w_t^n)^{-\frac{1}{\gamma}}+\frac{\Gamma^n(w_t^n)z_t^{n\top}\Sigma z_t^n}{[1+\pi^{n\prime}(w_t^n)^\top z_t^n w_t^n]^2}+(1-\gamma)\frac{\hat{z}_t^{h\top}\hat{\sigma}_t^{n\top}\sigma z_t^n}{[1+\pi^{n\prime}(w_t^n)^\top z_t^n w_t^n]^2}\Big] \\ &-\frac{\gamma(1-\gamma)}{2}\frac{\hat{z}_t^{h\top}\hat{\sigma}_t^{n\top}\hat{\sigma}_t^n\hat{z}_t^h}{[1+\pi^{n\prime}(w_t^n)^\top z_t^n w_t^n]^2}+\frac{1}{2}(w_t^n)^2\frac{q^{h\prime\prime}(w_t^n;\theta)}{q^h(w_t^n;\theta)}\frac{z_t^{n\top}\Sigma z_t^n}{[1+\pi^{n\prime}(w_t^n)^\top z_t^n w_t^n]^2} \\ &+\lambda\Big[\frac{q^n(w_t^n)}{q^h(w_t^n;\theta)}\Big(1+\frac{\chi}{\lambda}\hat{z}^{h\top}\epsilon\Big)^{1-\gamma}-1\Big]=\rho.\end{aligned} \tag{A.211}$$

Define $\varepsilon\equiv 1/\lambda$. Then the left-hand side of the ODE (A.205) can be rewritten as the sum of a collection of terms that are independent of ε and the jump term that depends on ε. That is,

$$\frac{1}{\varepsilon}\Big[\frac{q^n(w_t^n)}{q^h(w_t^n;\theta)}\Big(1+\varepsilon\chi\hat{z}_t^{h\top}\epsilon\Big)^{1-\gamma}-1\Big]+O(1)=\rho. \tag{A.212}$$

For small ε (or equivalently, large λ), we can apply the following binomial expansion

$$\Big(1+\varepsilon\chi\hat{z}_t^{h\top}\epsilon\Big)^{1-\gamma}=1+(1-\gamma)\varepsilon z_t^{h\top}\epsilon+O\left(\varepsilon^2\right). \tag{A.213}$$

Thus, the first term on the left-hand side of (A.212) can be written as

$$\begin{aligned}&\frac{1}{\varepsilon}\left[\frac{q^n(w_t^n)}{q^h(w_t^n;\theta)}\Big(1+(1-\gamma)\varepsilon\hat{z}_t^{h\top}\epsilon+O(\varepsilon^2)\Big)-1\right]\\&=\frac{1}{\varepsilon}\left[\frac{q^n(w_t^n)}{q^h(w_t^n;\theta)}-1\right]+(1-\gamma)\frac{q^n(w_t^n)}{q^h(w_t^n;\theta)}\hat{z}_t^{h\top}\epsilon+O(\varepsilon).\end{aligned} \tag{A.214}$$

When $\lambda\to\infty$, $\varepsilon\to 0$. The only way the left-hand side of the ODE does not diverge as $\varepsilon\to 0$ is that

$$\frac{q^n(w_t^n)}{q^h(w_t^n;\theta)}-1=O(\varepsilon). \tag{A.215}$$

Therefore,

$$\lim_{\lambda\to\infty}\frac{q^n(w_t^n)}{q^h(w_t^n;\theta)}=1 \tag{A.216}$$

for all w_t^n. Thus, the limiting ratio is a constant and does not depend on w_t^n. Hence, we have proved that the Brownian hedging component of (A.208) goes to zero as $\lambda\to\infty$.

Next, we want to show that the last component (hedging against reversal) also goes to zero when $\lambda\to\infty$). Equivalently, we want to show that

$$\lim_{\lambda\to\infty}\frac{q^n(w_t^n)}{q^h(w_t^n;\theta)}\left(1+\hat{y}_t^{h\top}\frac{\chi\delta}{\lambda}\right)^{-\gamma}-1=0. \tag{A.217}$$

Given χ and δ,

$$\lim_{\lambda\to\infty}\left(1+\hat{y}^{h\top}\frac{\chi\delta}{\lambda}\right)^{-\gamma}=1. \tag{A.218}$$

Combined with (A.216), it is thus immediate that (A.217) holds. Therefore,

$$\lim_{\lambda\to\infty}\mathcal{C}^{\mathrm{dyn}}(w_t^n;\theta)=\mathcal{C}^{\mathrm{myo}}(w_t^n;\delta). \tag{A.219}$$

B Connection to HHHKL

As explained in Section 6.2, we have

$$\sigma_{Pt}^{\xi}=\frac{\bar{\sigma}}{1-P^{\xi\prime}(w_t^{\xi})^\top Y_t^{\xi}}Y_t^{\xi}P^{\xi\prime}(w_t^{\xi})^\top. \tag{B.220}$$

Then

$$\sigma_{Pt}^{\xi}+\sigma=\bar{\sigma}\left[\mathbf{I}+\frac{Y_t^{\xi}P^{\xi\prime}(w_t^{\xi})^\top}{1-P^{\xi\prime}(w_t^{\xi})^\top Y_t^{\xi}}\right]. \tag{B.221}$$

Hence, the endogenous variance-covariance matrix of dollar returns can be written as

$$\begin{aligned}\Pi_t^\xi &\equiv (\sigma_{Pt}^\xi + \sigma)^\top (\sigma_{Pt}^\xi + \sigma) \\ &= \bar{\sigma}^2 \left[\mathbf{I} + \frac{Y_t^\xi P^{\xi\prime}(w_t^\xi)^\top}{1 - P^{\xi\prime}(w_t^\xi)^\top Y_t^\xi}\right]^\top \left[\mathbf{I} + \frac{Y_t^\xi P^{\xi\prime}(w_t^\xi)^\top}{1 - P^{\xi\prime}(w_t^\xi)^\top Y_t^\xi}\right] \\ &= \bar{\sigma}^2 \left[\mathbf{I} + \frac{P^{\xi\prime}(w_t^\xi) Y_t^{\xi\top}}{1 - P^{\xi\prime}(w_t^\xi)^\top Y_t^\xi}\right] \left[\mathbf{I} + \frac{Y_t^\xi P^{\xi\prime}(w_t^\xi)^\top}{1 - P^{\xi\prime}(w_t^\xi)^\top Y_t^\xi}\right].\end{aligned} \tag{B.222}$$

Taking the inverse of Π_t^ξ and using the Sherman-Morrison formula, we get

$$\begin{aligned}(\Pi_t^\xi)^{-1} &= \frac{1}{\bar{\sigma}^2} \left[\mathbf{I} + \frac{Y_t^\xi P^{\xi\prime}(w_t^\xi)^\top}{1 - P^{\xi\prime}(w_t^\xi)^\top Y_t^\xi}\right]^{-1} \left[\mathbf{I} + \frac{P^{\xi\prime}(w_t^\xi) Y_t^{\xi\top}}{1 - P^{\xi\prime}(w_t^\xi)^\top Y_t^\xi}\right]^{-1} \\ &= \frac{1}{\bar{\sigma}^2} \left[\mathbf{I} - Y_t^\xi P^{\xi\prime}(w_t^\xi)^\top\right] \left[\mathbf{I} - P^{\xi\prime}(w_t^\xi) Y_t^{\xi\top}\right] \\ &= \frac{1}{\bar{\sigma}^2} \left[\mathbf{I} - P^{\xi\prime}(w_t^\xi) Y_t^{\xi\top} - Y_t^\xi P^{\xi\prime}(w_t^\xi)^\top + P^{\xi\prime}(w_t^\xi)^\top P^{\xi\prime}(w_t^\xi) Y_t^\xi Y_t^{\xi\top}\right].\end{aligned} \tag{B.223}$$

We can define

$$\boldsymbol{\beta} \equiv \begin{bmatrix} Y_t^\xi & P^{\xi\prime}(w_t^\xi) \end{bmatrix}_{I\times 2} \tag{B.224}$$

and

$$\mathcal{E}_{sub} \equiv \begin{bmatrix} P^{\xi\prime}(w_t^\xi)^\top P^{\xi\prime}(w_t^\xi) & -1 \\ -1 & 0 \end{bmatrix}_{2\times 2} \tag{B.225}$$

Then it is easy to verify that

$$(\Pi_t^\xi)^{-1} = \frac{1}{\bar{\sigma}^2}\left(\mathbf{I} + \boldsymbol{\beta}\mathcal{E}_{sub}\boldsymbol{\beta}^\top\right) \tag{B.226}$$

Substituting into the investor's individual demand (48), we get

$$\begin{aligned}\hat{Y}_t^\xi =& \frac{1}{\gamma^A(w_t^\xi)\bar{\sigma}^2}(\mathbf{I} + \boldsymbol{\beta}\mathcal{E}_{sub}\boldsymbol{\beta}^\top)(\bar{D} + \mu_{Pt}^\xi - rP_t^\xi) + \frac{1}{\gamma^A(w_t^\xi)} \frac{q^{\xi\prime}(w_t^\xi)}{q^\xi(w_t^\xi)} Y_t^\xi \\ &+ \frac{\lambda^\xi}{\gamma^A(w_t^\xi)}(\Pi_t^\xi)^{-1} \frac{q^{-\xi}(w_t^\xi + Y_t^{\xi\top} J_t^\xi)}{q^\xi(w_t^\xi)\left(1 + \frac{1}{\hat{w}_t^\xi}\hat{Y}_t^{\xi\top} J_t^\xi\right)^\gamma} J_t^\xi.\end{aligned} \tag{B.227}$$

B.1 Factor Structure for Variance-Covariance Matrix

We now add another factor structure for the variance-covariance matrix of cash flows. Specifically, we suppose

$$\Sigma \equiv \sigma^\top \sigma = \bar{\sigma}^2(\mathbf{I} + \psi\psi^\top), \tag{B.228}$$

where $\bar{\sigma}$ is a scalar, $\mathbf{I}$ is an $I \times I$ identity matrix and ψ is $I \times K$. By eigen-decomposition, the diffusion σ can be written as

$$\sigma = \bar{\sigma}(\mathbf{I} + \nu\Lambda\nu^\top), \tag{B.229}$$

where ν are the eigenvectors of $\psi\psi^\top$ corresponding to its non-zero eigenvalues (size $I \times K$), and $\Lambda = \mathrm{diag}(\lambda_1, \dots, \lambda_K)$ where $\lambda_i = -\bar{\sigma} + \sqrt{\bar{\sigma}^2 + s_i^2}$ where s_i^2 denotes eigenvalues of $\psi\psi^\top$.

Then the endogenous variance-covariance matrix of dollar returns can be written as

$$\begin{aligned}
\Pi_t^\xi &\equiv (\sigma_{Pt}^\xi + \sigma)^\top (\sigma_{Pt}^\xi + \sigma) \\
&= \bar{\sigma}^2 \left[\mathbf{I} + \nu\Lambda\nu^\top + \frac{Y_t^\xi P^{\xi\prime}(w_t^\xi)^\top}{1 - P^{\xi\prime}(w_t^\xi)^\top Y_t^\xi} \right]^\top \left[\mathbf{I} + \nu\Lambda\nu^\top + \frac{Y_t^\xi P^{\xi\prime}(w_t^\xi)^\top}{1 - P^{\xi\prime}(w_t^\xi)^\top Y_t^\xi} \right] \\
&= \bar{\sigma}^2 \left[\mathbf{I} + \nu\Lambda\nu^\top + \frac{P^{\xi\prime}(w_t^\xi) Y_t^{\xi\top}}{1 - P^{\xi\prime}(w_t^\xi)^\top Y_t^\xi} \right] \left[\mathbf{I} + \nu\Lambda\nu^\top + \frac{Y_t^\xi P^{\xi\prime}(w_t^\xi)^\top}{1 - P^{\xi\prime}(w_t^\xi)^\top Y_t^\xi} \right].
\end{aligned} \tag{B.230}$$

Let $A \equiv \mathbf{I} + \nu\Lambda\nu^\top$. Taking the inverse of Π_t^ξ and using the Sherman-Morrison formula, we have

$$\begin{aligned}
(\Pi_t^\xi)^{-1} &= \frac{1}{\bar{\sigma}^2} \left[\mathbf{I} + \nu\Lambda\nu^\top + \frac{Y_t^\xi P^{\xi\prime}(w_t^\xi)^\top}{1 - P^{\xi\prime}(w_t^\xi)^\top Y_t^\xi} \right]^{-1} \left[\mathbf{I} + \nu\Lambda\nu^\top + \frac{P^{\xi\prime}(w_t^\xi) Y_t^{\xi\top}}{1 - P^{\xi\prime}(w_t^\xi)^\top Y_t^\xi} \right]^{-1} \\
&= \frac{1}{\bar{\sigma}^2} \left[A + \frac{Y_t^\xi P^{\xi\prime}(w_t^\xi)^\top}{1 - P^{\xi\prime}(w_t^\xi)^\top Y_t^\xi} \right]^{-1} \left[A + \frac{P^{\xi\prime}(w_t^\xi) Y_t^{\xi\top}}{1 - P^{\xi\prime}(w_t^\xi)^\top Y_t^\xi} \right]^{-1} \\
&= \frac{1}{\bar{\sigma}^2} \left[A^{-1} - \frac{A^{-1} Y_t^\xi P^{\xi\prime}(w_t^\xi)^\top A^{-1}}{1 - P^{\xi\prime}(w_t^\xi)^\top Y_t^\xi + P^{\xi\prime}(w_t^\xi)^\top A^{-1} Y_t^\xi} \right] \left[A^{-1} - \frac{A^{-1} P^{\xi\prime}(w_t^\xi) Y_t^{\xi\top} A^{-1}}{1 - P^{\xi\prime}(w_t^\xi)^\top Y_t^\xi + Y_t^{\xi\top} A^{-1} P^{\xi\prime}(w_t^\xi)} \right].
\end{aligned} \tag{B.231}$$

By Woodbury identity,

$$A^{-1} = \mathbf{I} - \nu(\Lambda^{-1} + \nu^\top\nu)^{-1}\nu^\top. \tag{B.232}$$

Substituting into above, we get

$$\begin{aligned}
(\Pi_t^\xi)^{-1} = \frac{1}{\bar{\sigma}^2} &\left[\mathbf{I} - \nu(\Lambda^{-1} + \nu^\top\nu)^{-1}\nu^\top - \frac{A^{-1} Y_t^\xi P^{\xi\prime}(w_t^\xi)^\top A^{-1}}{1 - P^{\xi\prime}(w_t^\xi)^\top Y_t^\xi + P^{\xi\prime}(w_t^\xi)^\top A^{-1} Y_t^\xi} \right] \\
&\left[\mathbf{I} - \nu(\Lambda^{-1} + \nu^\top\nu)^{-1}\nu^\top - \frac{A^{-1} P^{\xi\prime}(w_t^\xi) Y_t^{\xi\top} A^{-1}}{1 - P^{\xi\prime}(w_t^\xi)^\top Y_t^\xi + Y_t^{\xi\top} A^{-1} P^{\xi\prime}(w_t^\xi)} \right]
\end{aligned} \tag{B.233}$$

Let $d_1 \equiv 1 - P^{\xi\prime}(w_t^\xi)^\top Y_t^\xi + P^{\xi\prime}(w_t^\xi)^\top A^{-1} Y_t^\xi$ and $d_2 \equiv 1 - P^{\xi\prime}(w_t^\xi)^\top Y_t^\xi + Y_t^{\xi\top} A^{-1} P^{\xi\prime}(w_t^\xi)$.

Then,

$$
\begin{aligned}
(\Pi_t^\xi)^{-1} = \frac{1}{\bar{\sigma}^2}\Big\{ & \mathbf{I} - 2\nu(\Lambda^{-1}+\nu^\top\nu)^{-1}\nu^\top - \frac{A^{-1}P^{\xi\prime}(w_t^\xi)Y_t^{\xi\top}A^{-1}}{d_2} \\
& + \nu(\Lambda^{-1}+\nu^\top\nu)^{-1}\nu^\top\nu(\Lambda^{-1}+\nu^\top\nu)^{-1}\nu^\top + \frac{\nu(\Lambda^{-1}+\nu^\top\nu)^{-1}\nu^\top A^{-1}P^{\xi\prime}(w_t^\xi)Y_t^{\xi\top}A^{-1}}{d_2} \\
& - \frac{A^{-1}Y_t^\xi P^{\xi\prime}(w_t^\xi)^\top A^{-1}}{d_1} + \frac{A^{-1}Y_t^\xi P^{\xi\prime}(w_t^\xi)^\top A^{-1}\nu(\Lambda^{-1}+\nu^\top\nu)^{-1}\nu^\top}{d_1} \\
& + \frac{P^{\xi\prime}(w_t^\xi)^\top A^{-1}A^{-1}P^{\xi\prime}(w_t^\xi)}{d_1 d_2}A^{-1}Y_t^\xi Y^{\xi\top}A^{-1}\Big\}.
\end{aligned} \tag{B.234}
$$

Let us define

$$
\boldsymbol{\beta}^{full} \equiv \Big[\nu \quad A^{-1}Y_t^\xi \quad A^{-1}P^{\xi\prime}(w_t^\xi)\Big]_{I\times(K+2)} \tag{B.235}
$$

and

$$
\mathcal{E}^{full} \equiv \begin{bmatrix} (\Lambda^{-1}+\nu^\top\nu)^{-1}\nu^\top\nu(\Lambda^{-1}+\nu^\top\nu)^{-1} - 2(\Lambda^{-1}+\nu^\top\nu)^{-1} & \frac{(\Lambda^{-1}+\nu^\top\nu)^{-1}\nu^\top A^{-1}P^{\xi\prime}(w_t^\xi)}{d_2} & 0 \\ \frac{P^{\xi\prime}(w_t^\xi)^\top A^{-1}\nu(\Lambda^{-1}+\nu^\top\nu)^{-1}}{d_1} & \frac{P^{\xi\prime}(w_t^\xi)^\top A^{-1}A^{-1}P^{\xi\prime}(w_t^\xi)}{d_1 d_2} & -\frac{1}{d_1} \\ 0 & -\frac{1}{d_2} & 0 \end{bmatrix} \tag{B.236}
$$

which is of size $(K+2)\times(K+2)$. Then

$$
(\Pi_t^\xi)^{-1} = \mathbf{I} + \boldsymbol{\beta}^{full}\mathcal{E}^{full}\boldsymbol{\beta}^{full\top}. \tag{B.237}
$$

C The CARA case

We also consider the case with CARA investors. For these investors, their value functions are independent of the investors' total wealth w_t. Thus, w_t is not a state variable in this case. An investor maximizes

$$
-\mathbb{E}_t\Big[\int_t^\infty e^{-\rho(\tau-t)}e^{-\frac{\gamma}{r}\hat{c}_\tau}d\tau\Big], \tag{C.238}
$$

subject to the wealth dynamic

$$
d\hat{w}_t^\xi = r\hat{w}_t^\xi dt + \hat{Y}_t^{\xi\top}(dD_t - rP_t^\xi dt) - \hat{c}_t^\xi dt. \tag{C.239}
$$

Note that the asset prices depend only on the state of the economy ξ. That is, in a given state of the economy, the asset prices are constant. Thus, the investor's dollar excess returns from investing in the risky assets become $dD_t - rP_t^\xi dt$. Substituting (4) into the investor's wealth dynamic (C.239) yields

$$d\hat{w}_t^\xi = \Big[r\hat{w}_t^\xi - \hat{c}_t^\xi + \hat{Y}_t^{\xi\top}(\bar{D} - rP_t^\xi)\Big]dt + \hat{Y}_t^{\xi\top}\sigma^\top dB_t. \tag{C.240}$$

The investor's problem is to choose the optimal consumption and asset holdings $\{\hat{c}_t^\xi, \hat{Y}_t^\xi\}$ to maximize their utility (C.238) subject to the wealth dynamic (C.240) while taking prices P_t^ξ as given. Let $V^\xi(\hat{w}_t^\xi)$ denote the investor's value function in state ξ. The investor's HJB equation can be written as

$$\rho V^\xi = \max_{\hat{c}_t^\xi, \hat{Y}_t^\xi} \Big\{ - e^{-\frac{\gamma}{r}\hat{c}_t^\xi} + V_{\hat{w}}^\xi \mu_t^{\hat{w},\xi} + \frac{1}{2}V_{\hat{w}\hat{w}}^\xi (\sigma_t^{\hat{w},\xi})^\top \sigma_t^{\hat{w},\xi} + \lambda^\xi(V^{-\xi} - V^\xi)\Big\}, \tag{C.241}$$

where

$$\mu_t^{\hat{w},\xi} = r\hat{w}_t^\xi - \hat{c}_t^\xi + \hat{Y}_t^{\xi\top}(\bar{D} - rP_t^\xi), \tag{C.242}$$

$$\sigma_t^{\hat{w},\xi} = \sigma\hat{Y}_t^\xi. \tag{C.243}$$

We conjecture (and later verify) that the investor's value function takes the form

$$V^\xi(\hat{w}_t^\xi) = -e^{-\gamma\hat{w}_t^\xi} q^\xi, \tag{C.244}$$

where q^ξ is a constant. The long-term CARA investor's value function over wealth has the same negative-exponential form as the utility function over consumption, with the risk-aversion coefficient being γ rather than γ/r. Substituting the conjectured value function form into the HJB equation, we can rewrite the HJB equation as

$$\begin{aligned} -\rho e^{-\gamma\hat{w}_t^\xi} q^\xi = \max_{\hat{c}_t^\xi, \hat{Y}_t^\xi} \Big\{ & - e^{-\frac{\gamma}{r}\hat{c}_t^\xi} + \gamma e^{-\gamma\hat{w}_t^\xi} q^\xi \Big[r\hat{w}_t^\xi - \hat{c}_t^\xi + \hat{Y}_t^{\xi\top}(\bar{D} - rP_t^\xi)\Big] \\ & - \frac{1}{2}\gamma^2 e^{-\gamma\hat{w}_t^\xi} q^\xi \hat{Y}_t^{\xi\top}\Sigma\hat{Y}_t^\xi + \lambda^\xi\Big[- e^{-\gamma[\hat{w}_t^\xi + \hat{Y}_t^{\xi\top}(P_t^{-\xi} - P_t^\xi)]} q^{-\xi} + e^{-\gamma\hat{w}_t^\xi} q^\xi\Big]\Big\}. \end{aligned} \tag{C.245}$$

Taking the first-order condition with respect to $\hat{c}_t$ yields

$$\frac{\gamma}{r} e^{-\frac{\gamma}{r}\hat{c}_t^\xi} = \gamma e^{-\gamma\hat{w}_t^\xi} q^\xi, \tag{C.246}$$

which implies that the investor's optimal consumption $\hat{c}_t^\xi$ at time t if the economy is in state ξ is given by

$$\hat{c}_t^\xi = r\hat{w}_t^\xi - \frac{r}{\gamma}(\log q^\xi + \log r). \tag{C.247}$$

Taking the first-order condition with respect to $\hat{Y}_t^\xi$ yields

$$\begin{aligned} & \gamma e^{-\gamma\hat{w}_t^\xi} q^\xi(\bar{D} - rP_t^\xi) - \gamma^2 e^{-\gamma\hat{w}_t^\xi} q^\xi \Sigma\hat{Y}_t^\xi + \lambda^\xi \gamma e^{-\gamma[\hat{w}_t^\xi + \hat{Y}_t^{\xi\top}(P_t^{-\xi} - P_t^\xi)]} q^{-\xi}(P_t^{-\xi} - P_t^\xi) \\ & = 0. \end{aligned} \tag{C.248}$$

Rearranging, we obtain the investor's optimal asset holdings $\hat{Y}_t^\xi$ as

$$\hat{Y}_t^\xi = \frac{1}{\gamma}\Sigma^{-1}(\bar{D} - rP_t^\xi) + \frac{\lambda^\xi}{\gamma}\frac{q^{-\xi}}{q^\xi}e^{-\gamma\hat{Y}_t^{\xi\top}(P_t^{-\xi}-P_t^\xi)}\Sigma^{-1}(P_t^{-\xi} - P_t^\xi). \tag{C.249}$$

The investor's individual demand can be written as

$$\begin{aligned}\hat{Y}_t^\xi =& \frac{1}{\gamma}\Sigma^{-1}[\bar{D} - rP_t^\xi + \lambda^\xi(P_t^{-\xi} - P_t^\xi)] \\ &+ \frac{\lambda^\xi}{\gamma}\Big[\frac{q^{-\xi}}{q^\xi}e^{-\gamma\hat{Y}_t^{\xi\top}(P_t^{-\xi}-P_t^\xi)} - 1\Big]\Sigma^{-1}(P_t^{-\xi} - P_t^\xi),\end{aligned} \tag{C.250}$$

where $\bar{D} - rP_t^\xi + \lambda^\xi(P_t^{-\xi} - P_t^\xi)$ corresponds to the investor's expected dollar excess returns at time t when the economy is in state ξ.

Substituting the investor's optimal consumption $\hat{c}_t^\xi$ given by (C.247) and the investor's optimal asset holdings $\hat{Y}_t^\xi$ given by (C.249) into the HJB equation, we have

$$\begin{aligned}-\rho =& r(\log q^\xi + \log r - 1) + \gamma\hat{Y}_t^{\xi\top}(\bar{D} - rP^\xi) - \frac{1}{2}\gamma^2\hat{Y}_t^{\xi\top}\Sigma\hat{Y}_t^\xi \\ &+ \lambda^\xi\Big[1 - \frac{q^{-\xi}}{q^\xi}e^{-\gamma\hat{Y}_t^{\xi\top}(P_t^{-\xi}-P_t^\xi)}\Big]\end{aligned} \tag{C.251}$$

In equilibrium, because there is a unit measure of symmetric investors, the aggregate asset holdings by investors $Y_t^\xi = \hat{Y}_t^\xi$. Market clearing requires that

$$Q_t^\xi + Y_t^\xi = S \tag{C.252}$$

where Q_t^ξ equals hedgers' aggregate asset holdings, and is given by (A.6). From (A.6) and (C.249),

$$\begin{aligned}&\Big[\Sigma + \lambda^\xi(P_t^{-\xi} - P_t^\xi)(P_t^{-\xi} - P_t^\xi)^\top\Big]^{-1}\Big[\frac{\bar{D} - rP_t^\xi + \lambda^\xi(P_t^{-\xi} - P_t^\xi)}{\alpha} - \Sigma u^\xi\Big] \\ &+ \frac{1}{\gamma}\Sigma^{-1}(\bar{D} - rP_t^\xi) + \frac{\lambda^\xi}{\gamma}\frac{q^{-\xi}}{q^\xi}e^{-\gamma\hat{Y}_t^{\xi\top}(P_t^{-\xi}-P_t^\xi)}\Sigma^{-1}(P_t^{-\xi} - P_t^\xi) = S\end{aligned} \tag{C.253}$$

As explained earlier, the risky assets' prices depend only on the state of the economy ξ. Hence, we can define the state-dependent constant

$$\pi^\xi \equiv \frac{\bar{D}}{r} - P_t^\xi \tag{C.254}$$

Substituting (C.254) into (C.249), (C.251) and (C.253), and noting that $\hat{Y}_t^\xi = Y_t^\xi$, we have the following system of equations

$$Y_t^\xi = \frac{r}{\gamma}\Sigma^{-1}\pi^\xi + \frac{\lambda^\xi}{\gamma}\frac{q^{-\xi}}{q^\xi}e^{-\gamma Y_t^{\xi\top}(\pi^\xi-\pi^{-\xi})}\Sigma^{-1}(\pi^\xi - \pi^{-\xi}) \tag{C.255}$$

and

$$-\rho = r(\log q^{\xi} + \log r - 1) + r\gamma Y_t^{\xi\top}\pi^{\xi} - \frac{1}{2}\gamma^2 Y_t^{\xi\top}\Sigma Y_t^{\xi} + \lambda^{\xi}\Big[1 - \frac{q^{-\xi}}{q^{\xi}} e^{-\gamma Y_t^{\xi\top}(\pi^{\xi}-\pi^{-\xi})}\Big] \quad (C.256)$$

as well as

$$\begin{aligned} &\Big[\Sigma + \lambda^{\xi}(\pi^{\xi} - \pi^{-\xi})(\pi^{\xi} - \pi^{-\xi})^{\top}\Big]^{-1}\Big[\frac{r\pi^{\xi} + \lambda^{\xi}(\pi^{\xi} - \pi^{-\xi})}{\alpha} - \Sigma u^{\xi}\Big] \\ &+ \frac{r}{\gamma}\Sigma^{-1}\pi^{\xi} + \frac{\lambda^{\xi}}{\gamma}\frac{q^{-\xi}}{q^{\xi}} e^{-\gamma Y_t^{\xi\top}(\pi^{\xi}-\pi^{-\xi})}\Sigma^{-1}(\pi^{\xi} - \pi^{-\xi}) = S \end{aligned} \quad (C.257)$$

The equilibrium prices P_t^{ξ} of the risky assets at time t when the economy is in state ξ are

$$P_t^{\xi} = \frac{\bar{D}}{r} - \pi^{\xi}, \quad (C.258)$$

where $\xi \in \{n, s\}$. For all ξ, Y_t^{ξ}, π^{ξ} and q^{ξ} satisfy the sysem of equations

$$Y_t^{\xi} = \frac{r}{\gamma}\Sigma^{-1}\pi^{\xi} + \frac{\lambda^{\xi}}{\gamma}\frac{q^{-\xi}}{q^{\xi}} e^{-\gamma Y_t^{\xi\top}(\pi^{\xi}-\pi^{-\xi})}\Sigma^{-1}(\pi^{\xi} - \pi^{-\xi}), \quad (C.259)$$

and

$$-\rho = r(\log q^{\xi} + \log r - 1) + r\gamma Y_t^{\xi\top}\pi^{\xi} - \frac{1}{2}\gamma^2 Y_t^{\xi\top}\Sigma Y_t^{\xi} + \lambda^{\xi}\Big[1 - \frac{q^{-\xi}}{q^{\xi}} e^{-\gamma Y_t^{\xi\top}(\pi^{\xi}-\pi^{-\xi})}\Big], \quad (C.260)$$

as well as

$$\begin{aligned} &\Big[\Sigma + \lambda^{\xi}(\pi^{\xi} - \pi^{-\xi})(\pi^{\xi} - \pi^{-\xi})^{\top}\Big]^{-1}\Big[\frac{r\pi^{\xi} + \lambda^{\xi}(\pi^{\xi} - \pi^{-\xi})}{\alpha} - \Sigma u^{\xi}\Big] \\ &+ \frac{r}{\gamma}\Sigma^{-1}\pi^{\xi} + \frac{\lambda^{\xi}}{\gamma}\frac{q^{-\xi}}{q^{\xi}} e^{-\gamma Y_t^{\xi\top}(\pi^{\xi}-\pi^{-\xi})}\Sigma^{-1}(\pi^{\xi} - \pi^{-\xi}) = S. \end{aligned} \quad (C.261)$$

D The economics of the Myopic Slope vs. the Dynamic Slope

Figure D.1 explores the comparative statics of the three parameters (return size δ, persistence $1/\lambda$, and reversal share χ) for the two hedging components. Panel A and B of Figure D.1 plot the intertemporal hedging with respect to the (Poisson) reversal, which is relatively easier to explain. Investors understand that at some point the shift in residual supply will be reversed; when this occurs they realize a higher return, but, at the same time, their investment opportunity set gets worse. Investors hedge this by cutting their asset exposure, explaining the negative “reversal hedging.” Finally, it is intuitive that this negative effect gets stronger for larger shocks (larger δ) and larger reversal share (χ). To the extreme of $\chi = 0$ there is no hedging component due to reversal (Panel B).

Panel C and D then illustrate the hedging demand with respect to Brownian shocks, with

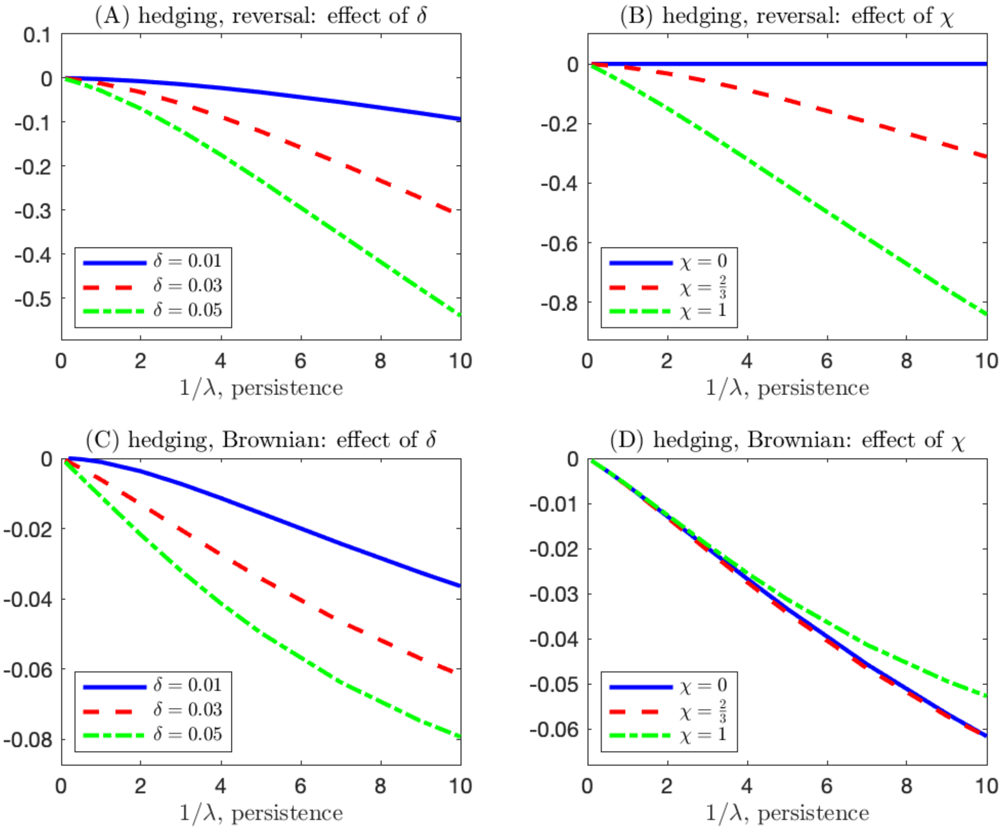

Figure D.1: Dynamic slope $\mathcal{C}$ and its decomposition for CRRA investors: myopic, Brownian hedging, and hedging against reversal components. We plot the decomposition for different size and persistence of the return shock. The parameter values are $\bar{D} = [0.27, 0.26]$, $\sigma = \text{diag}(0.485, 0.485)$, $\alpha = 1$, $\gamma = 3$, $r = 0.03$, $\rho = 0.08$, $s = [1, 1]$, and we consider $\delta = 0.01, 0.03, 0.05$ and $\chi = 0, \frac{2}{3}, 1$.

risk aversion $\gamma > 1$.[27] From (42), the sign of Brownian hedging depends on the sign

$$sgn\left\{\frac{q^{h\prime}\left(w_t^n\right)}{q^h\left(w_t^n\right)} - \frac{q^{n\prime}\left(w_t^n\right)}{q^n\left(w_t^n\right)}\right\} = sgn\left\{\left(\frac{q^h\left(w_t^n\right)}{q^n\left(w_t^n\right)}\right)'_{w^n}\right\}. \tag{D.262}$$

The simple expression in (D.262) captures the intuition of Brownian hedging. If the relative investment opportunities between two states $\frac{q^h(w_t^n)}{q^n(w_t^n)}$ decreases with the aggregate state w_t^n, then because the asset return is positively correlated with the Brownian shock, the investor should reduce her hedging demand of the asset with respect to (aggregate) Brownian shocks. Now, in line with Merton (1971) $q^n(w_t^n)$ is increasing in w_t^n for $\gamma > 1$,while the investment opportunity offered by the "deal" in Definition 2 features an extra expected return δ that is independent of w_t^n. To the extreme, when $\delta \to \infty$ the "deal" fully dominates the stochastic investment opportunity available in the economy and $q^h(w)$ becomes a constant. As a result,

[27] It is well known since Merton (1971) that all else equal the sign of this component flips if $\gamma < 1$.

$\frac{q^h(w_t^n)}{q^n(w_t^n)}$ tends to be decreasing in w, giving rise to a negative Brownian hedging demand.

Several additional points are worth making on the comparative statics revealed in Figure D.1. First, it is intuitive that when the reversal share χ gets larger the Brownian component gets smaller. Second, when $\lambda \to \infty$, the investor is as if receiving a fixed extra return δdt on his wealth over the interval $[t, t + dt]$ and then immediately followed by the equilibrium investment opportunity; hence $\frac{q^h(w)}{q^n(w)}$ should be a constant independent of w, which explains zero hedging demand Measured in all panels at the corner of when $1/\lambda \to 0$. Third, unlike the reversal component in Panel D, the Brownian component does not disappear as $\delta \to 0$ so that the dynamic slope stays strictly below the myopic one.

Finally, although in our later calibration in Section E.1 the effect of the (two) dynamic hedging components—which in sum is only about 1.4% of the dynamic slope—is quantitatively small,[28] from Figure D.1 they could be potentially large for a shock that is large and persistent.

E Details of Calibration and Sensitivity Analysis

In this part, we give details on the calibration exercises which are the basis of Section 6.1. In our first calibration, we calibrate to the study of Ben-David et al. (2021) focusing on factor-level elasticities. In the second one, we are interested in an economy where the residual supply shock affects only very few assets out of many. Hence, we consider the case of index inclusion calibrating our model to the recent study of Greenwood and Sammon (2024).

E.1 Fund flows and the Factor-Elasticity

In this part, we calibrate our model to Ben-David et al. (2021) study on factor-level elasticities. Then, using the obtained set of parameters as our baseline, we assess how our measures respond to a change of various parameters.

E.1.1 Baseline calibration

Our baseline parameters are obtained by calibrating our model to Ben-David et al. (2021). They show that when investment styles are highly correlated with Morningstar ratings, changes in fund ratings result in style-level fund flows which lead to price effect. In our calibration, we set $I = 2$. We interpret the two assets as two portfolios of different investment styles. In particular, we consider the first asset as the small value portfolio and the second asset as the large growth portfolio. The parameters $\bar{D}$ and σ pertain to the assets' fundamentals, and govern the first and second moments of portfolio returns. We set $\bar{D} = [0.27, 0.26]$ and $\sigma = diag(0.485, 0.485)$ to match the historical mean returns and return volatilities of the small value portfolio and the large growth portfolio as obtained from Ken French's data library.

We normalize the shocks in the normal state to zero, that is $u^n = [0, 0]$. We normalize the shock to the second asset to zero in the shock state and set $u^s = [0.15, 0]$ such that the model-implied difference in flows between the two assets (interpreted as the top and the

[28]It is about 0.05 out of circa 3.60 in the factor elasticity exercise, with calibrated parameters $1/\lambda = 2$, $\delta = 0.03$ and $\chi = 2/3$. See more details in Section E.1.

Incr. Return ($\Delta\mathbb{E}_t(R)$)	0.01	0.03	0.05	0.08	0.10
Myopic Slope	3.21	3.21	3.21	3.22	3.22
Dynamic Slope	3.20	3.17	3.13	3.05	3.00
Measured Slope	1.39	1.24	1.10	0.93	0.83
Measured Slope/Dynamic Slope	43%	39%	35%	30%	28%
Implied Price Elasticity	1.50	1.42	1.35	1.24	1.17
Dynamic minus Measured (DmM)	1.81	1.93	2.02	2.13	2.17
Shock to risk/DmM	114%	112%	111%	111%	112%
Diff Hedging Brownian/DmM	-15%	-13%	-12%	-11%	-10%
Diff Hedging Reversal/DmM	1%	1%	1%	0%	-2%

Table E.1: Comparative statics with respect to expected incremental return $\Delta\mathbb{E}_t(R)$, for the factor elasticity calibration. The colored (with bold font) column contains our benchmark parametrization: $\bar{D} = [0.27, 0.26]$, $\sigma = \text{diag}(0.485, 0.485)$, $\alpha = 1$, $\gamma = 3$, $r = 0.03$, $\rho = 0.08$, $S = [1, 1]$, $u^n = [0, 0]$, $u^s = [0.15, 0]$, $\lambda = 0.5$, and $w = 2.11$. The remaining columns show how our effects vary with the expected incremental return.

bottom styles) is about 13%, consistent with the reported style-based fund flows in Ben-David et al. (2021). We set $\lambda^n = 0$, that is the economy transitions from the normal state to the shock state with intensity 0 and thus the normal state is the steady state in our calibration. Furthermore, we set $\lambda^s \equiv \lambda = 0.5$ (and hence a persistence 2 of the residual supply shock) to match the empirical half life of fund flows as reported by Ben-David et al. (2021).

We set $r = 0.03$, which roughly equals the historical average risk-free rate. The investors' time discount rate ρ governs the aggregate asset holdings by hedgers. We interpret the hedgers in our model to include households, mutual funds and ETFs, foreign investors, etc. We set $\rho = 0.08$, which implies that hedgers hold 59% of the assets. Finally, we set $\gamma = 3$ which is taken from the literature. We normalize the hedgers' absolute risk aversion α and the supply of each asset to one. Our baseline parameters are provided in Table 1.

Using the baseline parameters, we simulate the distribution of investors' aggregate wealth. In the baseline, the mean wealth of investors is 2.11. In our model, the hedgers' endowment shocks give rise to trading needs by the hedgers. As the economy switches from the normal state to the shock state, the hedgers wish to hold less of the assets leading to increased expected returns. At the mean investors' wealth level, our calibration implies an incremental return of $\Delta\mathbb{E}_t(R) = 3\%$, of which roughly 2/3 comes from the reversal.

It is worthwhile mapping our measured slope to the empirical demand elasticity à la Koijen and Yogo (2019), defined as the percentage of capitalization traded divided by the price impact. In Section E.2.2, we provide a detailed discussion of how our measured slope translates to demand elasticity. Using the baseline parameters, the corresponding demand elasticity is 1.4. In Ben-David et al. (2021), the empirical factor elasticity is around 0.2. While our seven-fold difference is inline with the general observation that frictionless asset pricing models struggle to rationalize the empirically small factor elasticity, our difference is smaller than it is perhaps previously thought.[29]

[29] While the exact numbers widely vary, Gabaix and Koijen (2022) suggest that the empirical estimates are several order of magnitudes smaller than their conceptual counterpart.

Risk Aversion (γ)	0.7	1.5	2.0	3.0	3.5	log
Myopic Slope	9.59	5.37	4.34	3.21	2.84	7.28
Dynamic Slope	9.26	5.30	4.30	3.17	2.80	7.10
Measured Slope	1.43	1.48	1.41	1.24	1.15	1.49
Measured Slope/Dynamic Slope	15%	28%	33%	39%	41%	21%
Implied Price Elasticity	1.34	1.55	1.55	1.42	1.34	1.46
Dynamic minus Measured (DmM)	7.82	3.82	2.89	1.93	1.65	5.61
Shock to risk/DmM	97%	106%	109%	112%	112%	101%
Diff Hedging Brownian/DmM	4%	-6%	-9%	-13%	-13%	0%
Diff Hedging Reversal/DmM	-1%	0%	0%	1%	1%	-1%

Table E.2: Comparative statics with respect to risk aversion coefficient γ. The table shows comparative statics with respect to the investors' risk aversion coefficient γ, for the factor elasticity exercise. The colored (with bold font) column contains our benchmark parametrization: $\bar{D} = [0.27, 0.26]$, $\sigma = \text{diag}(0.485, 0.485)$, $\alpha = 1$, $r = 0.03$, $\rho = 0.08$, $S = [1, 1]$, $u^n = [0, 0]$, $u^s = [0.15, 0]$, $\lambda = 0.5$, and $w = 2.11$. The remaining columns show how our effects vary with the investors' risk aversion coefficient, with u^s recalibrated such that the implied expected return shocks remain constant at $\Delta\mathbb{E}_t(R) = 0.025$.

E.1.2 Results and slope analysis

Tables E.1-2 show the results of our baseline calibration and its slope to various parameters. In each table the colored column contains our benchmark parametrization, while the other columns show how our effects vary. For each set of parameters, we calculate the myopic slope, the dynamic slope and the measured slope, and decompose the difference to our three effects. Implied elasticity is a scaled version of our measured slope to make it comparable to the estimates collected by Gabaix and Koijen (2022) from the empirical literature.

Table E.1 focuses on the non-linear effect of the size of the expected return shocks implied by the shift in residual supply. Table E.2 illustrates the effect of varying relative-risk-aversion of investors. Table 2 concentrates on the effect of the persistence of the demand shift.[30]

There are a number of observations to make that are robust across our parameterizations.

First, consistent with the illustration in Panel A and B in Figure 2, we find that dynamic slope is always smaller than myopic slope. That is, Merton's intertemporal hedging tend to make the dynamic demand curve flatter compared to its static equivalent. However, this effect is small.

Second, in each of our parametrizations, measured slope significantly undershoots dynamic slope. In particular, measured slope is only 40% of demand slope in our baseline calibration. This fraction is shrinks to 15% when investors are less risk-averse than log. This illustrates the main observation of this paper. Because demand shifts change the slope of the demand curve in equilibrium, the econometrician relying on these shifts for estimating the demand slope is shooting a moving target. This calibration suggests that this results in significant underestimation of the targeted concept.

Table E.1 illustrates the non-linearity of this effect by varying the size of the shock. Note

[30]In Tables E.2-2, for each column we recalibrate the parameters u^ξ to ensure that the implied expected return shocks remain constant at $\Delta\mathbb{E}_t(R) = 0.025$.

that the fraction of measured slope to dynamic slope is shrinking when shocks are larger implying larger mismeasurement.

Third, the endogenous risk effect is responsible for the largest part of this mismeasurement. In fact, because the amplified intertemporal hedging component is going against the endogenous risk effect whenever $\gamma > 1$, in all these cases the endogenous risk effect is more than the 100% of the total under-measurement. This share is growing with risk-aversion and investor's wealth and shrinking with the persistence of the shock ranging from 102% (in our only parameterization with $\gamma < 1$) to 140%.

Fourth, while the GE amplified intertemporal hedging component is the same sign as Merton's intertemporal hedging effect, it is several magnitudes higher. In absolute terms, it can reach 34% of the total mismeasurement. Its absolute magnitude is larger for less persistent shocks, larger wealth and for greater risk-aversion.

Finally, the hedging effect against the reversal can be sizable reaching 31% for large shocks and 15% for persistence shocks relative to total mismeasurement.

E.2 Index Inclusion: Micro-Elasticity

In our second application, we are interested in an economy where the residual supply shock affects only very few assets. To make this point clear, we picture a market with a large number of fundamentally identical assets. That is, all I assets have identical average dividend $\bar{D}$, and $[\Sigma]_{ii} = \sigma^2$ and $[\Sigma]_{ij} = \kappa\sigma^2$ and their supply is $[s]_i = s$ for all i and $j \neq i$. The only difference across these assets is in hedgers' endowment. In particular, suppose that in the normal state, hedgers endowment for the first I_1 asset differs from the rest:

$$[u^n]_{i=1}^{I-1} = \upsilon - 1, \; [u^n]_{i=I_1+1}^{I} = \upsilon_2.$$

where $\upsilon_1 \neq \upsilon_2$ are scalars.

Our interpretation is that the first I_1 asset is in an index. We are interested in a shock for which $[u^n]_{i=2}^{I_1} = [u^s]_{i=2}^{I_1}$ while $[u^s]_1 = \upsilon_2$, $[u^s]_I = \upsilon_1$. That is, the first asset is excluded from and the last asset is included in the index.

Note that we intentionally design this example, so that the shock itself has a minimal effect on the structure of the economy—effectively, we only swap the labels of the first and the last assets. Hence, one might expect that the shock should have a negligible effect on the equilibrium outcome; hence, the informativeness of shifts to residual supply should perhaps be restored. We show that this is not the case.

In this part, we calibrate to Greenwood and Sammon (2024), which studies the measured slope of index inclusion and deletion events. We set $I = 502$ and $I_1 = 501$ such that the index in our calibration corresponds to the S&P 500 Index. Asset 1 and asset 2 switch in and out of the S&P 500 Index, while assets 3 to 501 are always in the index and the last asset is always out of the index. $\bar{D}_i$ is the expected cash flow of asset i, where $i \in \{1, 2, \ldots, I\}$. σ_i is the asset's cash flow volatility. κ is the pairwise cash flow correlation between asset i and asset j, where $i \neq j \in \{1, 2, \ldots, I\}$. We calibrate $\bar{D}_i$ and σ_i to match historical return moments. We set $\kappa = 0.3$ such that roughly 75% of an asset's return volatility is idiosyncratic (see Campbell, Lettau, Malkiel and Xi (2001)).

As in the our calibration of the factor-elasticity, we normalize the hedgers' absolute risk aversion α to one. We set $r = 0.03$, which roughly equals the historical risk-free rate. We also set the investors' time discount rate $\rho = 0.08$, the same as in the factor-elasticity calibration. ρ affects the aggregate asset holdings by hedgers, which are interpreted as mutual funds and ETFs, pension funds, etc. In our calibration, hedgers hold 45% of index assets and 2% of non-index assets. We set $\gamma = 3$, which is taken from the literature.

We normalize the supply of each asset to $1/\sqrt{I} = 0.0446$. To match the fund flows due to index inclusion and deletion in Greenwood and Sammon (2024), we normalize hedgers' endowment shock v_1 to zero for index assets, and set their endowment shock for non-index assets $v_2 = 0.0107$. These give us a fund flow of roughly 13% which is in-line with industry research cited by Greenwood and Sammon (2024). Finally, we set the intensity of state transitions $\lambda = 0.2$, which targets the conditional probability of index inclusion or deletion. λ is computed as the number of stocks included (or excluded) in a given year, divided by the number of stocks within the band on watch for inclusion (or exclusion). We conduct sensitivity analysis with respect to λ and γ.

Using the baseline parameters, we simulate the distribution of investors' aggregate wealth. In the baseline, the mean wealth of investors is 4.75. In our model, the index inclusion/deletion changes hedgers' relative endowment shock across the affected assets, giving rise to trading needs by the hedgers. As expected for such a local shock, the implied mean incremental return is very small, $\Delta\mathbb{E}_t(R_1) \approx 0.01\%$. One of the main points of this calibration is that even if identification relies on a demand shock implying tiny effects on expected returns, the implied mismeasurement of demand slope due to general equilibrium effects is not diminishing.

E.2.1 Results and slope analysis

Tables E.3-E.4 show the results of our baseline calibration and its slope to various parameters. As before, in each table the colored column contains our benchmark parametrization, while the other columns show how our effects vary.

Table E.3 illustrates the effect of varying relative-risk-aversion of investors. Table 2 concentrates on the effect of the persistence of the demand shift. Table E.4 highlights the effect of state dependence: how our measures change for smaller than mean wealth of investors.[31]

The results are remarkably similar to our index inclusion experiment even in a quantitative sense. Dynamic Slope is somewhat smaller than myopic slope. Measured slope is a fraction of its theoretical counterpart (amounting only to 26% when risk-aversion is small). The bulk of the effect is due to the enodgenous change in the covariance matrix, but the GE amplified intertemporal effect remaining also significant.

In this calibration, we also present how our effects depend on the state of the economy, i.e. the level of aggregate wealth of investors, w_t on Table E.4. This illustrates that all our objects of interest, myopic slope, dynamic slope and measured slope are highly state dependent. Measured slope is only 44% of demand slope in our baseline calibration for the micro elasticity calibration exercise. As the investor wealth increases from 4.25 to 4.75, this fraction also increases reaching 57%. The corresponding implied price elasticity is monotonically increasing in investor wealth, suggesting that a lower wealth share of the investor sector decreases the implied price elasticity measured by econometricians. In practice, many investors are often "sleepy" and do not engage in active trading. This reduces the effective investor wealth, causing the implied price elasticity to be smaller.

[31] In Tables E.3 and 2, for each column we recalibrate the parameters u^{ξ} to ensure that the implied expected return shocks remain constant at $\Delta\mathbb{E}_t(R) = 0.01\%$.

Risk Aversion (γ)	0.9	1.5	2.0	3.0	3.5	log
Myopic Slope	33.13	17.45	12.52	7.81	6.55	27.20
Dynamic Slope	33.13	17.45	12.52	7.81	6.55	27.20
Measured Slope	8.55	5.96	4.80	3.42	2.99	7.51
Measured Slope/Dynamic Slope	26%	34%	38%	44%	46%	28%
Implied Price Elasticity	719	569	500	399	357	648
Dynamic minus Measured (DmM)	24.58	11.49	7.72	4.39	3.55	19.68
Shock to risk/DmM	98%	107%	111%	116%	118%	100%
Diff Hedging Brownian/DmM	2%	-7%	-11%	-16%	-18%	0%
Diff Hedging Reversal/DmM	0%	0%	0%	0%	0%	0%

Table E.3: Comparative statics with respect to risk aversion coefficient γ. The table shows comparative statics with respect to the investors' risk aversion coefficient, for the micro elasticity calibration. The colored (with bold font) column contains our benchmark parametrization: $\bar{D}_i = 0.0369$, $\sigma_i = 0.0692$, $\kappa = 0.3$, $\alpha = 1$, $\gamma = 3$, $r = 0.03$, $\rho = 0.08$, $S_i = 0.0446$, $v_1 = 0$, $v_2 = 0.0107$, $\lambda = 0.2$ and $w = 4.75$. The remaining columns show how our effects vary with the risk aversion coefficient, with v_2 recalibrated such that the implied expected return shocks remain constant at $\Delta\mathbb{E}_t(R) = 0.01$.

Investor Wealth (w)	1.00	2.50	3.50	4.25	4.75
Myopic Slope	3.20	5.11	6.66	7.81	8.51
Dynamic Slope	3.20	5.11	6.66	7.81	8.51
Measured Slope	1.48	2.08	2.65	3.42	4.87
Measured Slope/Dynamic Slope	46%	41%	40%	44%	57%
Implied Price Elasticity	79	218	322	399	443
Dynamic minus Measured (DmM)	1.71	3.03	4.01	4.39	3.64
Shock to risk/DmM	109%	114%	115%	116%	118%
Diff Hedging Brownian/DmM	-9%	-14%	-15%	-16%	-18%
Diff Hedging Reversal/DmM	0%	0%	0%	0%	0%

Table E.4: Comparative statics with respect to investor wealth w. The table shows comparative statics with respect to the investors' aggregate wealth w, for the micro elasticity calibration. The colored (with bold font) column contains our benchmark parametrization: $\bar{D}_i = 0.0369$, $\sigma_i = 0.0692$, $\kappa = 0.3$, $\alpha = 1$, $\gamma = 3$, $r = 0.03$, $\rho = 0.08$, $S_i = 0.0446$, $v_1 = 0$, $v_2 = 0.0107$, $\lambda = 0.2$ and $w = 4.75$. The remaining columns show how our effects vary with the risk aversion coefficient, with v_2 recalibrated such that the implied expected return shocks remain constant at $\Delta\mathbb{E}_t(R) = 0.01$.

It is also useful to observe that the hedging effect against the reversal is the only one which is significantly smaller in index inclusion than in our factor-elasticity calibration. It is so, because this is the only one which diminishes with the size of the shock.

Under the baseline parameters, the index-inclusion calibration yields an implied price elasticity à la Koijen and Yogo (2019) of 399, which is substantially lower than the commonly cited value of around 6250 from Petajisto (2009). For comparison, the empirical micro elasticity estimated by Greenwood and Sammon (2024) is approximately 2.7.

E.2.2 Translating Measured Slope to Elasticity

As explained earlier, the demand elasticity is defined as the percentage of capitalization traded divided by the percentage change in price, that is

$$\mathcal{E}(w_t;\lambda,u^s)=\frac{\frac{w^s\hat{y}_i^s}{S_iP_i^s}-\frac{w^n\hat{y}_i^n}{S_iP_i^n}}{-\frac{P_i^s-P_i^n}{P_i^n}},\tag{E.263}$$

where we have suppressed time index for simplicity. $\hat{y}_i^n$ and P_i^n are the investor's portfolio share and the equilibrium price of asset i in the normal state. $\hat{y}_i^s$ and P_i^s are portfolio share and price in the shock state. The investor has wealth w^n in the normal state and w^s in the shock state. Notice that (E.263) can be rewritten as

$$\mathcal{E}(w_t;\lambda,u^s)=\frac{1}{y_i^n}\frac{[w^n\,\mathrm{diag}(P^n)^{-1}y^n]_i}{S_i}\frac{\frac{P_i^nw^s}{P_i^sw^n}\hat{y}_i^s-\hat{y}_i^n}{\frac{P_i^n-P_i^s}{P_i^n}}.\tag{E.264}$$

The middle term on the right-hand side is the fraction of asset i's capitalization held by investors. The last term on the right-hand side is at the same order of magnitude as the measured slope. The equation thus suggests that the demand elasticity is inversely related to investors' portfolio share of asset i.

The implied demand elasticity $\mathcal{E}(w_t;\lambda,u^s)$ depends on the shock persistence $1/\lambda$. For our comparative statics, we fix the incremental expected return $\Delta\mathbb{E}_t(R)\equiv\mu_{Rt}^s-\mu_{Rt}^n+\lambda\,\mathrm{diag}(P_t^s)^{-1}(P_t^n-P_t^s)$ at the level implied in the baseline. Fixing $\Delta\mathbb{E}_t(R)$, the change in asset prices $\mathrm{diag}(P_t^s)^{-1}(P_t^n-P_t^s)$ is higher when λ is low, that is when the shock is more persistent. In other words, transitory shocks tend to have smaller price impact. Consequently, when $1/\lambda$ is small, the denominator in (E.263) is also small, magnifying the implied demand elasticity. Notice that our measured slope $\mathcal{M}(w_t;\lambda,u^s)$ adjusts for this horizon effect due to shock persistence.

E.3 Single Asset, Log Investors Calibration

In Section 4.2, we examine a simple case with a single asset and log investors. The baseline parameter values are shown in Table E.5.

For the single asset, log investors case, we calibrate to Gabaix and Koijen (2022), which studies the macro elasticity. Corresponding to their framework, the single asset in our calibration can be interpreted as the stock market. $\bar{D}$ is the expected cash flow of the stock market, and σ is the cash flow volatility. We calibrate $\bar{D}$ and σ to match historical return moments, with a calibrated mean return of 8.9% and return volatility of 16.4%.

Moreover, as in our calibration of the factor-elasticity and micro-elasticity, we normalize the hedgers' absolute risk aversion α to one. We set $r=0.03$, which roughly equals the historical risk-free rate, and set $\rho=0.08$ to be consistent with the factor-elasticity and micro-elasticity calibrations. We further normalize the asset supply to one and u^n (hedgers' endowment in the normal state) to zero.

Finally, the endowment in the shock state $u^s=0.01$ is chosen such that the implied fund flow is roughly 1%, as documented by Gabaix and Koijen (2022). In Figure 1, we plot the demand slope of a log investor, $\mathcal{C}^{\log}$ and the corresponding measured slope $\mathcal{M}^{\log}|_{\delta=\Delta\mathbb{E}_t(R)}$

Para.	Description	Value	Targeted Moments
$\bar{D}$	Expected cash flow of assets	0.415	historical returns
σ	Cash flow volatility	0.57	historical return volatilities
α	Hedgers' absolute risk aversion	1	normalization
r	Risk-free rate	0.03	historical average interest rate
ρ	Investors' time discount rate	0.08	investor holdings
S	Asset supply	1	normalization
u^n	Hedgers' endowment in normal state	0	normalization
u^s	Hedgers' endowment in shock state	0.4	fund flows Ben-David et al. (2021)
λ	Intensity of state transitions	0.5	persistence Ben-David et al. (2021)

Table E.5: Parameters and Calibration Details for the Single Asset, Log Investors Case. The table reports the parameter values used in the single asset, log investors calibration exercise.

against the implied change in percentage incremental expected return $\Delta\mathbb{E}_t(R)$ by varying the u^s shock. The intensity parameter λ matches the inertia parameter in Gabaix and Koijen (2022) who also consider a case of short-term vs. long-term elasticity when funds are inertial. The half-life in their calibration is 0.7, which translates into $\lambda = 1$ in our model. In Figure 1, we consider two limit cases with $\lambda \to 0$ and $\lambda \to \infty$ respectively.

The implied price elasticity of the single asset, log investors calibration is 2, compared to the macro elasticity estimate of 0.2 in Gabaix and Koijen (2022).