A Macroeconomic Framework for Quantifying Systemic Risk

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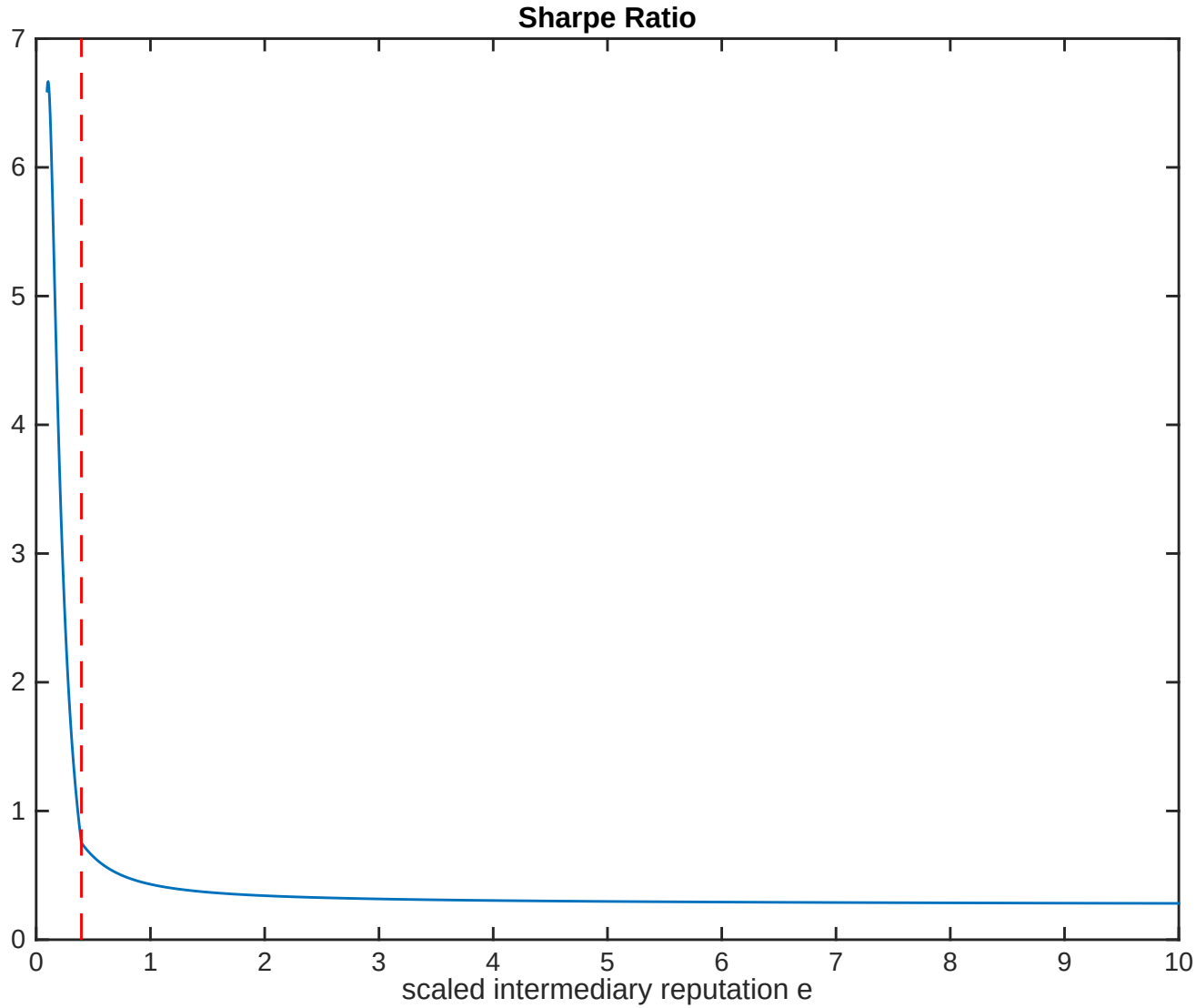
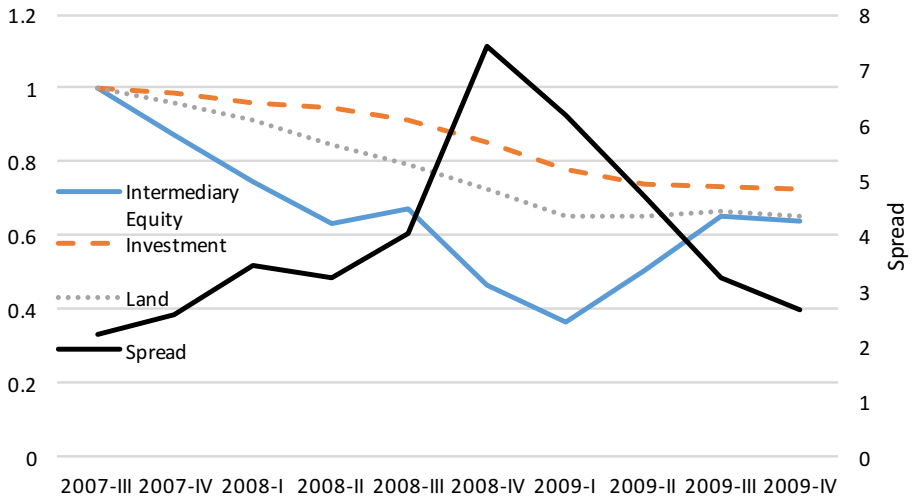
Bank of Canada, August 2017

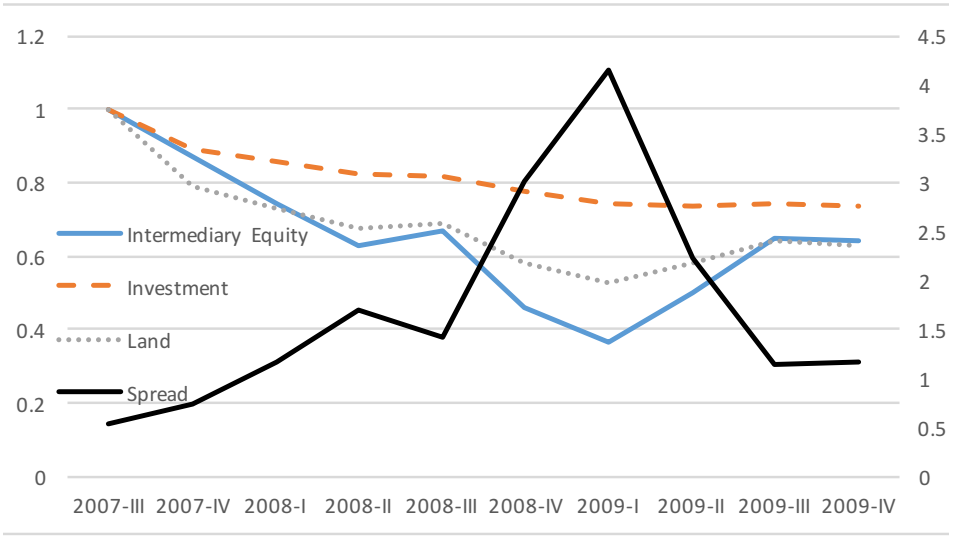
Financial Crisis in the Model



Note: Capital constraint binds for $e < 0.3957$

Matching Recent Crisis: *Data(L) and Model(R)*





Outline

1. Nonlinear macro model of a financial crisis
   - Recent work on financial intermediaries: He-Krishnamurthy, Brunnermeier-Sannikov, Rampini-Viswanathan, Adrian-Boyarchenko, Gertler-Kiyotaki
   - Our approach: occasionally binding constraint; global solution method (similar to Brunnermeier-Sannikov, Adrian-Boyarchenko)

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   - Match conditional moments of the data, conditioning on negative (i.e., recession) states
3. Quantify systemic risk
   - Systemic risk: the state where financial intermediation is widely disrupted to affect real activities severely
     - ⋆ In the model, states where capital constraint binds, crisis state
   - What is the ex-ante (e.g., initial conditions of 2007Q2) likelihood of crisis states? (... low)
   - What makes the probability higher?
   - Economics of stress tests (as opposed to accounting of stress tests)

Agents and Technology

- Two classes of agents: households and bankers
  - Households:

$$\mathbb{E}\left[\int_0^\infty e^{-\rho t}\left(c_t^y\right)^{1-\phi}\left(c_t^h\right)^{\phi} dt\right],$$

- Two types of capital: productive capital K_t and housing capital H.
  - Fixed supply of housing $H \equiv 1$
  - Price of capital q_t and price of housing P_t determined in equilibrium

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- Production $Y = AK_t$, with A being constant
- Fundamental shocks: stochastic capital quality shock dZ_t. TFP shocks

$$\frac{dK_t}{K_t} = i_t dt - \delta dt + \sigma dZ_t$$

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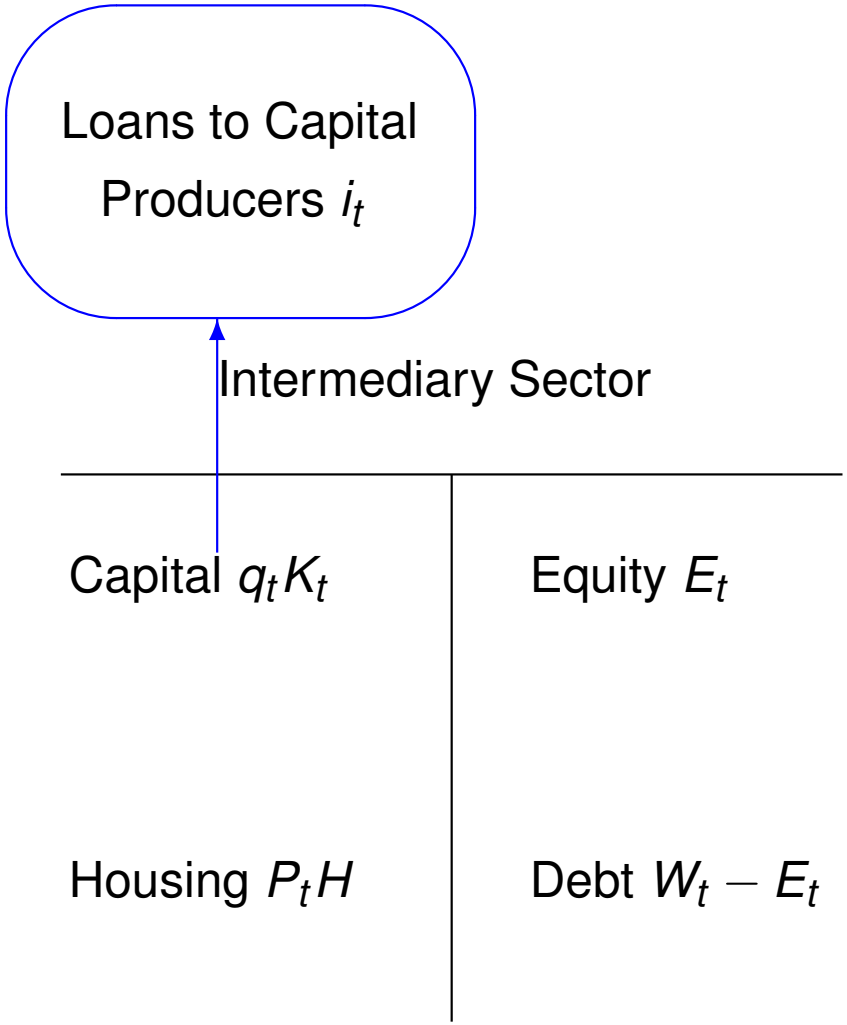
$$\frac{dK_t}{K_t} = i_t dt - \delta dt + \sigma dZ_t$$

- Investment/Capital i_t, quadratic adjustment cost

$$\Phi(i_t, K_t) = i_t K_t + \frac{\kappa}{2}(i_t - \delta)^2 K_t$$

$$\max_{i_t} \; q_t i_t K_t - \Phi(i_t, K_t) \;\Rightarrow\; i_t = \delta + \frac{q_t - 1}{\kappa}$$

Aggregate Balance Sheet

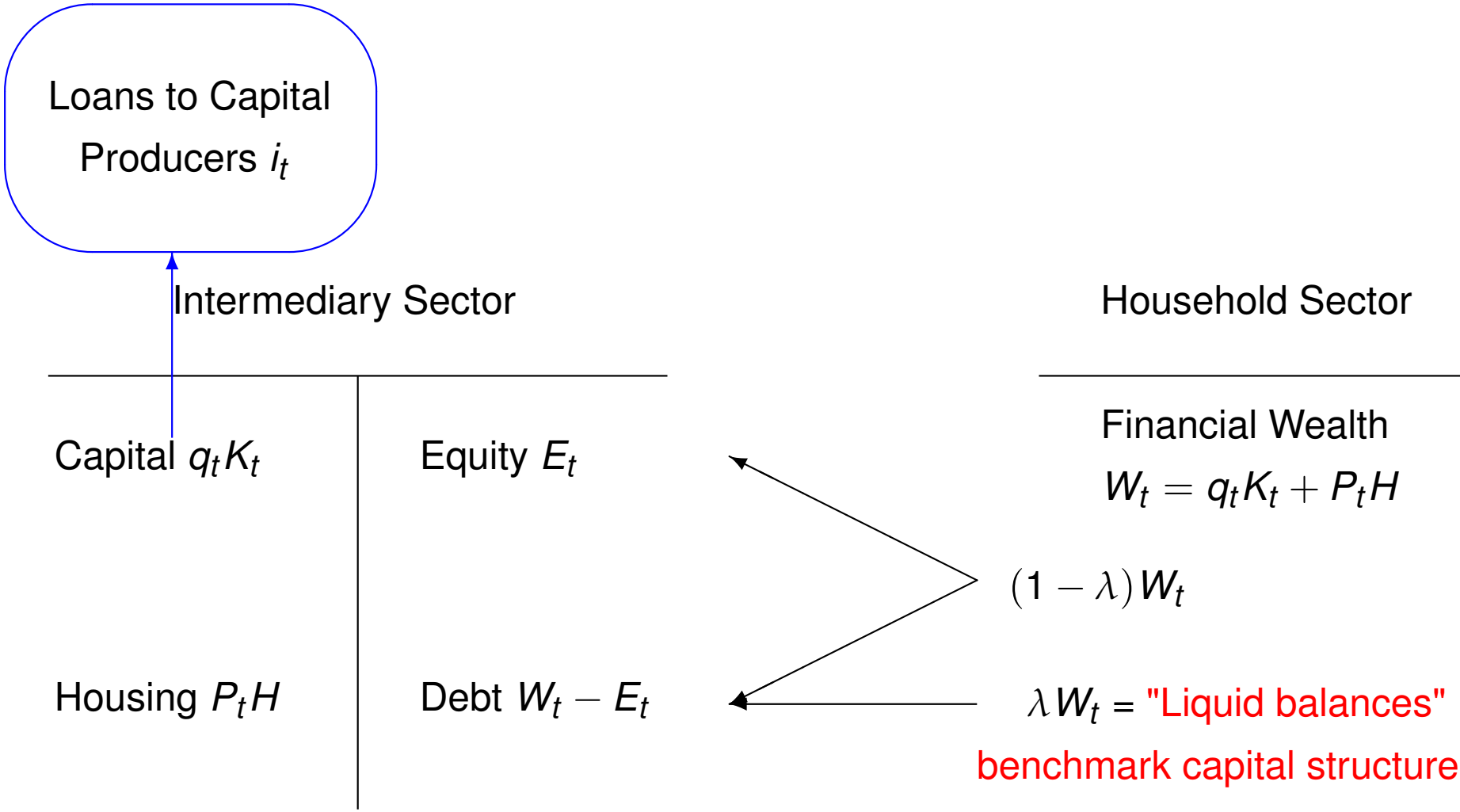


Household Sector

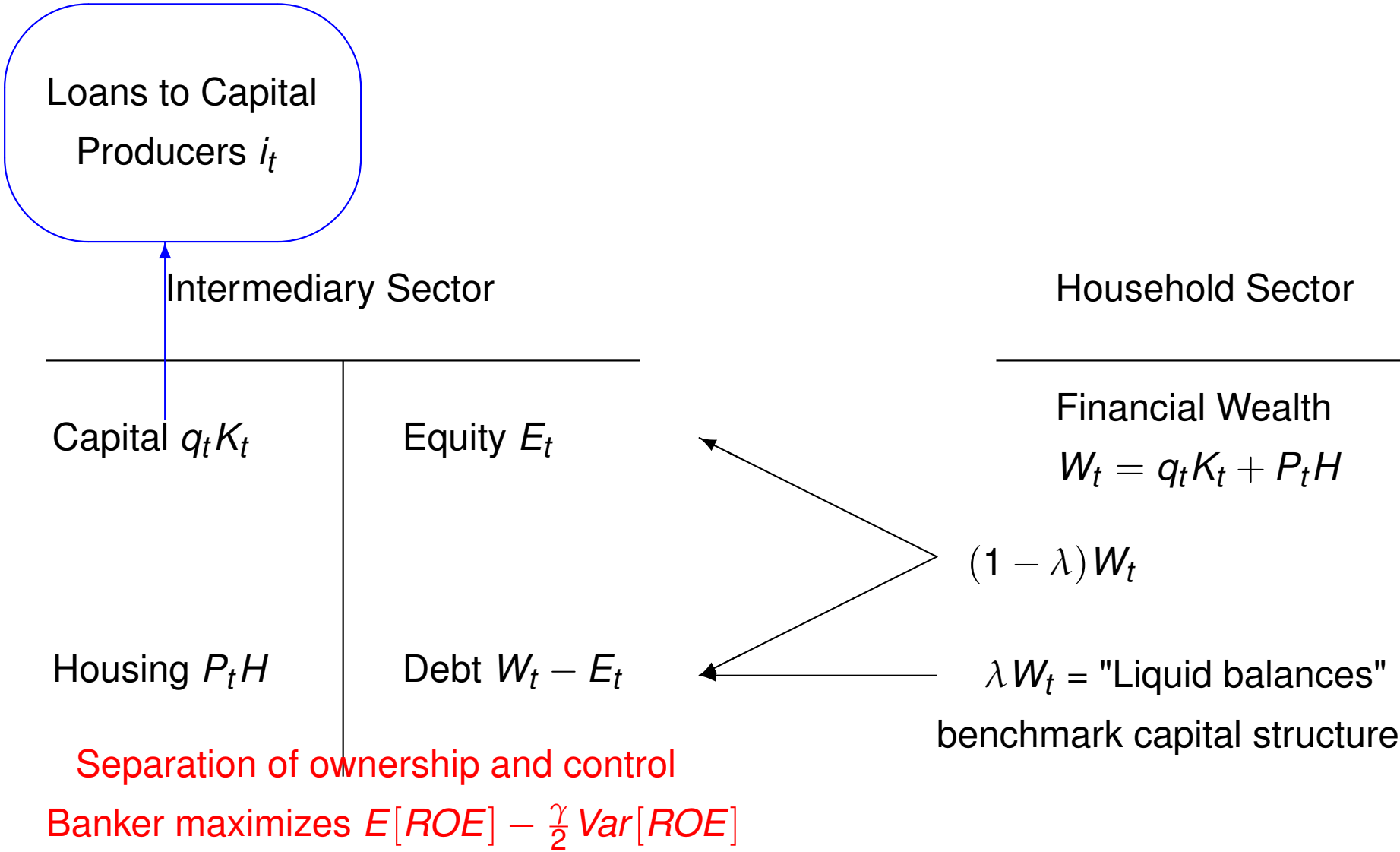
Financial Wealth

$W_t = q_t K_t + P_t H$

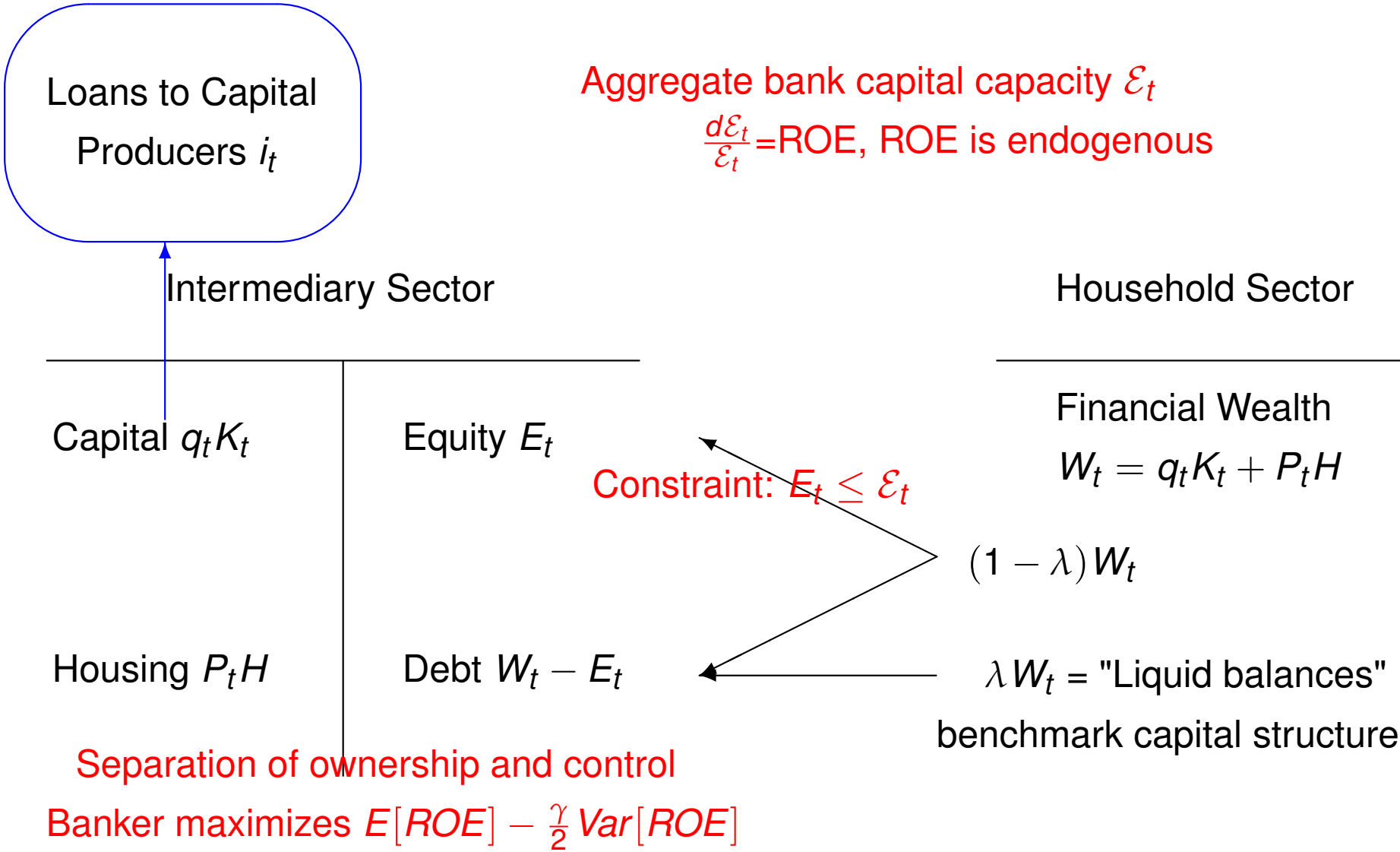
Aggregate Balance Sheet



Equity Matters



Intermediary Equity Constraint



Single Bank/Banker Choice of Portfolio and Leverage

Capital $q_t k_t$	$equity_t$
Housing $P_t h_t$	$debt_t$

Portfolio share in capital: $\alpha_t^k = \frac{q_t k_t}{equity_t}$

Portfolio share in housing : $\alpha_t^h = \frac{P_t h_t}{equity_t}$

Borrowing (no constraint): $debt_t = q_t k_t + P_t h_t - equity_t = (\alpha_t^k + \alpha_t^h - 1) equity_t$

Bank Choice of Portfolio and Leverage

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Return on bank equity ROE: $d\tilde{R}_t = \alpha_t^k dR_t^k + \alpha_t^h dR_t^h - (\alpha_t^k + \alpha_t^h - 1) r_t dt$

Banker (log preference) solves: $\max_{\alpha_t^k, \alpha_t^h} \mathbb{E}_t[d\tilde{R}_t - r_t dt] - \frac{\gamma}{2} Var_t[d\tilde{R}_t]$; m parameter

Bank Choice of Portfolio and Leverage

Capital $q_t k_t$	$equity_t$
Housing $P_t h_t$	$debt_t$

Properties

- (k, h) scales with *equity*
- (k, h) increasing in $\mathbb{E}_t[d\tilde{R}_t - r_t dt]$
- (k, h) decreasing in $Var_t[d\tilde{R}_t]$

Portfolio share in capital: $\alpha_t^k = \frac{q_t k_t}{equity_t}$

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General Equilibrium

Intermediary Sector		Household Sector
Capital $q_t K_t$	Equity E_t	Financial Wealth
Housing $P_t H$	Debt $W_t - E_t$	$W_t = q_t K_t + P_t H$

Portfolio share in capital: $\alpha_t^k = \frac{q_t K_t}{E_t} = \frac{q_t K_t}{\min[\mathcal{E}_t, (1-\lambda) W_t]}$

Portfolio share in housing: $\alpha_t^h = \frac{P_t H}{E_t} = \frac{P_t H}{\min[\mathcal{E}_t, (1-\lambda) W_t]}$

- Given E_t, the equilibrium portfolio shares are pinned down by GE
- But portfolio shares must also be optimally chosen by banks, pinning down prices

$$\max_{\alpha_t^k, \alpha_t^h} \ \mathbb{E}_t[d\tilde{R}_t - r_t dt] - \frac{\gamma}{2} Var_t[d\tilde{R}_t]$$

- Asset prices affect real side through investment (q_t)

Equity Capital Constraint

- Representative household with W_t, split between bonds (at least) λW_t and equity (at most) $(1-\lambda)W_t$
- Benchmark capital structure: λW_t of Debt, $(1-\lambda)W_t$ of Equity
  - if there is no capital constraint ($\mathcal{E}_t$ is infinite)...

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- Benchmark capital structure: λW_t of Debt, $(1-\lambda)W_t$ of Equity
  - if there is no capital constraint ($\mathcal{E}_t$ is infinite)...
- Intermediary equity capital:

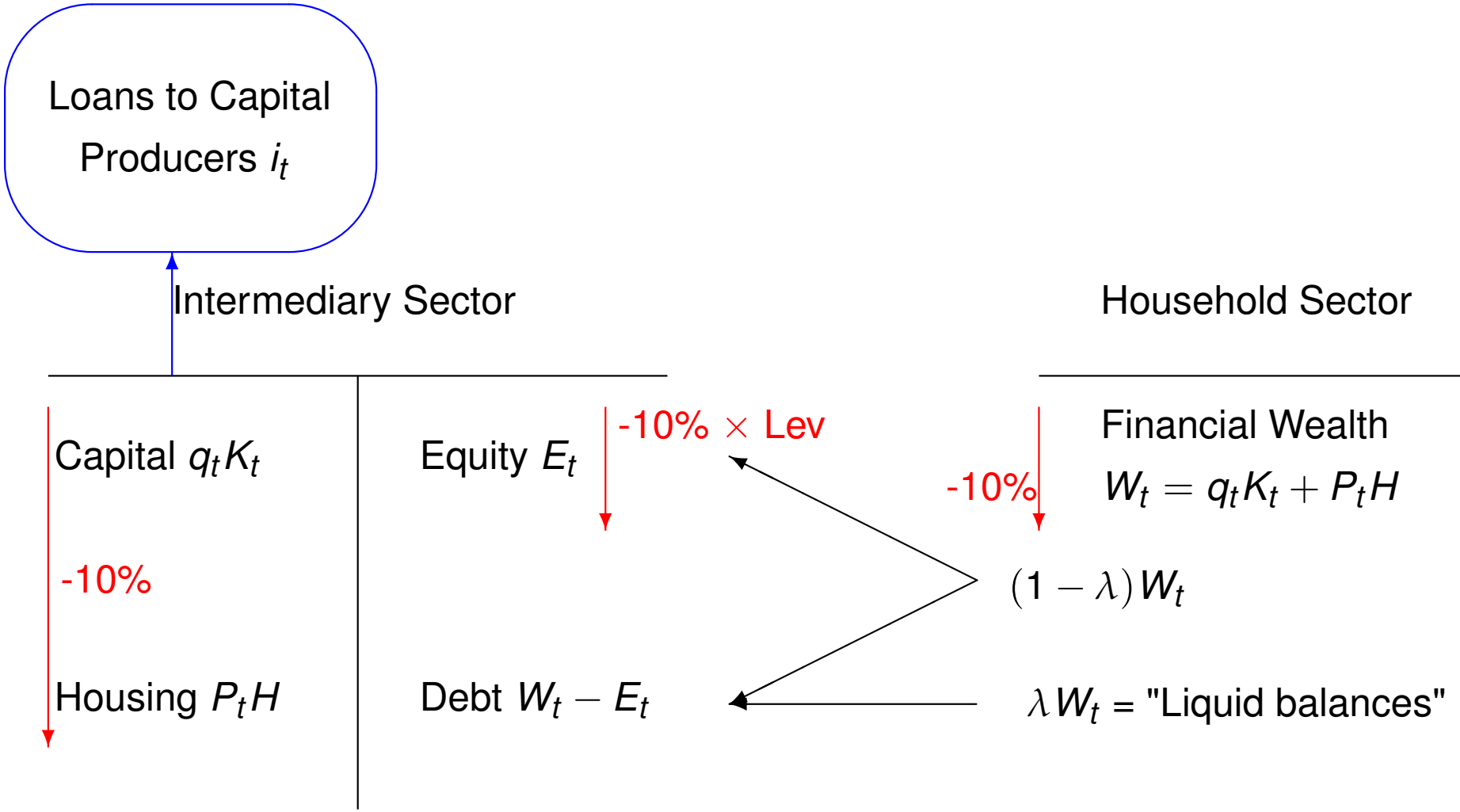
$$E_t = \min\left[\mathcal{E}_t, (1-\lambda)W_t\right]$$

- Suppose a -10% shock to real estate and price of capital:
- $W_t \downarrow 10\%$ (Household wealth = aggregate wealth)
- Capital capacity: $\frac{d\mathcal{E}_t}{\mathcal{E}_t} = d\tilde{R}_t + ...$ and $\mathcal{E}_t \downarrow$ more than 10%:
  - Return on equity = $d\tilde{R}_t < -10\%$: equity is levered claim on assets
  - leverage is endogenous in the model

Micro foundation of Capital Constraint

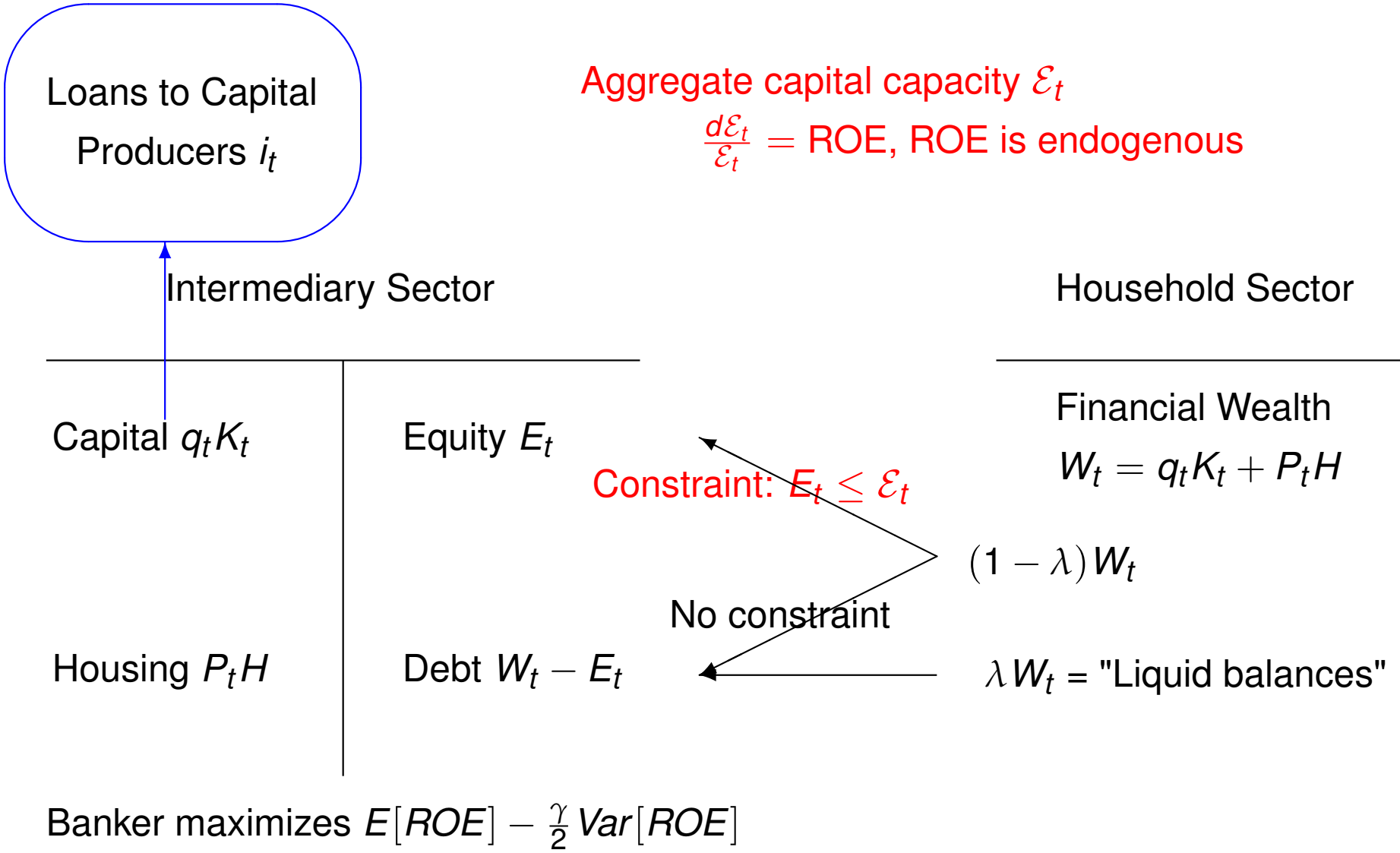
- We develop theory in He-Krishnamurthy (2012, Restud), and applied to MBS market in He-Krishnamurthy (2013, AER)
- Two-agents **endowment** economy, **Households** with wealth W_t^h cannot hold MBS assets but can delegate their money to **Bankers** with wealth W_t
- With agency friction, households are only willing to contribute at most mW_t as outside equity capital, so risk-sharing rule cannot fall below $1 : m$
  - "Skin in the game" idea
- When banker's net worth W_t is low, capital constraint is binding
- Binding capital constraint is a binding **Incentive Compatibility** constraint in delegation/agency contracting problem
  - IC binds after a series of bad shocks where banker's net worth W_t is low
- Banker's net worth W_t evolves with fund performance, just like reputation or equity capacity ϵ_t

Equity Dynamics in GE



Banker maximizes $E[ROE] - \frac{\gamma}{2} Var[ROE]$

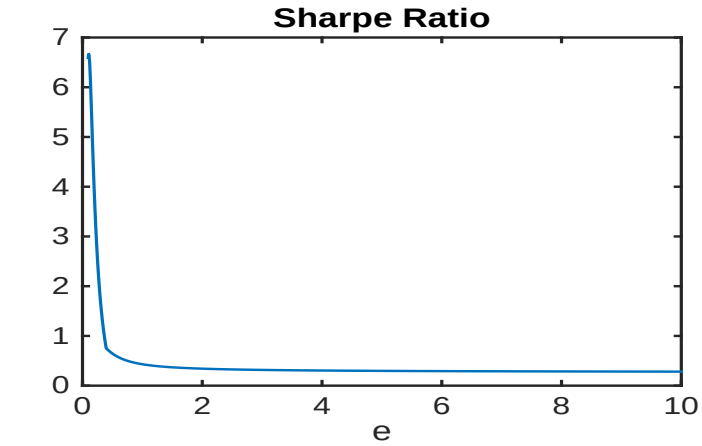
Equity Constraint Amplifies Shocks

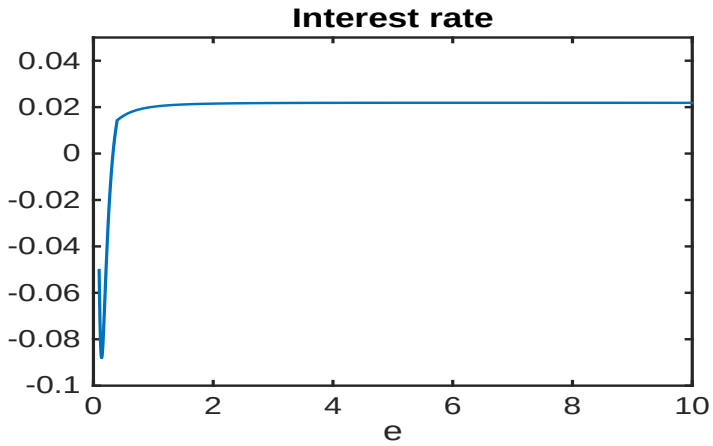


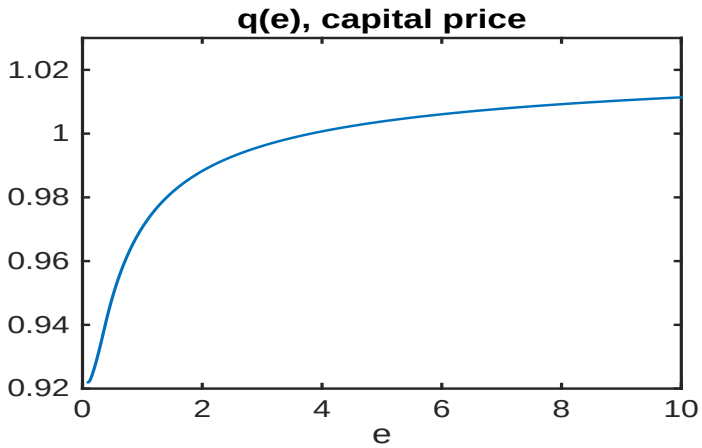
Calibration: Baseline Parameters

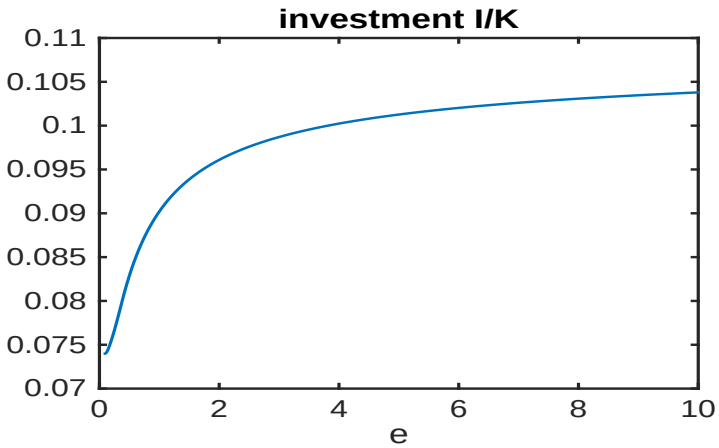
	Parameter	Choice	Targets (Unconditional)
Panel A: Intermediation			
γ	Banker risk aversion	2	Average Sharpe ratio)
λ	Debt ratio	0.75	Average intermediary leverage
η	Banker exit rate	13%	Prob. of crisis (model,data = 3%)
B	Entry barrier	6.5	Highest Sharpe ratio
β	Entry cost	2.8	Average land price vol (model,data=14%)
Panel B: Technology			
σ	Capital quality shock	3%	Consumption volatility (model=1.4%) Note: Model investment vol = 4.5%
δ	Depreciation rate	10%	Literature
κ	Adjustment cost	3	Literature
A	Productivity	0.133	Average investment-to-capital ratio
Panel C: Others			
ρ	Time discount rate	2%	Literature
ϕ	Housing share	0.4	Housing-to-wealth ratio (bank or household)

Results(1): State variable is $e_t = \mathcal{E}_t / K_t$





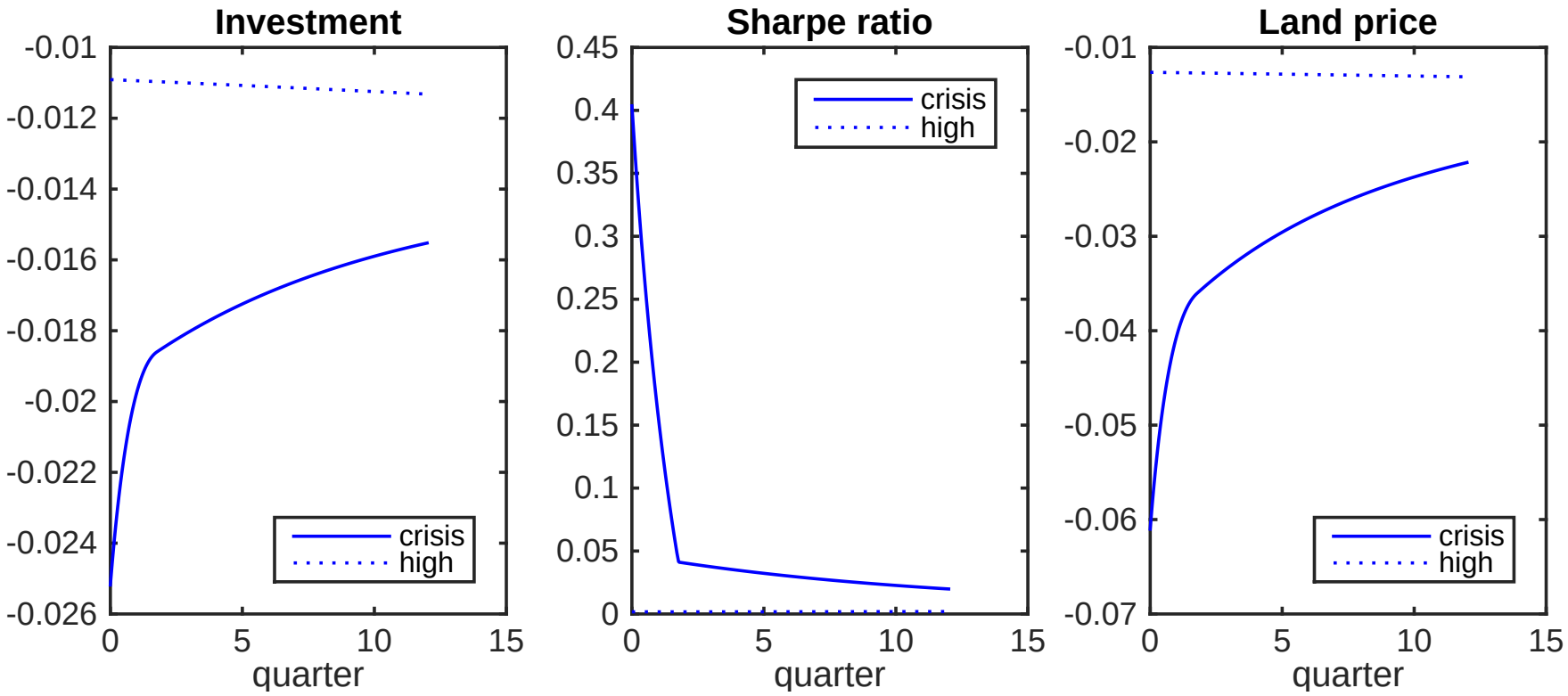




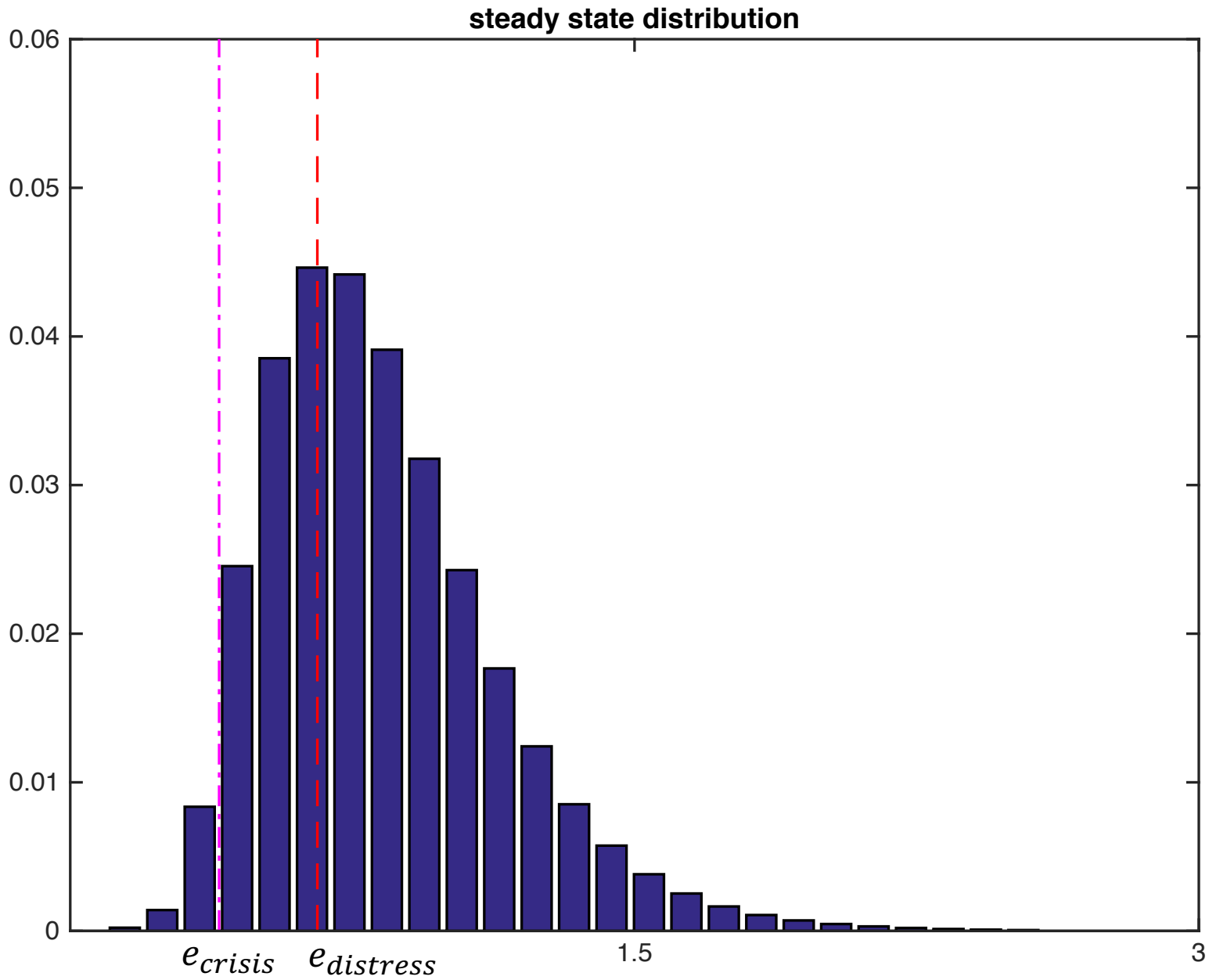
- Capital constraint binds for $e < 0.3957$
- Without the possibility of the capital constraint, all of these lines would be flat. Model dynamics would be i.i.d., with vol=3%

State-dependent Impulse Response: -1% Shock ($= \sigma dZ_t$)

VARdata



Steady State Distribution



Nonlinearities in Model and Data

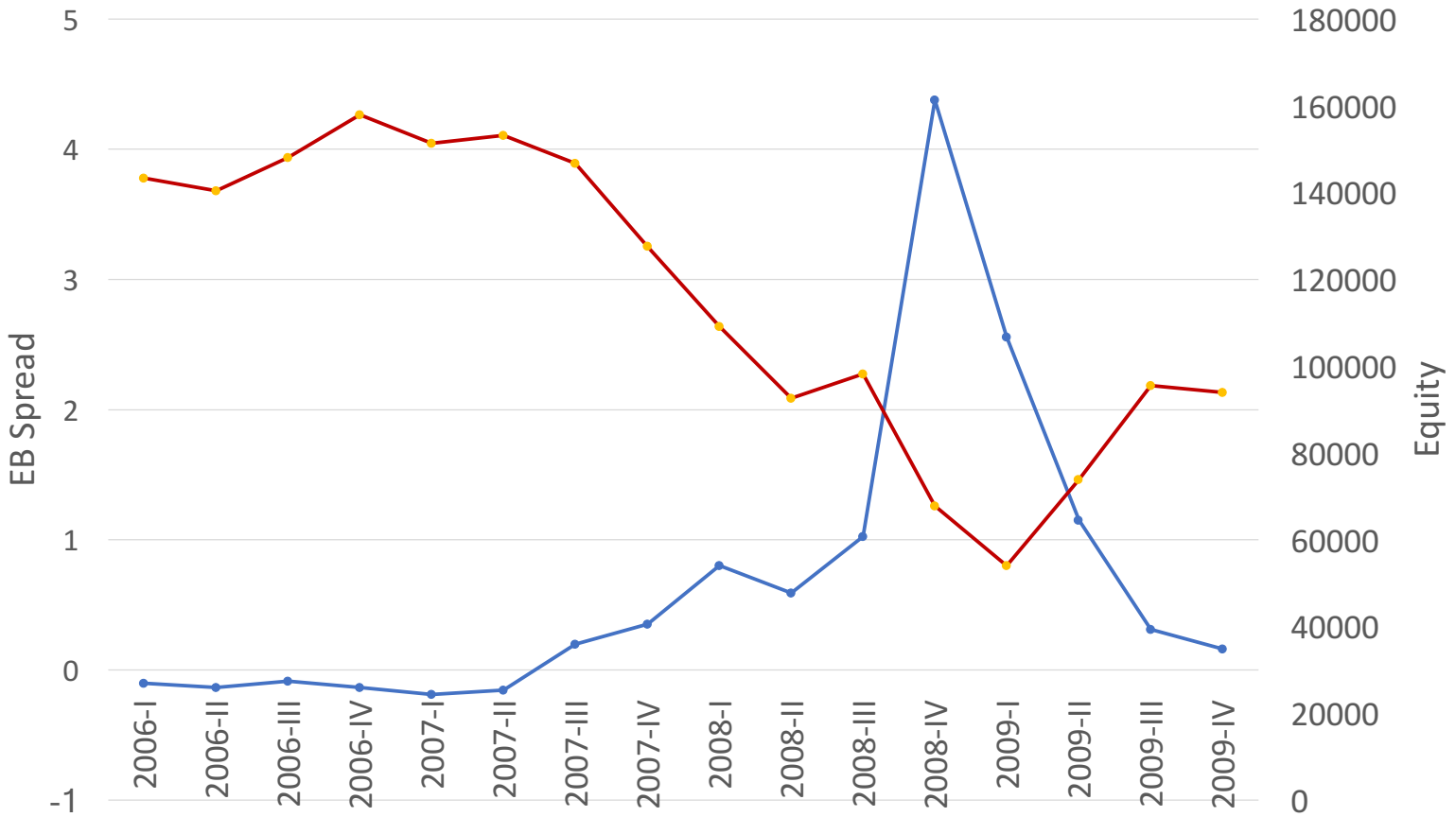
Model:

- Distress states = worst 33% of realizations of e ($e < 0.66$)
- Compute conditional variances, covariances of intermediary equity growth with other key variables

Data:

- Distress states = worst 33% of realizations of (risk premium in) credit spread
  - We use Gilchrist-Zakrajsek (2011) Excess Bond Premium, which we convert to a Sharpe ratio
  - Excess Bond Premium: risk premium of corporate bonds, presumably reflects distress of financial sector
  - Similar results if using NBER recessions
- Compute conditional variances, covariances of intermediary equity growth with other key variables

EBS and Intermediary Equity time series



- Intermediary equity: market equity of commercial banks and broker/dealer sectors (SIC codes 6000-6299)

Distress Classification

Distress Periods	NBER Recessions
1975Q1 - 1975Q4	11/73 - 3/75
1982Q3 - 1982Q4	7/81 - 11/82
1986Q1 - 1987Q1	
1989Q1 - 1990Q1	
	7/90 - 3/91
1992Q3 - 1993Q1	
2000Q1 - 2003Q1	3/01 - 11/01
2007Q4 - 2009Q3	12/07 - 6/09
2010Q2 - 2010Q4	
2011Q3 - 2013Q1	

Covariances in Data

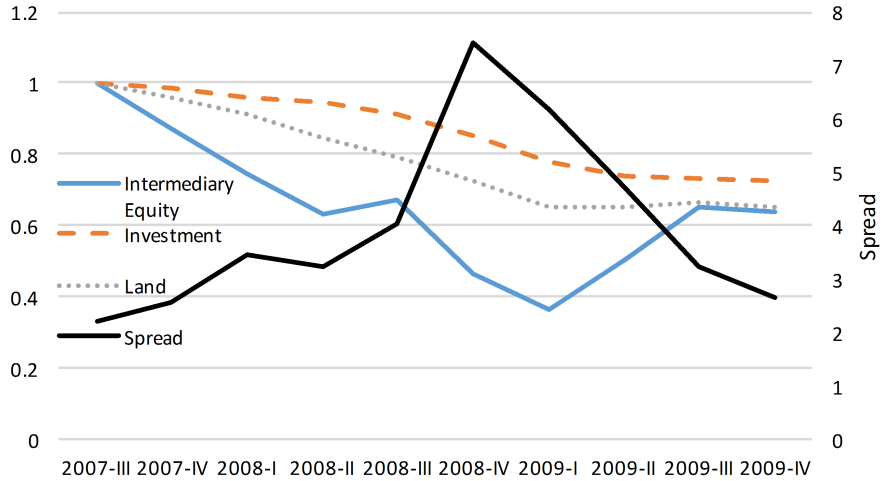
	EB	NBER Recession	NBER+,-2Qs	NBER+, Drop Crisis
Panel A: Distress Periods				
vol(Eq)	25.73	28.72	27.14	22.11
vol(I)	7.71	7.24	6.93	4.70
vol(C)	1.72	1.79	1.83	1.37
vol(PL)	15.44	15.11	10.51	8.10
vol(EB)	65.66	107.16	85.04	36.23
cov(Eq, I)	1.02	1.10	0.60	0.20
cov(Eq, C)	0.20	0.10	0.07	-0.04
cov(Eq, PL)	2.38	3.12	1.88	0.11
cov(Eq, EB)	-8.50	-19.03	-11.32	1.66
Panel B: Non-distress Periods				
vol(Eq)	20.54	19.42	18.90	19.15
vol(I)	5.79	5.92	4.75	4.99
vol(C)	1.24	1.29	1.09	0.91
vol(PL)	9.45	10.51	10.26	8.63
vol(EB)	16.56	29.95	29.33	30.95
cov(Eq, I)	-0.07	-0.06	-0.18	-0.14
cov(Eq, C)	-0.01	0.01	0.00	-0.01
cov(Eq, PL)	-0.43	-0.23	-0.31	-0.59
cov(Eq, EB)	0.60	0.19	0.02	0.54

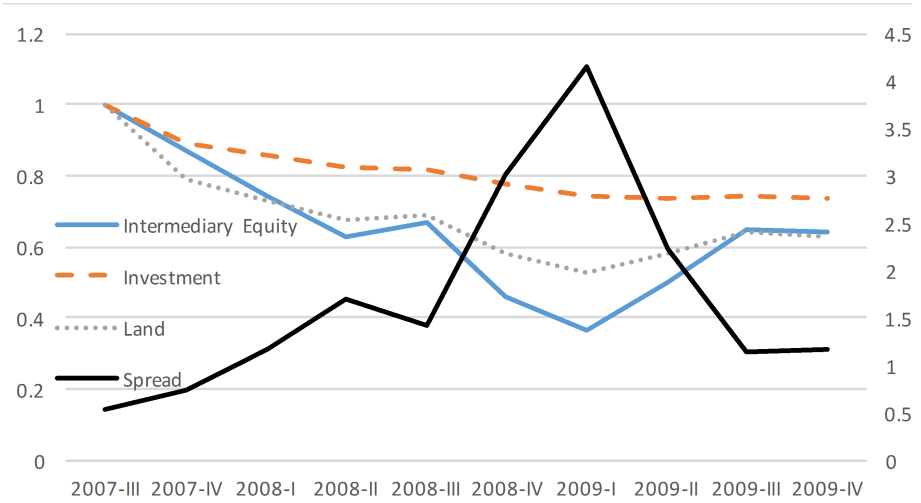
Matching State-Dependent Covariances

	Distress		Non Distress	
	Data	Baseline	Data	Baseline
$vol\,(Eq)$	25.73%	21.74	20.54	5.45
$vol\,(I)$	7.71%	6.01	5.79	4.97
$vol\,(C)$	1.72%	5.55	1.24	2.20
$vol\,(LP)$	15.44%	15.16	9.45	7.98
$vol\,(EB)$	66.66%	71.51	16.56	11.67
$cov\,(Eq, I)$	1.02%	0.95	-0.07	0.27
$cov\,(Eq, C)$	0.20%	-0.98	-0.01	-0.09
$cov\,(Eq, LP)$	2.38%	2.86	-0.43	0.43
$cov\,(Eq, EB)$	-8.50%	-8.94	0.60	-0.24

- Note: without the capital constraint, all volatilities would be 3%, and have no state dependence.

Matching Recent Crisis: *Data(L) and Model(R)*





- Based on EBS classification, economy crossed the 33% boundary ($e = 1.27$) between 2007Q3 and 2007Q4. Assume $e = 0.66$ in 2007Q3.
- Then choose $(Z_{t+1} - Z_t)$ shocks to match realized intermediary equity series.

07QIV	08QI	08QII	08QIII	08QIV	09QI	09QII	09QIII	09QIV
-5.0%	-1.5	-1.5	-0.9	-2.2	-2.6	-2.5	-0.7	-0.7

  - Total -16.3%. Capital constraint binds after 08Q3—systemic risk state
  - In the model (data), land price falls by 47% (32%)
  - In the model (data), investment falls by 23% (25%)

Probability of Systemic Event

- Based on EBS classification, we cross the 33% boundary ($e = 0.66$) between 2007Q3 and 2007Q4
- What is the likelihood of the constraint binding ("systemic crisis") assuming $e = 0.66$ currently:
  - 3.0% in next 1 years
  - 16% in next 2 years
  - 44% in next 5 years

Small...

VIX


VIX
90
80
70
60
50
40
30
20
10
0
Jan-06
Mar-06
May-06
Jul-06
Sep-06
Nov-06
Jan-07
Mar-07
May-07
Jul-07
Sep-07
Nov-07
Jan-08
Mar-08
May-08
Jul-08
Sep-08
Nov-08
Jan-09
Mar-09
May-09
Jul-09
Sep-09
Nov-09

Stress testing: “Hidden" Leverage

- Financial sector aggregate leverage fixed at 3.77 in model
- Suppose “hidden" leverage: leverage was 4.10 but agents take as given price functions and returns at leverage=3.77
- Prob. of hitting crisis rises from 16% to 30%!

Stress testing

Key step: Need to map from stress scenario into underlying shock, dZ_t.

- Say stress scenario $\Rightarrow$ -30% Return on equity
- Naive partial eqbm: leverage of 4, $\sigma(Z_{t+0.25} - Z_t) = -30/4 = -7.5\%$.
- Feed in -7.5% shock into the model over one quarter.
- Result: Beginning at $e = 0.66$ in 2007Q3, economy is immediately moved into crisis region
- our model helps in figuring out the right shock dZ_t

In US stress tests, scenario was over 6 quarters. Feed in shocks quarter-by-quarter, over 6 quarters:

Return on Equity	6 QTR Shocks	Prob(Crisis within next 2 years)
-2%	-1.0%	10.9 %
-5%	-2.3%	19.1%
-10%	-3.7%	31.97%
-15%	-5.7%	59.85%
-25%	-7.5%	100.00%

Summary

- Fully stochastic model of a systemic crisis, with an equity capital constraint on the intermediary sector
- Calibrated model matches differential comovements in distress and non-distress periods for US data
  - Replicate 2007/2008 period with only intermediary capital shocks
- Tool to map macro-stress tests into probability of systemic states: “Macro-VaR"