A model of dynamic compensation and capital structure ☆

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ABSTRACT

This paper studies the optimal compensation problem between shareholders and the agent in the Leland (1994) capital structure model, and finds that the debt-overhang effect on the endogenous managerial incentives lowers the optimal leverage. Consistent with data, our model delivers a negative relation between pay-performance sensitivity and firm size, and the interaction between debt-overhang and agency issue leads smaller firms to take less leverage relative to their larger peers. During financial distress, a firm's cash flow becomes more sensitive to underlying performance shocks due to debt-overhang. The implications on credit spreads and debt covenants are also considered.

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1. Introduction

This paper embeds optimal contracting between the agent (manager) and shareholders into the cash flow framework commonly used in the literature of structural models of capital structure (Leland, 1994). By connecting these two literatures, I provide a general framework to study the impact of agency characteristics on firm valuation and capital structure. Moreover, the dynamic nature of this framework allows me to calibrate my model and, in turn, quantitatively assess the agency impact on the firm's leverage decision.

I characterize the optimal contract between shareholders and the agent explicitly. In determining the leverage level, debt bears an additional "debt-overhang" cost relative to the bankruptcy cost in standard models (a la Leland, 1994): By interpreting the agent's effort as a form of investment, shareholders implement diminishing effort (as cut back investment) during financial distress. As a result, the agency problem reduces the optimal leverage from 63.2% based on Leland (1994, with my calibration) to as low as 39.5%. Consistent with the data, my model predicts that small firms take less leverage relative to their large peers. The debt-overhang problem also implies that the firm's cash flow is more sensitive to underlying shocks, reinforcing the standard leverage effect.

Section 2 starts by offering a general analysis in solving the optimal contracting problem. The analysis hinges on the agent's constant absolute risk aversion (CARA, or exponential) preference. In contrast to Holmstrom and Milgrom (1987), in which the lump-sum compensation is

considered, the agent in my model has intermediate consumption flows and can privately save. To solve for the optimal contract, I employ the approach in Sannikov (2008) and take the agent's continuation value (continuation payoff or promised utility) as one state variable. The absence of wealth effect under CARA preference allows me to characterize the optimal contract by an ordinary differential equation (ODE) in Section 2.3. I derive the (second-best) firm value and the agent's pay-performance sensitivity (PPS, the dollar-to-dollar measure as in Jensen and Murphy, 1990) based on the optimal contracting. I also characterize the condition that ensures the empirical regularity of an inverse relation between the agent's pay-performance sensitivity and firm size.

Section 3 applies the optimal contracting result to the framework in Leland (1994). There, the firm growth is endogenously affected by the agent's effort, and in the optimal contract both the pay-performance sensitivity and the firm growth are decreasing in firm size. To investigate the impact of agency issues on capital structure, Section 3.3 introduces debt into the baseline model. For better comparison with Leland (1994) and other related work, I leave the debt contract to take the same form as in Leland (1994). Specifically, only consol bond is considered, and shareholders have the option to default when the firm profitability deteriorates. On the contracting side, I bond the agent and shareholders together through an optimal contract solved in Section 2.[1] Furthermore, shareholders and the agent can revise the compensation contract dynamically, so that the compensation contract is a best response to the capital structure.[2] Essentially, these simplifying assumptions capture the key notion that, in United States corporations, managers are responsible only to shareholders (e.g., Brealey, Myers, and Allen, 2006).

I then solve for the optimal capital structure and the optimal employment contract in Section 3.3. Compared with Leland (1994), my model features a debt-overhang problem. Specifically, in my endogenous firm growth framework in which the firm growth is controlled by the manager or shareholders or both, when close to bankruptcy shareholders assign diminishing incentives to the agent. This result is due to debt-overhang, i.e., reducing the positive net present value (NPV) effort investment in financially distressed firms. In other words, beyond the standard bankruptcy cost, debt bears another form of cost, as debt-overhang interferes with agency issues. As a result, my model produces lower optimal leverage ratios relative to the Leland (1994) benchmark.

The debt-overhang generates a negative relation between leverage and agent's working incentives, a prediction opposite to Cadenillas, Cvitanic, and Zapatero (2004),

in which the debt level and agent's incentives are positively related. In that paper, they study a dynamic compensation and capital structure model in which the agent controls both the drift (effort) and the volatility (project selection) of the firm value. There, the agent's compensation space is restricted to equity shares, and shareholders commit to this static compensation scheme. Setting a higher leverage directly reduces the value of the agent's equity compensation. Under their assumption of the agent having log utility, this induces a higher sensitivity of the agent's value to his performance and, therefore, stronger working incentives. In contrast, I show that in a dynamic model, if shareholders and the agent can revise the compensation contract ex post,[3] then there is an opposite effect in addition to these channels, and it is an empirical question of which force prevails under various economic circumstances.

Further, the interaction between agency issue and debt-overhang predicts that smaller firms take less leverage, which is consistent with empirical regularity. In my model, shareholders in small firms implement a higher effort (or, higher investment) without debt, which matches the empirically observed negative relation between pay-performance sensitivities and firm sizes. Because the presence of debt cuts back effort investment, debt-overhang becomes more severe in small firms. Taking this higher debt-overhang cost into account, small firms choose a lower optimal leverage. In my calibrations for small firms, the predicted optimal leverage ratios, with or without the agency issue, can have a sizeable difference (63.2% versus 39.5%).

In the literature, several attempts have been made to incorporate other important agency issues into the corporate security pricing setting. For instance, Leland (1998) studies the risk-shifting issue due to the endogenous choice of firm's volatility; there, the agent and shareholders are treated as one party. This paper focuses on debt-overhang.[4] Moreover, this paper distinguishes itself from the above mentioned literature in that I study the agency impact based on the optimal dynamic contracting approach. Even though it seems appealing to restrict the compensation contract space within commonly observed forms as in Cadenillas, Cvitanic, and Zapatero (2004), one might wonder whether the derived impact of agency problems is sensitive to specific contract forms.[5] The optimal contracting approach is free of this issue.

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[1] In my model, the agent, once bonded with shareholders by an optimal contract, has perfectly aligned interest with shareholders when dealing with debt holders. As a result, the default policy is independent of whether shareholders or the agent control the bankruptcy decision. This is different from Morellec (2004), in which the agent tends to keep the firm alive longer for more private benefit.

[2] This assumption can be justified by the fact that, under this CARA framework, the long-term contract is renegotiation-proof and can be implemented by a sequence of short-term contracts (see Fudenberg, Holmstrom, and Milgrom, 1990).

[3] This assumption is consistent with both the practice of resetting the strike price of stock options, and the empirical results in Bryan, Hwang, and Lilien (2000) who investigate the stock-based compensations in a panel of firms (see footnote 29 for more details).

[4] Based on the free cash flow problem, Morellec (2004) introduces a tension between the agent and shareholders, and the empire-building agent tends to set a lower leverage ratio for rent-maximizing purposes. Cadenillas, Cvitanic, and Zapatero (2004) study a different version of agency problem, in which they restrict the compensation contract space to be equity. Because the equity payoff ties to the debt face value, in their model the capital structure becomes a direct compensation scheme. In contrast, in my model the impact of leverage decision on the compensation contract is indirect.

[5] Technically speaking, in the aforementioned papers, either the volatility choice in Leland (1998) which is observable in Brownian setting, or the over-investment (observable) decision in Morellec (2004), can be easily resolved by optimal contracting. These extreme examples illustrate the sensitivity of agency costs to the contracting space.

This paper is also related to the ongoing continuous-time contracting literature. Sannikov (2008) studies a general dynamic agency problem without private savings. Williams (2006) focuses on the general hidden-state problem and solves an example with CARA preference. My paper, based on the continuation value approach advocated in Sannikov (2008), analyzes the general cash flow setup and focuses on the applications to corporate finance. DeMarzo and Fishman (2007), DeMarzo and Sannikov (2006), and Biais, Mariotti, Plantin, and Rochet (2007) solve a dynamic contracting problem with a risk-neutral agent, in which the limited liability restriction is imposed.[6] In contrast, this paper takes the Holmstrom and Milgrom (1987) framework, which not only allows for a risk-averse agent, but also easily accommodates a second state variable to capture the firm's time-varying profitability.[7] Compared with Holmstrom and Milgrom (1987), I allow for the agent's intermediate consumption, and therefore their approach is no longer applicable.

The rest of this paper is organized as follows. Section 2 derives an ODE that characterizes the optimal contracting, and Section 3 applies the contracting result to Leland (1994). Section 4 concludes. All proofs are in the Appendix.

2. General model and optimal contracting

In this section, I first study the optimal contracting problem in a model with general cash flow process. Then based on the implementation of the optimal contract, I discuss its implications on pay-performance sensitivities (PPS) in executive compensation.

2.1. General model

I study an infinite-horizon, continuous-time agency problem. The firm (shareholders) hires an agent to operate the business. The firm produces cash flows δ_t per unit of time, where δ_t follows the stochastic process

$$d\delta_t = \mu(\delta_t, a_t)\, dt + \sigma(\delta_t)\, dZ_t. \tag{1}$$

I also interpret δ_t as firm size in this paper. Through unobservable effort $a_t \in [0, \overline{a}]$, the agent controls the cash flow growth rate $\mu(\delta_t, a_t)$, where $\mu_a(\delta, a) \equiv \partial\mu(\delta, a)/\partial a > 0$ and $\mu_{aa}(\delta, a) \equiv \partial^2\mu(\delta, a)/\partial a^2 \leq 0$. The performances $\{\delta_t\}$ are contractible.

Shareholders (the principal) are risk-neutral, and they discount future cash flows at the constant market interest rate r. To focus on the optimal contracting,

throughout Section 2 the firm is unlevered. I will introduce debt holders in Section 3.

The agent, with a CARA instantaneous utility and a time discounting factor r, maximizes his expected life time utility

$$\mathbb{E}\left[\int_0^\infty -\frac{1}{\gamma} e^{-\gamma(c_t - g(\delta_t, a_t)) - rt}\, dt\right],$$

where $c_t \in \mathbb{R}$ is the agent's consumption rate and $g(\delta, a)$ is the agent's monetary effort cost with $g_a(\delta, a) = \partial g(\delta, a)/\partial a > 0$ and $g_{aa}(\delta, a) = \partial^2 g(\delta, a)/\partial a^2 > 0$. To ensure that pay-performance sensitivity is falling with firm size in the optimal contract, later in Section 2.4.2 I impose restrictions on the dependence of $\mu(\delta, a)$ and $g(\delta, a)$ on firm size δ.

I allow for the agent's private (unobservable) savings. The agent can borrow and save at the risk-free rate r in his personal savings account. The account balance, as well as the agent's actual consumption, is unobservable to shareholders. It is the agent's intermediate consumption, associated with the possibility of private savings, that distinguishes my analysis from the classic Holmstrom and Milgrom (1987).

2.2. Contracting problem

I distinguish the policies recommended by the contract, from the agent's actual policies. The latter is indicated by a "hat" on top of the relevant symbols.

The employment contract $\Pi = \{c; a\}$ specifies the agent's recommended consumption process c and the recommended effort process $\{a\}$. The process $\{c\}$ can also be interpreted as the wage process. Both elements are adapted to the filtration generated by $\{\delta\}$. In other words, they are functions of the agent's performance history. To simplify the analysis, unless otherwise stated, in this subsection I assume that the effort process $\{a\}$ takes interior solutions.

For simplicity I assume that the agent's initial wealth is 0.[8] Given $\Pi = \{c; a\}$, the agent's problem is

$$V_0(\Pi) = \max_{\{\hat{c}_t, \hat{a}_t\}}\ \mathbb{E}\left[\int_0^\infty -\frac{1}{\gamma} e^{-\gamma(\hat{c}_t - g(\delta_t, \hat{a}_t)) - rt}\, dt\right] \tag{2}$$

s.t. $dS_t = rS_t\, dt + c_t\, dt - \hat{c}_t\, dt, \quad S_0 = 0,$
    $d\delta_t = \mu(\delta_t, \hat{a}_t)\, dt + \sigma(\delta_t)\, dZ_t,$

where $V_0(\Pi)$ denotes the agent's time-0 value derived from the contract Π, $\{S_t\}$ denotes the balance in the agent's savings

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[6] For extensions, e.g., He (2009) studies the optimal executive compensation in a geometric Brownian cash flow setting, and Piskorski and Tchistyi (2010) study the optimal mortgage design by considering the exogenous regime switching in the investors' discount rate.

[7] For another example among various extensions of Holmstrom and Milgrom (1987), Schattler and Sung (1993) offer a general treatment for the continuous-time contracting with CARA preference, but under the original Holmstrom and Milgrom (1987) setting, i.e., a finite time horizon with a lump-sum compensation. Because the lump-sum compensation (consumption) occurs at the end of employment period, as opposed to flows studied in this paper, there is no issue of private savings in Schattler and Sung (1993).

[8] This assumption is innocuous given the CARA preference. If the agent's initial wealth W_0 is observable, then the contract could ask the agent to hand over his wealth to the principal (shareholders), who can plan for the agent subsequently through the contract. Even if the initial wealth W_0 is the agent's private information, the absence of wealth effect implies that, facing any contract, the agent takes the same effort policy as another hypothetical agent with 0 initial wealth (except consuming rW_0 more each period). For details, see the argument in Section 2.3.2 and Lemma 3. Therefore, the principal can easily design an optimal scheme to induce truth-telling, in that the contract promises the agent rW_0 more per period if at $t = 0$ the agent hands over W_0 to the principal.

account, and transversality condition $\lim_{T\to\infty}\mathbb{E}[e^{-rT}S_T]=0$ is imposed for all feasible policies. Both the consumption policy $\{c\}$ and effort policy $\{a\}$ are "recommended" only. For instance, the first constraint states that, the change of the agent's savings dS_t is the interest accrual rS_tdt plus the wage deposit c_tdt and minus the consumption withdrawal $\hat{c}_tdt$. To save, the agent can set his consumption $\hat{c}_t$ strictly below the wage c_t.

Suppose that the agent has a time-0 outside option v_0. Shareholders solve the following problem

$$\max_{\Pi} \quad \mathbb{E}^{\delta(\Pi)}\left[\int_0^\infty e^{-rt}(\delta_t-c_t)\,dt\right]$$
$$\text{s.t.} \quad V_0(\Pi)\geq v_0,$$

where $\mathbb{E}^{\delta(\Pi)}[\cdot]$ indicates the dependence of probability measure (over $\{\delta\}$) on the employment contract Π when the agent solves his problem (2). The second line is the agent's participation constraint. As in Holmstrom and Milgrom (1987), under this CARA framework without limited liability, the participation constraint always binds, and the outside option v_0 affects only the optimal contract by a constant transfer between these two parties.

I define the class of incentive-compatible and no-savings contracts as follows.

Definition 1. A contract $\Pi=\{c;a\}$ is incentive-compatible and no-savings if the solution to the agent's problem (2) is $\{c;a\}$.

That is, a contract Π is incentive-compatible and no-savings if the agent, once facing the contract Π, finds it optimal to exert the recommended effort (i.e., incentive-compatible) and follow the recommended consumption plans (i.e., no-savings).

Because shareholders have the same saving technology (with rate r) as the agent, Lemma 2 allows me to focus on incentive-compatible and no-savings contracts only, a result similar to the "Revelation Principle." Essentially, when shareholders can fully commit, they can save for the agent on his behalf.

Lemma 2. It is without loss of generality to focus on the incentive-compatible and no-savings contracts.

2.3. Model solution

2.3.1. Agent's continuation value

Following the literature, in solving the optimal contract, I take the agent's continuation value (continuation payoff, or promised utility) as the state variable. Formally, given the contract $\Pi=\{c;a\}$, the agent's continuation value is defined as

$$V_t(\Pi,\delta_t)=\mathbb{E}_t\left[\int_t^\infty -\frac{1}{\gamma}e^{-\gamma(c_s-g(\delta_s,a_s))-r(s-t)}\,ds\right]. \qquad (3)$$

This payoff is a function of the compensation contract Π and the current cash flow level δ_t. To be specific, it is the agent's payoff (given δ_t) obtained under the policies specified by Π: The agent exerts effort policy $\{a_s:s\geq t\}$ recommended by Π and consumes exactly his future wages $\{c_s:s\geq t\}$. In Section 2.3.3, I shall invoke the

important fact that these recommended policies have to be optimal among all policies in the agent's problem (2).

By the martingale representation theorem (e.g., Sannikov, 2008), Eq. (3) implies that the agent's continuation value evolves as

$$dV_t=rV_t\,dt-u(c_t,a_t)\,dt+\beta_t(-\gamma rV_t)[d\delta_t-\mu(\delta_t,a_t)\,dt], \qquad (4)$$

where the agent's instantaneous utility $u(c_t,a_t)$ is

$$u(c_t,a_t)=-e^{-\gamma(c_t-g(\delta_t,a_t))}/\gamma,$$

and $\{\beta\}$ is a progressively measurable process. Here, $-\gamma rV_t>0$ [note that $V_t<0$ in (3)] is a scaling factor that facilitates the economic interpretation of β_t later in Section 2.3.3.

To read the evolution of the continuation value in Eq. (4), the agent's expected total value change is

$$\mathbb{E}_t[dV_t+u(c_t,a_t)\,dt]=rV_t\,dt,$$

which is the required return for the agent. The key element in Eq. (4) lies in the volatility part. It is the volatility of the agent's continuation value that controls the agent's working incentives. Intuitively, as clear from reading Eq. (4), the volatility part $\beta_t(-\gamma rV_t)[d\delta_t-\mu(\delta_t,a_t)\,dt]$ directly links to the observable performance $d\delta_t$, and $\beta_t(-\gamma rV_t)$ measures the punishing or rewarding extent in the employment contract. As a result, in Section 2.3.3 I connect $\beta_t(-\gamma rV_t)$, which is the incentive imposed by the contract, to the implemented effort a_t.

2.3.2. Absence of wealth effect

The CARA preference plays a key role in solving for the optimal contract. In essence, the absence of wealth effect allows me to derive the agent's deviation value (when he deviates to other off-equilibrium nonzero savings) based only on the agent's equilibrium value V without savings.

Lemma 3. At any time $t\geq 0$, consider a deviating agent with savings S who faces the contract Π, and denote by $V_t(S;\Pi,\delta_t)$ his deviation continuation value. Then

$$V_t(S;\Pi,\delta_t)=V_t(0;\Pi,\delta_t)\cdot e^{-\gamma rS}=V_t\cdot e^{-\gamma rS}, \qquad (5)$$

where $V_t(0;\Pi,\delta_t)$ is the agent's continuation value V_t along the no-savings path defined in Eq. (3).

The intuition is simple. For a CARA agent without wealth effect, given the extra savings S, his new optimal policy is to take the optimal consumption-effort policy without savings but to consume an extra rS more for all future dates $s\geq t$. Because

$$u(c_s+rS,a_s)=e^{-\gamma rS}u(c_s,a_s),$$

this explains the factor $e^{-\gamma rS}$ in Eq. (5). Essentially, for CARA preference, the agent's problem is translation-invariant to his underlying wealth level. Without CARA preference, the agent's working incentives is wealth-dependent, and the deviation value representations, as simple as Eq. (5), are no longer available.

2.3.3. Equilibrium evolution of V

For incentive-compatible and no-savings contracts, the recommended consumption-effort policies specified in Π

have to be optimal among all policies. Based on this requirement, I now derive the necessary and sufficient conditions for the equilibrium evolution of V_t in Eq. (4).

No Savings. Fix the effort policy first. By the optimality of the agent's consumption-savings policy in problem (2), his marginal utility from consumption must equal his marginal value of wealth:

$$u_c(c_t, a_t) = \frac{\partial}{\partial S} V_t(0; \Pi, \delta_t).$$

Therefore, the necessary condition for Π to rule out private savings is

$$u_c(c_t, a_t) = e^{-\gamma(c_t - g(\delta_t, a_t))} = \frac{\partial}{\partial S} V_t(0; \Pi, \delta_t) = -\gamma r V_t$$

$$\Rightarrow u(c_t, a_t) = r V_t, \qquad (6)$$

where the third equation uses the functional form of $V_t(S; \Pi, \delta_t)$ in Eq. (5). Plugging this result into Eq. (4), one observes that $u(c_t, a_t)$ just offsets $r V_t$, and Eq. (4) becomes

$$dV_t = \beta_t(-\gamma r V_t)[d\delta_t - \mu(\delta_t, a_t)\, dt]. \qquad (7)$$

Therefore, the agent's continuation value V_t follows a martingale.

Two points are noteworthy. First, because $u(c_t, a_t) = -e^{-\gamma(c_t - g(\delta_t, a_t))}/\gamma$, the relation $u(c_t, a_t) = r V_t$ implies that the equilibrium consumption (or the agent's wage) is

$$c_t = g(\delta_t, a_t) - \frac{\ln \gamma r}{\gamma} - \frac{1}{\gamma}\ln(-V_t). \qquad (8)$$

Second, because $u_c(c_t, a_t) = -\gamma r V_t$ as shown in Eq. (6), the agent's marginal utility also follows a martingale. Naturally, this is a consequence of the agent's optimal consumption-savings policy, which is in direct contrast to the optimal contracting with observable savings as studied in Rogerson (1985) and Sannikov (2008). There, the principal can dictate the agent's consumption plan that is suboptimal from the agent's view.

Incentive compatibility. Now I turn to incentive provision to pinpoint the diffusion loading β_t. In Eq. (7), $\beta_t(-\gamma r V_t)$ measures the agent's continuation utility sensitivity with respect to the unexpected performance $d\delta_t - \mu(\delta_t, a_t)\, dt$. Now the role of the scaling factor $-\gamma r V_t$ becomes clear: Because $-\gamma r V_t$ is the agent's marginal utility u_c as shown in (6), by transforming "utilities" to "dollars," β_t directly measures the (monetary) compensation sensitivity with respect to his performance.

Consider the agent's effort decision. On the one hand, choosing $\hat{a}_t$ affects the agent's instantaneous utility $u(c_t, \hat{a}_t)$. On the other hand, $\hat{a}_t$ sets the drift of his performance $d\delta_t$, which affects his expected continuation payoff $\mathbb{E}_t[dV_t(\hat{a}_t)]$ in Eq. (7) via $\beta_t \cdot u_c \cdot \mu(\delta_t, \hat{a}_t)$. By balancing the impacts on his instantaneous utility and continuation payoff, the agent is solving

$$\max_{\hat{a}_t} u(c_t, \hat{a}_t) + \beta_t \cdot u_c \cdot \mu(\delta_t, \hat{a}_t).$$

Because $u(c_t, \hat{a}_t) = u(c_t - g(\hat{a}_t))$, I have $u_a = u_c \cdot (-g_a(\delta_t, a_t))$. Therefore, implementing $\hat{a}_t = a_t$ requires that

$$-g_a(\delta_t, a_t) + \beta_t \mu_a(\delta_t, a_t) = 0 \Rightarrow \beta_t = \frac{g_a(\delta_t, a_t)}{\mu_a(\delta_t, a_t)}, \qquad (9)$$

and it is easy to check that this first-order condition is also sufficient.[9]

Eq. (9) gives an equilibrium relation between the recommended effort a_t and the incentive loading β_t. Intuitively, $\mu_a(\delta_t, a_t)$ is the agent's effort impact on the instantaneous performance, and $\beta_t \mu_a(\delta_t, a_t)$ gives the agent's monetary marginal benefit of his effort. To be incentive-compatible, the marginal benefit must equal the agent's monetary marginal effort cost $g_a(\delta_t, a_t)$. And, because g (or μ) is convex (or concave) in a, one can show that the required incentive loading β_t is increasing in a_t. In other words, implementing a higher level of effort needs greater incentives.

In sum, for Π to be incentive-compatible and no-savings, it must be that [recall Eq. (7)]

$$dV_t = \frac{g_a(\delta_t, a_t)}{\mu_a(\delta_t, a_t)} \gamma r(-V_t)\sigma(\delta_t)\, dZ_t, \qquad (10)$$

where the innovation term in (7) is replaced by $\sigma(\delta_t)\, dZ_t$ due to Eq. (1).

So far, I have used the agent's first-order conditions (FOCs) regarding the recommended consumption and effort to derive necessary conditions for the dynamics of V_t. It is well known that with private savings, FOCs cannot guarantee the global optimality of the recommended policies (e.g., Kocherlakota, 2004, and He, 2010). However, for the case of CARA preference without wealth effect, FOCs are both necessary and sufficient, a result that I verify in the working paper version of this paper.[10]

2.3.4. Optimal contracting

Given the state variables δ and V, the shareholders' value function is

$$J(\delta, V) = \max \quad \mathbb{E}\left[\int_t^\infty e^{-r(s-t)}(\delta_s - c_s)\, ds \,\Big|\, \delta_t = \delta\right] \qquad (11)$$

$$\text{s.t.} \quad V_t(\Pi, \delta) = V.$$

The absence of wealth effect, thanks to the CARA preference, leads to the guess of

$$J(\delta, V) = f(\delta) - \frac{-1}{\gamma r}\ln(-\gamma r V),$$

where $-1/\gamma r \ln(-\gamma r V) > 0$ is the agent's certainty-equivalent given his continuation value V. I refer to the agent's certainty-equivalent as the agent's inside stake in later discussions.

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[9] The main driving force underlying Eq. (9) is the monetary effort cost specification, i.e., $u(c_t, a_t) = u(c_t - g(a_t))$, not the CARA preference. To see this, if I write $dV_t = r V_t\, dt - u(c_t, a_t)\, dt + \beta_t \cdot u_c \cdot [d\delta_t - \mu(\delta_t, a_t)\, dt]$ in Eq. (4), then β_t is still the monetary incentive loading measured in dollars, and the same argument gives the result in Eq. (9). However, as shown in Eq. (6), the CARA preference implies a convenient result that $u_c = -\gamma r V_t$, which makes the evolution of the agent's continuation value dependent on V itself (not consumption c).

[10] It is available at http://faculty.chicagobooth.edu/zhiguo.he/research/06132009newversionname.pdf.

Using Eqs. (1) and (10), the Hamilton-Jacobi-Bellman (HJB) equation for the shareholders' problem in (11) is

$$rJ(\delta,V) = \max_{a\in[0,\bar{a}]} \left\{ \begin{array}{c} \delta - c(a,\delta,V) + J_\delta \cdot \mu(\delta,a) + \frac{1}{2}J_{\delta\delta}\sigma(\delta)^2 \\ J_{\delta V}\frac{g_a(\delta_t,a_t)}{\mu_a(\delta_t,a_t)}\gamma r(-V_t)\sigma(\delta_t)^2 + \frac{1}{2}J_{VV}\gamma^2 r^2 V^2 \left[\frac{g_a(\delta_t,a_t)\sigma(\delta_t)}{\mu_a(\delta_t,a_t)}\right]^2 \end{array} \right\},$$

$$(12)$$

where $c(a,\delta,V)$ takes the form in Eq. (8). Plugging Eq. (8) into Eq. (12), and noting that $J_\delta = f'(\delta)$, $J_{\delta\delta} = f''(\delta)$, $J_{VV} = -(1/\gamma r)1/V^2$, and $J_{\delta V} = 0$, the following ODE is obtained for $f(\cdot)$:

$$rf(\delta) = \max_{a\in[0,\bar{a}]} \left\{ \delta + f'(\delta)\mu(\delta,a) + \frac{1}{2}f''(\delta)\sigma(\delta)^2 - g(\delta,a) - \frac{1}{2}\gamma r\left[\frac{g_a(\delta,a)\sigma(\delta)}{\mu_a(\delta,a)}\right]^2 \right\}.$$

$$(13)$$

Here, δ is cash inflow, the second and third terms capture the expected instantaneous change of $f(\delta)$ due to δ, and the last two terms are the effort-related costs. The optimal effort a^* is characterized by

$$\underset{a\in[0,\bar{a}]}{\arg\max} \left\{ f'(\delta)\mu(\delta,a) - g(\delta,a) - \frac{1}{2}\gamma r\left[\frac{g_a(\delta,a)\sigma(\delta)}{\mu_a(\delta,a)}\right]^2 \right\} \qquad (14)$$

Similar to Holmstrom and Milgrom (1987), in Eq. (13) there are two kinds of costs in implementing effort a. The first is the direct monetary effort cost $g(\delta,a)$, and the second is the risk-compensation term for the risk-averse agent to bear incentives:

$$\frac{1}{2}\gamma r\left[\frac{g_a(\delta_t,a_t)\sigma(\delta_t)}{\mu_a(\delta_t,a_t)}\right]^2. \qquad (15)$$

This additional agency-related cost, as in Holmstrom and Milgrom (1987), captures the key trade-off between incentive provision and risk-sharing in the optimal contract.

The solution to Eq. (13), combined with Eqs. (8) and (10), and certain problem-specific boundary conditions, characterizes the optimal contracting. In the working paper version of this paper, I provide a detailed verification argument to show rigorously that the derived contract is optimal.

2.4. Model implications

2.4.1. Firm value and agent's deferred compensation: an implementation

I interpret $f(\delta_t)$ as the firm value. From the derivation in Section 2.3.4, maximizing shareholders' value $J(\delta_t,V_t)$ is equivalent to maximizing the firm value $f(\delta_t)$ in this model, as both aim to minimize the agency cost.

The agency cost under the CARA setup has one particular feature. Given the promised continuation value V_t to the agent, the cost of delivering V_t, from the shareholders' view, is its certainty equivalent $(-1/\gamma r)\ln(-\gamma rV_t)$, plus some additional agency cost due to inefficient incentive-risk allocation. Under the CARA setup, this additional agency cost is independent of the agent's continuation value V_t.[11] In other words, the severity of agency problems are reflected only in the functional form of $f(\cdot)$ as a

solution to the ODE Eq. (13), not the agent's continuation value per se.

From the view of implementation, the firm value $f(\delta_t)$ is the sum of the (common) shareholders' value $J(\delta_t,V_t)$, plus the agent's inside stake, which is measured by his certainty equivalent $-(1/\gamma r)\ln(-\gamma rV_t)$. Specifically, in implementing the optimal contract, shareholders set up a deferred-compensation fund inside the firm with a balance

$$W_t = -\frac{1}{\gamma r}\ln(-\gamma rV_t), \qquad (16)$$

and shareholders adjust this balance continuously according to the evolution of V_t in Eq. (10).[12] By keeping the agent's stake inside the firm, the firm (market) value becomes the total value enjoyed by both the agent and shareholders. To the extent that in practice the agent's non-marketable rent (e.g., future wages) is small relative to the firm value, this treatment is a close approximation.[13]

Under the optimal employment contract, the deferred compensation fund W_t follows:

$$dW_t = \frac{g_a(\delta_t,a_t)}{\mu_a(\delta_t,a_t)}\sigma(\delta_t)\,dZ_t + \frac{1}{2}\gamma r\left[\frac{g_a(\delta_t,a_t)\sigma(\delta_t)}{\mu_a(\delta_t,a_t)}\right]^2\,dt. \qquad (17)$$

Here, the first diffusion term provides incentives, and the second drift term captures risk compensation. Interestingly, under the optimal contract, the agent's consumption c_t cancels with the interest rW_t earned by the deferred-compensation fund and the effort cost reimbursement g_t [check Eq. (8)].

2.4.2. Pay-performance sensitivity and size dependence

By interpreting the deferred-compensation balance W_t as the agent's financial wealth, I can derive the agent's pay-performance sensitivity in this model. In the literature, the executive's (dollar-to-dollar) pay-performance sensitivity (PPS) has received great attention since Jensen and Murphy (1990). The central question, whose answer is just PPS, is that: "how much does the executive's wealth change when the firm value changes by one dollar?"

In the current continuous-time framework, the agent's pay-performance sensitivity can be measured by the response of the balance of W_t, to a unit shock of the firm

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(footnote continued)
the lower the agent's continuation value, the more likely the agent is terminated and, therefore, the higher the agency cost.

[12] When I introduce bankruptcy in Section 3, it is important to ensure that this deferred compensation has priority over debt in the event of default. As in Westerfield (2006), this balance can also be interpreted as the committed separation payment if either party wants to renege in the future. Theoretically, the CARA framework cannot rule out the possibility of $W_t < 0$. I interpret this case as the agent to take a personal debt, and the debt is netted out in calculating the total firm value.

[13] In this implementation shareholders conduct all the savings for the agent, as his wealth is kept inside the firm. Another equivalent implementation is to put W_t into the agent's personal account and allow for two-way transfers between the agent and shareholders according to Eq. (10). This corresponds to the case in which the agent's entire rent is non-marketable, and the firm's market value becomes $J(\delta,V)$.

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[11] This is different from other dynamic agency models with inefficient termination (e.g., DeMarzo and Fishman, 2007) in which the agency cost is linked to the agent's continuation value directly. There,

value.[14] Specifically, by neglecting all drift terms, I have [recall that $\beta_t(\delta_t,a_t) = g_a(\delta_t,a_t)/\mu_a(\delta_t,a_t)$ in Eq. (9)]

$$PPS = \frac{dW_t}{df(\delta_t)} = \frac{\beta_t(\delta_t,a_t^*)\sigma(\delta_t)dZ_t}{f'(\delta_t)\sigma(\delta_t)dZ_t} = \frac{\beta_t(\delta_t,a_t^*)}{f'(\delta_t)} = \frac{g_a(\delta_t,a_t^*)}{\mu_a(\delta_t,a_t^*)}\frac{1}{f'(\delta_t)}. \tag{18}$$

Intuitively, PPS is the ratio between β_t, which is the agent's dollar incentive, and $f'(\delta_t)$, which captures the value change (in dollars) of the firm. The optimal effort a_t^* in Eq. (18) is endogenously determined by the optimization problem in Eq. (14).

The result in Eq. (18) implies that the agent's pay-performance sensitivity depends on firm size δ_t. My later calibration aims to replicate the well-known empirical regularity that PPS is negatively related to firm size (e.g., Murphy, 1999).[15] To this end, I now impose some structure on my model to study the general pattern of relation between PPS and firm size.

When does PPS decrease with firm size? Suppose that

$$\mu(\delta_t,a_t) = \mu_0(\delta_t) + a_t\delta_t^{\mu_1}, \quad \sigma(\delta_t) = \sigma\delta_t^{\sigma_1},$$

and $\quad g(\delta_t,a_t) = g_0(\delta_t) + \frac{\theta}{2}a_t^2\delta_t^{g_1},$

which imply that

$$\mu_a(\delta_t,a_t) = \delta_t^{\mu_1} \quad \text{and} \quad g_a(\delta_t,a_t) = \theta a_t\delta_t^{g_1}, \tag{19}$$

Here, μ_1, σ_1, θ, and g_1 are constants. This general specification encompasses Baker and Hall (2004), who argue that the effort impact on the firm growth (which I refer to as effort benefit) might be size-dependent, i.e., $\mu_1 > 0$. I focus on μ_1, g_1 and σ_1 which characterize the dependence of the agent's effort benefit, direct monetary effort cost, and indirect risk-compensation cost on firm size, respectively.

Given this structure, the first-order condition for Eq. (14) (assuming an interior solution a_t^*) is

$$f'(\delta)\delta_t^{\mu_1} - \theta a_t^*\delta_t^{g_1} - \gamma r\sigma^2\theta^2 a_t^*\delta_t^{2(g_1+\sigma_1-\mu_1)} = 0,$$

which implies the optimal effort as

$$a_t^* = \frac{f'(\delta)\delta_t^{\mu_1}}{\theta\delta_t^{g_1} + \gamma r\sigma^2\theta^2\delta_t^{2(g_1+\sigma_1-\mu_1)}}. \tag{20}$$

Plugging Eqs. (19) and (20) into Eq. (18), one can show that $f'(\delta)$ cancels, and

$$PPS = \frac{1}{1 + \theta\gamma r\sigma^2\delta_t^{g_1+2\sigma_1-2\mu_1}}. \tag{21}$$

Therefore, the necessary and sufficient condition for a negative relation between PPS and firm size δ_t is

$$g_1 + 2\sigma_1 - 2\mu_1 > 0. \tag{22}$$

In other words, when the size-dependence of the effort cost (either direct cost part g_1 or indirect cost part σ_1) is sufficiently large, or the size-dependence of effort benefit is sufficiently small, the firm should design an incentive contract whose power is decreasing in firm size.

When I apply the optimal contracting results to the Leland framework in Section 3, I set

$$\mu(\delta,a) = (\phi+a)\delta, \quad \sigma(\delta) = \sigma\delta, \quad \text{and} \quad g(\delta,a) = \frac{\theta}{2}a^2\delta.$$

Here, $g_1 = \sigma_1 = \mu_1 = 1$, and $g_1 + 2\sigma_1 - 2\mu_1 = 1$. Therefore the PPS in the optimal contract is

$$PPS = \frac{1}{1 + \theta\gamma r\sigma^2\delta_t},$$

which is falling with firm size.

Several attempts are made in the literature to estimate these parameters. It is well known (as the leverage effect) that the large firm has a greater dollar variance but a smaller return variance, i.e., $\sigma_1 \in (0,1)$. Cheung and Ng (1992) fit an EGARCH (exponential general autoregressive conditional heteroskedasticity) model with CEV (constant elasticity of variance) specification to a large sample of individual stocks (as opposed to certain stock index, which is common in this literature) and find that σ_1 falls in the range of 0.84 (in the 1960s) and 0.94 (in the 1980s).[16] This estimation is subject to the caveat that δ_t is being approximated by the firm's stock price. For μ_1 and g_1, Baker and Hall (2004) assume that the agent's effort cost is independent of firm size (i.e., $g_1 = 0$) and find that $\mu_1 = 0.4$. If I instead set $g_1 = 1$, then one can show that the effort benefit measure that Baker and Hall (2004) are estimating is effectively $\mu_1 - 0.5$. Therefore, the estimate for μ_1 in my model (with $g_1 = 1$) is approximately 0.9 (close to the choice of $\mu_1 = 1$ in Section 3). The bottom line is, the condition Eq. (22) that guarantees a negative relation between PPS and firm size holds for these estimates, which extends indirect support to my model.

CRRA (power) preference. My entire analysis hinges on the assumption of CARA (constant-absolute-risk-aversion, exponential) preference, which implies that the agent's risk absorbing capacity is independent of his wealth level. As an important ingredient for PPS, the risk absorbing capacity directly determines the risk compensation cost in Eq. (15), which in turn pins down the size-dependence of PPS in Eq. (21). What can we say if instead

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[14] Strictly speaking, in the executive compensation literature, the pay-performance sensitivity is with respect to the shareholders' value, which should exclude the agent's non-marketable stake. There are two reasons that this treatment is inessential: (1) the magnitude of PPS is small (1−5%), and (2) empirically, the executive's PPS mainly comes from his or her inside holdings, which are marketable. For other definitions of performance sensitivities (e.g., pay-performance elasticities) and an agency model distinct from the Holmstrom and Milgrom (1987) framework, see Edmans, Gabaix, and Landier (2009).

[15] Typically, the PPS in executive compensation literature considers only the chief executive officer's incentive holdings. My model takes this interpretation as well, so that the agent is the single top manager of the firm. Readers can also interpret the manager here as a team of top managers, and the relevant PPS measure becomes the inside holdings of the firm's officers and directors. Even though it is theoretically possible that a larger firm might have more top managers who, as a team, hold more inside shares, empirically the opposite holds. For instance, Holderness, Kroszner, and Sheehan (1999) show a negative relation between the total ownership of officers and directors and firm size.

[16] One important determination of σ_1 is the correlation among the projects taken by the firm. Conditional on firm size, σ_1 tends to be lower for conglomerate firms (so the projects have diversified activities) than specialized firms (so the projects are highly correlated), which suggests different implications for PPS across these two types of firms. I thank the referee Hayne Leland for this excellent point.

the agent has a CRRA (constant relative risk aversion, power) preference?

Unfortunately, the wealth effect in the CRRA preference complicates the optimal contracting significantly, and not much is known about the solution to that problem.[17] In the literature, several attempts are made to accommodate this question. For example, Baker and Hall (2004) solve a static optimal contracting problem as if the agent has an exponential utility. However, they specify the agent's absolute risk aversion parameter $\gamma(W_t) = \gamma_0/W_t$ (where γ_0 is a positive constant) to be proportional to the inverse of his wealth $1/W_t$, as if the agent has a power utility. Here I take this simple approach as well.

It is important to note that the agent's wealth W_t is not necessarily proportional to firm size δ_t. Ideally I would like to derive the path of W_t endogenously from the model, but it is not available in CARA setting.[18] Because the question at hand is a calibration question, I resort help from data. Empirically, it is well known that, managers in larger firms, although get higher pay in terms of dollar amounts, have lower inside stakes (Murphy, 1999). Most of literature (e.g., Baker and Hall, 2004) use the manager's total compensation $Comp_t$ to approximate his wealth W_t. In fact, Gabaix and Landier (2008) calibrate that $Comp_t \propto \delta_t^{1/3}$, i.e., the elasticity between pay level and firm size is 1/3. Given this result, I set the agent's absolute risk aversion parameter to be

$$\gamma(W_t) = \hat{\gamma}_0 \delta_t^{-1/3},$$

where $\hat{\gamma}_0$ is another positive constant potentially different from γ_0. Plugging this result into Eq. (21) yields

$$PPS = \frac{1}{1 + \theta\hat{\gamma}_0 r\sigma^2 \delta_t^{g_1 + 2\sigma_1 - 2\mu_1 - 1/3}}.$$

Therefore, under the CRRA preference, the necessary and sufficient condition that ensures a negative relation between PPS and firm size δ_t becomes

$$g_1 + 2\sigma_1 - 2\mu_1 > \tfrac{1}{3}. \tag{23}$$

This condition holds for the case $g_1 = \sigma_1 = \mu_1 = 1$ that is studied in Section 3, as well as for the empirical estimates of $\{g_1, \sigma_1, \mu_1\}$ discussed in the previous subsection. Thus, even taking into account the fact that the agent might have a risk-aversion decreasing with his wealth (as implied by CRRA preferences), my model has qualitatively similar predictions under reasonable parameterizations.

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[17] Edmans, Gabaix, Sadzik, and Sannikov (2009) consider a multiplicative effort cost model and impose some modification on the timing structure to make the linear contract optimal in every instant. They solve a long-term optimal contracting problem in closed-form if the optimal contract aims to implement a maximum target effort (exogenously given). In the optimal contract, the implemented effort is the constant target effort level, and the return PPS (i.e., the log change of manager's compensation to the log change of firm value) is also a constant independent of firm size.

[18] In the theoretical result with CARA preference, W_t in Eq. (17) can be negative, which is inappropriate to define $\gamma(W_t) = \gamma_0/W_t$.

3. Revisiting Leland, 1994: optimal capital structure

This section applies the optimal contracting results derived in Section 2 to Leland (1994) with debt holders, and studies the interaction between dynamic compensation and capital structure.

3.1. Model specification

Following Leland (1994), I consider the case that

$$d\delta_t = (\phi + a_t)\delta_t\, dt + \sigma\delta_t\, dZ_t,$$

where ϕ and σ are constants. In the language of Eq. (1), $\mu(\delta, a) = (\phi + a)\delta$ and $\sigma(\delta) = \sigma\delta$. Here, ϕ is the baseline growth level, and by exerting effort the agent can accelerate the firm growth. The agent's effort cost takes the form $g(\delta, a) = (\theta/2)a^2\delta$, which is quadratic in a and linear in size δ.

Recall that in Section 2.1 I restrict the agent's action space to a bounded interval $[0, \overline{a}]$, and the calibration in the unlevered firm might call for a binding effort $a_t = \overline{a}$ in the optimal contract. In fact, under the parametrization considered later, the first-best solution has $a_t^{FB} = \overline{a}$. To characterize the first-best solution, I simply set $\gamma = 0$ in Eq. (13) (so there is no agency problem), and, as a result,

$$rf^{FB}(\delta) = \max_{a \in [0,\overline{a}]} \left\{ \delta + f^{FB\prime}(\delta)(\phi + a)\delta + \frac{1}{2}f^{FB\prime\prime}(\delta)\sigma^2\delta^2 - \frac{\theta}{2}a^2\delta \right\}, \tag{24}$$

where $f^{FB}(\delta)$ is the first-best firm value without agency problems. Because all model elements are proportional to δ, I guess that $f^{FB}(\delta) = A^{FB}\delta$, where A^{FB} is a constant to be solved. Plugging into Eq. (24) yields

$$rA^{FB} = \max_{a \in [0,\overline{a}]} \left\{ 1 + A^{FB}(\phi + a) - \frac{\theta}{2}a^2 \right\},$$

which jointly determines a^{FB} and A^{FB}. In the Appendix (Section A.3) I give the exact condition under which a^{FB} binds at $\overline{a}$.

Independent of whether a^{FB} binds at $\overline{a}$ or not, the scale-invariance of this model implies that, in the first-best case, the firm's cash flow – as well as the firm value – follows a geometric Brownian motion. Due to its analytical convenience, this setup has become the workhorse in the literature of structural models of capital structure (e.g., Leland, 1994; Goldstein, Ju, and Leland, 2001).

3.2. Optimal contracting in an unlevered firm

Before introducing debt into this framework, I apply the optimal contracting results obtained in Section 2 to an unlevered firm. To implement effort a_t, Eq. (9) implies that the agent's incentive slope $\beta_t = \theta a_t$. Then Eq. (13) becomes

$$rf(\delta) = \max_{a \in [0,\overline{a}]} \left\{ \delta + f'(\delta_t) \cdot (\phi + a)\delta + \frac{1}{2}f''(\delta_t)\sigma^2\delta^2 - \frac{\theta}{2}a^2\delta \right. $$
$$\left. - \frac{1}{2}\gamma r\theta^2 a^2\sigma^2\delta^2 \right\}. \tag{25}$$

Simple calculation yields the optimal effort as (the optimal effort might bind at $\overline{a}$ along the optimal path)[19]:

$$a_t^* = \min\left(\frac{f'(\delta_t)}{\theta(1+\theta\gamma r\sigma^2\delta_t)}, \overline{a}\right). \qquad (26)$$

As discussed in Section 2.4.2, when the optimal effort level is interior, PPS is decreasing in firm size δ_t as

$$PPS = \frac{1}{1+\theta\gamma r\sigma^2\delta_t}, \qquad (27)$$

Fundamentally, this result is due to the fact that, as the firm grows, the quadratic risk-compensation cost $\frac{1}{2}\gamma r\theta^2 a^2\sigma^2\delta_t^2$ is in the order of δ_t^2, while the incentive benefit is in the order of δ_t [check Eq. (25); in the Appendix (Section A.5) I show that when $\delta_t \to \infty$, $f'(\delta_t) \to 1/(r-\phi)$]. This exactly reflects the common wisdom that managers in larger firms have lower powered incentive schemes due to risk considerations.[20]

Another appealing feature, which is closely related to the pattern of PPS falling in firm size, is that the endogenous firm growth rate $\phi + a_t^*$ is decreasing in firm size δ_t as well (see Section 3.4.3 for a numerical example). The negative relation between firm size and growth is studied in, for instance, Cooley and Quadrini (2001). In this model, because incentivizing the agent is more costly in larger firms, the optimal contract implements a lower effort in larger firms, leading to a lower growth.

3.3. Optimal employment contract and leverage

3.3.1. Additional assumptions

Suppose that the firm issues debt to take advantage of tax shields. Relative to the standard bilateral contracting framework between investors (the principal) and the agent, now I have heterogenous investors—shareholders and debt holders. To abstract from complicated contracting issues among three parties, I make the following assumptions. First, the debt contract takes the form studied in Leland (1994), i.e., only the consol bond (with a constant coupon rate C) is considered, and shareholders (with their perfectly aligned agent when they are dealing with debt holders) can endogenously default when the firm's financial status deteriorates. Second, shareholders

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[19] When a binds at $\overline{a}$, the same incentive loading $\beta_t = \theta\overline{a}$ applies: Investors can set a higher incentive loading, but it is costly to do so because the agent is risk-averse. And, because firm value is increasing in the cash flow level δ, one can formally show that in this model $f(\delta)$ is always positive, therefore a^* never binds at zero. For formal proofs, see the Appendix (Section A.4).

[20] For instance, (Murphy, 1999, p. 2531) states: "The inverse relation between company size and pay-performance sensitivities is not surprising, since risk-averse and wealth-constrained CEOs of large firms can feasibly own only a tiny fraction of the company…The result merely underscores that increased agency problems are a cost of company size that must weighed against the benefits of expanded scale and scope." Of course, this reasoning is precise when the manager's risk-absorbing capacity is independent of firm size, which holds only for CARA preference. However, this statement is probably better interpreted as following: Even though the managers in larger firms might have greater risk-absorbing capacity (presumably because they receive higher pay), their greater risk-absorbing capacity cannot offset the higher total risk in larger firms. For a related discussion of CRRA preference, see Section 2.4.2.

can fulfill the promise of the agent's continuation value at bankruptcy as a part of employment contract. In other words, when bankruptcy occurs, the agent is guaranteed with the deferred-compensation fund W defined in Eq. (16). This assumption is commented upon in Section 3.4.2.

Another important timing assumption is that, in this model, shareholders design the employment contract as an optimal response to the leverage decision. Theoretically, this is consistent with the fact that, in this CARA framework, a long-term optimal contract can be implemented by a sequence of short-term contracts (Fudenberg, Holmstrom, and Milgrom, 1990). Essentially, in the CARA framework studied here, shareholders and the agent can revise the contract (as long as both parties agree to do so) once the debt is issued, which generates the debt-overhang problem analyzed in Section 3.4.2.[21]

These assumptions represent a minimum, but essential, departure from Leland (1994). They reflect the key economic rationale regarding the manager's objective in corporations in United States: Managers are supposed to be responsible to shareholders only (Brealey, Myers, and Allen, 2006).

3.3.2. Equity value and endogenous default

Similar to the case of unlevered firms, the shareholders' value function is $J^E(\delta, V) = f^E(\delta) + (1/\gamma r)\ln(-\gamma rV)$. The separability between δ and V hinges on the assumption that in the leveraged firm the shareholders can always keep the promise of delivering the continuation payoff V to the agent, even when the firm goes bankrupt at $\delta = \delta_B$. In the implementation, the promise is guaranteed by the deferred-compensation fund, which has priority over debt in the event of bankruptcy.

As in Section 2.3.4, by writing down the HJB equation for $J^E(\delta, V)$, the following ODE is reached for the equity value $f^E(\delta)$, where the control is over a_t and the bankruptcy boundary δ_B:

$$rf^E(\delta) = \max_{a \in [0,\overline{a}], \delta_B} \left\{ \delta - C(1-\tau) + f^{E'}(\delta) \cdot (\phi+a)\delta + \frac{1}{2}f^{E''}(\delta)\sigma^2\delta^2 \right.$$
$$\left. -\frac{1}{2}\theta a^2\delta - \frac{1}{2}\gamma r\theta^2 a^2\sigma^2\delta^2 \right\}, \qquad (28)$$

where C is the coupon rate and τ is the corporate tax rate. The equity value $f^E(\delta)$ is the sum of (common) shareholders' value $J^E(\delta, V)$ and the agent's deferred-compensation fund $-(1/\gamma r)\ln(-\gamma rV)$. Compared with Eq. (25) without debt, Eq. (28) has an additional cash outflow $C(1-\tau)$ as the after-tax coupon payment. Similar to Eq. (26), the optimal effort policy is

$$a_t^* = \min\left(\frac{f^{E'}(\delta_t)}{\theta(1+\theta\gamma r\sigma^2\delta_t)}, \overline{a}\right). \qquad (29)$$

Plugging it into Eq. (28) yields the ODE to characterize the optimal contracting.

When δ_t falls to a certain level, say, δ_B, shareholders refuse to serve the coupon payment by simply declaring

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[21] In this CARA setting, the resulting optimal contract is renegotiation-proof, as the Pareto boundary is always downward-sloping, i.e., $J_V = 1/\gamma rV < 0$ (V is negative in this model).

bankruptcy. This is captured by the value-matching boundary condition

$$f^E(\delta_B) = 0 \qquad (30)$$

and the smooth-pasting condition

$$f^{E'}(\delta_B) = 0. \qquad (31)$$

Both conditions are standard in this literature (e.g., Leland, 1994). These conditions are a result of maximizing the shareholders' value $J^E(\delta, V)$. But because these policies are toward debt holders, it is equivalent to maximizing $f^E(\delta)$, i.e., the joint (ex post) surplus enjoyed by shareholders and the agent.[22]

For the boundary condition on the other end, when δ_t takes a sufficiently large value $\overline{\delta} \to \infty$, the bankruptcy event is negligible. In the Appendix (Section A.5) I show that

$$f^E(\overline{\delta}) \simeq \overline{f}(\overline{\delta}) - \frac{C(1-\tau)}{r}, \qquad (32)$$

where $\overline{f}(\cdot)$ captures the firm value under a Gordon growth model with a growth rate ϕ [see Eq. (34)], and $C(1-\tau)/r$ is the value for a perpetual after-tax coupon payment. Then one can numerically solve for $f^E(\cdot)$ and δ_B, based on Eq. (30)–(32). For detailed numerical methods, see the Appendix (Section A.5).

3.3.3. Debt value and capital structure

Given the implemented effort policy $a^*(\delta)$ in Eq. (29), one can evaluate the consol bond with a promised coupon rate C. Because debt holders anticipate the optimal contracting between shareholders and the agent, the value of the corporate debt, $D(\delta)$, satisfies

$$rD(\delta) = C + D_\delta(\delta) \cdot (\phi + a^*(\delta))\delta + \tfrac{1}{2}D_{\delta\delta}(\delta)\sigma^2\delta^2,$$

with $D(\delta_B) = (1-\alpha)f(\delta_B)$, where $\alpha < 1$ is the percentage bankruptcy cost and $D(\overline{\delta}) \to C/r$ as $\overline{\delta} \to \infty$. Here I simply assume that, once bankruptcy occurs, debt holders pay the bankruptcy cost $\alpha f(\delta_B)$ and then keep running the project as an unlevered firm.[23]

Given the time-0 cash flow level δ_0, shareholders choose coupon C to maximize the total levered firm value $f^E(\delta_0; C) + D(\delta_0; C)$ before the debt issuance. They then design the optimal contract with an agent who has an outside option v_0. As discussed in Section 3.3.1, this timing assumption is equivalent to allowing shareholders and the agent to revise the employment contract ex post after the debt issuance.

The firm's optimal leverage ratio is defined as

$$LR(\delta_0) \equiv \frac{D(\delta_0; C^*(\delta_0))}{f^E(\delta_0; C^*(\delta_0)) + D(\delta_0; C^*(\delta_0))}.$$

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[22] This result implies that, despite the agency conflicts between the agent and shareholders, under the optimal contract they have perfectly aligned interests with respect to the policy toward debt holders. In other words, the default policy does not depend on whether shareholders or the agent is in charge of the bankruptcy decision. This differs from Morellec (2004), in which the agent tends to keep the firm alive longer for more private benefits.

[23] Also, the new agent's outside option is $v_0 = -1/\gamma r$, so $W_0 = 0$. The results in the paper are insensitive to the treatment of unlevered firm after the bankruptcy.

Table 1
Baseline parameters.

Model parameters	Parameter values
Risk aversion (γ)	5
Effort cost (θ)	35
Lower bound growth (ϕ)	−0.5%
Upper bound effort ($\overline{a}$)	5%
Volatility (σ)	25%
Interest rate (r)	5%
Bankruptcy cost (α)	25%
Marginal tax rate (τ)	20%

I take volatility $\sigma = 25\%$, interest rate $r = 5\%$, bankruptcy cost $\alpha = 25\%$, and tax rate $\tau = 20\%$ from Leland (1998). I set $\gamma = 5$ which is the median value used in Haubrich (1994). The firm growth parameters $\phi = -0.5\%$ and $\overline{a} = 5\%$ are chosen to match the average firm growth rate used in the literature. The effort cost $\theta = 35$ is set to (roughly) match the PPS documented in the empirical literature.

In Leland (1994), the scale-invariance implies that the optimal leverage ratio LR is independent of firm size δ_0. However, in this model the quadratic risk-compensation eliminates the scale invariance. In fact, in the following calibration exercises, I mainly investigate the divergent leverage decisions for firms with different sizes.

3.3.4. Parameterization

Table 1 tabulates the baseline parametrization. Interest rate $r = 5\%$, bankruptcy cost $\alpha = 25\%$, and tax rate $\tau = 20\%$ (considering personal tax effect) are typical in the literature (e.g., Leland, 1998).

I also record the average growth rate in the 50-year simulation, and this measure helps pin down ϕ and $\overline{a}$. In the literature with constant coefficients, Goldstein, Ju, and Leland (2001) calibrate a slightly negative μ, and Leland (1998) chooses the growth rate $\mu = 1\%$. Under the choice of $\phi = -0.5\%$ and $\overline{a} = 5\%$ ($\overline{a}$ is irrelevant for levered firms as the optimal effort $\{a^*\}$ never binds at 5%; see Fig. 2), the simulated average growth rates fit these numbers squarely across various firm sizes (see Table 2).

Because in my calibration the optimal effort never binds at $\overline{a}$ in levered firms, the pay-performance sensitivity is $1/1 + \theta\gamma r\sigma^2\delta_t$ as in Eq. (27).[24] Based on this result, I choose the agency-related parameters [risk aversion $\gamma = 5$ which is the median value used in Haubrich (1994), and effort cost $\theta = 35$] and the starting firm size δ_0 to match the PPS documented in the empirical literature. Jensen and Murphy (1990) report a PPS of 0.3% in their sample (1969–1983), while Hall and Liebman (1998) document a higher PPS with mean 2.5%. Aggarwal and Samwick (1999) control for the firm risk and report a mean PPS of 6.94% from the ordinary least square regression. For the size-dependence pattern of PPS, Murphy (1999) finds that for Standard & Poors 500 firms, the PPS is approximately 1% for large firms, 1.5% for Midcap firms, and 3% for small firms. Hall and Liebman (1998) find a mean PPS around 2.5%, and in page 676 they

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[24] The presence of debt does not affect the expression of PPS in Eq. (27). To see this, the agent's performance is measured as the change of equity value $f^E(\delta)$. However, $f^E(\delta)$ cancels in Eq. (27) when the optimal effort $a^*(\delta)$ takes an interior solution [check the derivation in Eq. (21)].

Table 2
optimal capital structure for firms with different sizes.

	Initial cash flow level (firm size) δ_0				
	50	100	150	200	250
Panel A: Optimal debt policies					
Optimal coupon C^*	24.50	61.35	106.00	147.62	203.15
Default boundary δ_B	7.97	22.79	40.58	57.28	79.98
Scaled default boundary δ_B/δ_0 (percent)	15.95	22.79	27.05	28.64	31.99
Panel B: Valuation and leverage					
Debt value $D(\delta_0)$	446.05	997.44	1640.02	2267.70	3090.52
Leverage ratio $\frac{D(\delta_0)}{D(\delta_0)+f^E(\delta_0)}$ (percent)	39.35	47.70	53.57	56.10	61.32
Credit spreads (basis points)	49.22	115.12	146.34	150.98	157.34
Panel C: Simulation results					
Average growth (percent)	1.95	0.50	0.12	−0.05	−0.17
Average volatility (percent)	25.04	24.98	25.04	25.03	25.01
Pay-performance sensitivity (percent)	5.33	2.52	1.69	1.29	0.99

The parameters are $r = 5\%$, $\sigma = 25\%$, $\theta = 35$, $\gamma = 5$, $\phi = -0.5\%$, $\overline{a} = 5\%$, $\alpha = 25\%$, and $\tau = 20\%$. Credit spreads are calculated as $(C/D-r) \times 10{,}000$. I simulate the model for 50 years to obtain the average growth rate and volatility for $d\delta/\delta$, given the initial δ_0. I also calculate the agent's average pay-performance sensitivity based on Eq. (27).
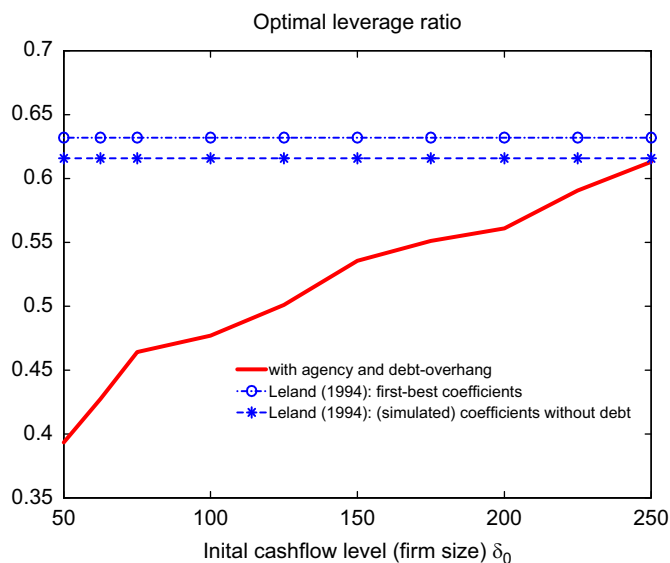

Fig. 1. Optimal leverage ratio as a function of initial cash flow level (firm size). I plot the two benchmark leverage ratios under Leland (1994). The first one is based on the first-best coefficients ($\mu = 4.5\%$ and $\sigma = 25\%$), which gives a leverage ratio 63.21% plotted in the dashed line with circles. The second one is based on the time series averages in simulating the unlevered firm in Section 3.2 ($\mu = 3.31\%$ and $\sigma = 25.02\%$; for simulation details, see Footnote 25). This case yields a leverage ratio 61.59% plotted in the dotted line with asterisks. The parameters are $r = 5\%$, $\sigma = 25\%$, $\theta = 35$, $\gamma = 5$, $\phi = -0.5\%$, $\overline{a} = 5\%$, $\alpha = 25\%$, and $\tau = 20\%$.

note that "the largest firms in our sample (market value over $10 billion) have a median Jensen and Murphy statistic (PPS) that is more than an order of magnitude smaller than the smallest firms in our sample (market value less than $500 million)."

3.4. Results and discussions

3.4.1. Optimal leverage ratio

Because debt-overhang adversely affects the firm's endogenous growth, relative to Leland (1994) firms take less leverage for their optimal capital structure in this model. Fig. 1 shows the optimal leverage ratio (the solid line) for firms with different sizes. For better comparison, Fig. 1 also provides two benchmark optimal leverage ratios predicted by the Leland (1994) model, with exogenous constant growth. The dashed line with circles (63.21%) is the first-best case, in which the cash flow growth rate is $\overline{a} + \phi = 4.5\%$. However, as in Section 3.2, the agency problem alone reduces the firm growth, which would lead to a lower leverage ratio even without debt-overhang. To address this issue, I take the results of
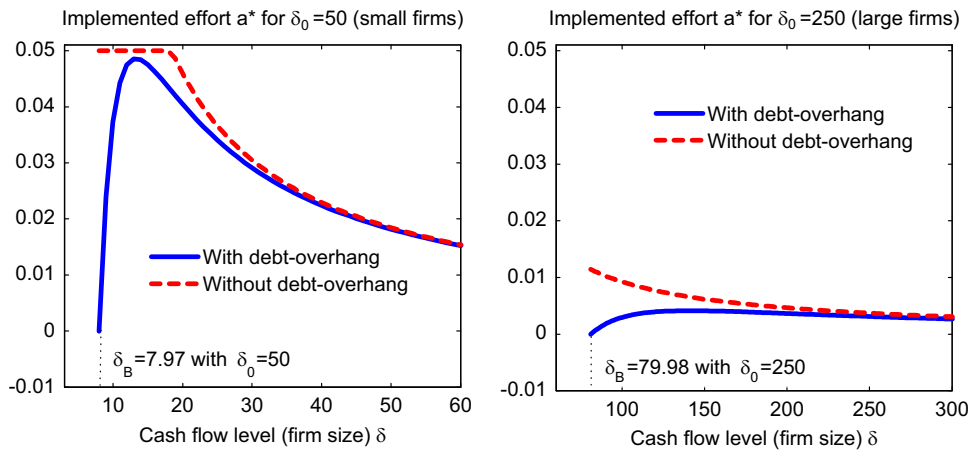
Fig. 2. Implemented effort policies with debt-overhang for small ($\delta_0 = 50$) and large ($\delta_0 = 250$) firms under optimal leverage decisions. The optimal effort policy $a^*(\delta)$ with debt-overhang is shown in the solid line, and the optimal effort policy in an unlevered firm without debt-overhang is shown in dashed line. I also mark the optimal endogenous default boundary, where shareholders optimally (ex post) default and implement a zero effort. The parameters are $r = 5\%$, $\sigma = 25\%$, $\theta = 35$, $\gamma = 5$, $\phi = -0.5\%$, $\bar{a} = 5\%$, $\alpha = 25\%$, and $\tau = 20\%$.

unlevered firms from Section 3.2, simulate the model, and obtain time series averages of the cash flow growth and volatility.[25] I then use these estimates as inputs to calculate the Leland leverage ratio, which is graphed in the dotted line with asterisks (61.59%) in Fig. 1.

The optimal leverage ratios are reported in Table 2 along with other measures. For each initial cash flow level δ_0, I calculate the sample mean of growth and volatility of $d\delta/\delta$ over one hundred years in five hundred simulations (see Panel C in Table 1). I also report the sample average of pay-performance sensitivity based on Eq. (27); these numbers fit the empirical estimates discussed in Section 3.3.4 squarely. For small firms ($\delta_0 = 50$), the optimal leverage ratio falls from 61.59% (or 63.21% in the first-best case) to 39.35%.[26] In contrast, the optimal leverage ratio for large firms is close to the result under Leland (1994). I will come back to this cross-sectional result shortly.

[25] Specifically, to match the relevant range for δ in levered firms, I set the initial $\delta_0 = 250$ and stop the process once δ reaches 7.97 (which is the lowest default boundary in the calibration). Also, the simulation length is 50 years to mitigate the impact of initial condition. I then average the time series mean of growth rate and volatility across five hundred simulations, which gives an average growth rate (volatility) as 3.31% (25.02%). Other treatment gives similar results.
[26] This reduction is comparable to other modifications of the Leland (1994) benchmark. For instance, by combining both the "callable" feature of the debt and upward capital restructuring together, Goldstein, Ju, and Leland (2001) reduce the optimal leverage from 49.8% to 37.1% in their baseline case.
It is not easy to accommodate the dynamic capital restructuring into my framework, as this model does not have the convenient scale-invariance property. Certainly, as in Goldstein, Ju, and Leland (2001), the possibility of raising leverage in the future should reduce the firm's initial leverage. In fact, dynamic capital restructuring would have an interesting impact to this model if the restructuring was downward, i.e., reducing debt when the firm's cash flow level drops. This is because in this model almost all the action is on the downside where debt overhang cuts down the efficient effort. If I only allow for upward restructuring as in Goldstein, Ju, and Leland (2001), the interaction effect should be small.

3.4.2. Debt-overhang

In Fig. 1, the optimal leverage ratio is lower relative to Leland (1994). The reason is debt-overhang, where I interpret the agent's effort as a form of investment; see Hennessy (2004) for a similar mechanism. In my model, shareholders design an employment contract to maximize the ex post equity value, and the smooth-pasting condition Eq. (31) implies that $f^E(\delta)$ goes to zero as δ approaches the default boundary δ_B. It implies that, once the firm is close to bankruptcy, shareholders gain almost nothing by improving the firm's performance. Then, according to Eq. (29) which says that the optimal effort a^* is proportional to $f^E(\delta)$, shareholders implement diminishing effort (through providing diminishing incentives) during financial distress. As a result, in addition to the traditional bankruptcy cost, in this model the debt bears another form of cost due to debt-overhang.

This mechanism is illustrated in Fig. 2. The left panel plots the implemented effort investment a^* in solid line as a function of firm's financial status δ for small firms ($\delta_0 = 50$). As explained shortly, the debt-overhang problem is more severe for small firms. I also plot the optimal effort policy without debt-overhang (the dashed line), which corresponds to the case of unlevered firms studied in Section 3.2.

Relative to the effort policy without debt-overhang plotted in the dotted line, an abrupt drop of implemented effort is evident when the firm is in the verge of bankruptcy ($\delta \to \delta_B = 7.97$). From the view of social welfare, in this situation a higher effort is desirable, because it helps avoid the costly bankruptcy once δ hits δ_B. However, it is not in the shareholders' interest to ask the agent to work hard. When the firm approaches bankruptcy, shareholders obtain zero marginal value from improving δ. Consequently, they implement diminishing effort, a typical symptom of debt-overhang.

It is worth emphasizing that the driving forces of the debt-overhang result are the endogenous nature of firm growth, and the smooth-pasting condition of the shareholders' value; both ingredients are generic in practice.

Therefore, if in reality the management gives place to existing shareholders (such as the board) during financial distress, then debt-overhang is still present without the intermediate link of diminishing management incentives.

Leverage versus management incentives. The debt-overhang generates a negative relation between leverage and agent's working incentives. This result contrasts to Cadenillas, Cvitanic, and Zapatero (2004), in which the log agent's compensation space is restricted to equity shares, and shareholders commit to this compensation scheme. In comparison, in this optimal dynamic contracting setup, I do not place any restriction on the contracting space, and I allow for dynamically revising the employment contract between the agent and shareholders. Finally, as discussed in Section 3.3.1, part of the implementation of the optimal contract requires the firm to set the agent's deferred compensation aside as cash. This way, shareholders commit to fully insulate the agent's compensation from bankruptcy. I now discuss these assumptions by relating them to "inside debt" investigated in Sundaram and Yermack (2007).

Inside debt. Several interesting remarks can be made regarding the above assumptions, which point to the robustness of the debt-overhang result. First, in reality, although there are certain revising activities such as resetting the strike price of executives' previously awarded options, modifying compensation contract is not frictionless. For instance, managers' pensions – as a form of deferred compensation – are calculated according to certain prespecified formulae. More importantly, these pensions represent unsecured, unfunded debt claims against firm assets (inside debt as advocated in Sundaram and Yermack, 2007, and Edmans and Liu, 2011),[27] which is not a senior claim against a cash-based deferred compensation fund as in my implementation. As a result, this portion of compensation scheme can potentially alleviate the debt-overhang problem in this paper and the risk-shifting problem in Edmans and Liu (2011).[28]

Nevertheless, for inside debt to work, one also needs shareholders' and the agent's commitment on other compensation schemes to prohibit undoing the inside debt. The important point is that, to rule out ex post revising, shareholders and the agent need commitment with debt holders on all compensation schemes, not on one or some schemes. Put differently, as long as shareholders can modify the residual compensation freely, they can still undo the inside debt, and theoretically return to the optimal contract without

commitment. This implies that my theoretical results are robust to the practice of inside debt. In addition, some indirect empirical evidence (Bryan, Hwang, and Lilien, 2000) is consistent with this view of "undoing" or "dynamically revising" the employment contract, which can be a potential topic for future empirical studies.[29]

3.4.3. Size-heterogeneity

My model offers another explanation why small firms take less leverage relative to their large peers, a stylized fact documented in the literature (e.g., Frank and Goyal, 2005). The mechanism here is rooted in divergent severities of the aforementioned debt-overhang problem for different size firms. In this model, to be consistent with the inverse relation between PPS and firm size, larger firms implement lower effort. This implies that, for larger firms, debt-overhang – which reduces the profitable effort investment – is less of concern. Consequently, larger firms issue more debt to maximize the ex ante firm value.

This point is illustrated in the right panel in Fig. 2, where effort policies are plotted with and without debt-overhang for large firms ($\delta_0 = 250$). For better comparison, the right panel adopts the same scale as the left panel where small firms are considered. Debt-overhang is moderate for large firms. As shown, at their relatively high default boundary $\delta_B = 79.98$, the optimal effort even without debt-overhang is low (only about 1.14%). Therefore, the drop of a_t^* when larger firms approach bankruptcy – the exact force of debt-overhang – is less dramatic compared with smaller firms (the left panel). In sum, in smaller firms the debt-overhang cost is greater, leading to a lower predicted leverage ratio.

It is important to add that this result is not driven by CARA preference. Instead, CARA preference is used only as the analytical tool to match the empirical pattern of pay-performance sensitivity, and it is the negative relationship between PPS and firm size that implies lower debt-overhang costs in large firms.

3.4.4. Default policy and credit spreads

Table 2 also reports the endogenous default policies. The scaled default boundary δ_B/δ_0 are lower (so shareholders default later) than the Leland (1994) benchmark.[30] The intuition for shareholders to postpone bankruptcy is as follows. In this model, a firm with recent

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[27] This means that when the firm becomes insolvent, pension beneficiaries have the same priority as other unsecured creditors. However, footnote 10 in Sundaram and Yermack (2007) gives an example of the "secular" trust fund, which secures an executive's pension in a bankruptcy-proof form.

[28] These papers are closely related to the early theoretical work by John and John (1993) who consider a risk-shifting problem. Essentially, for the agent to maximize the firm's value, the compensation should be less aligned with shareholders when the leverage is higher, which predicts a negative relation between leverage and PPS. There, commitment is also essential, even though it appears not so in a two-period model. In contrast, in this model, the agent is always perfectly aligned with shareholders in terms of incentives thanks to potential revising.

[29] Bryan, Hwang, and Lilien (2000) analyze the *Incentive-Intensity* (the change in the value of annual stock-based compensation per change in equity value) and *Mix* (ratio of the value of annual stock-based compensation to cash compensation) measures, which are based on the annual stock-based grants only (as opposed to cumulative inside holdings that relate to the agent's wealth). They find that both *Incentive-Intensity* and *Mix* decrease with firm leverage. Under the debt-overhang framework studied in this paper, to the extent that working incentives generated by inside debt is increasing with leverage, these empirical results are consistent with the dynamic revising activity. Also, decreasing *Mix* with leverage implies that financially troubled firms pay the agent more cash compensation, a result consistent with my model if the juniority of pensions force the firm to start paying out deferred compensation to the agent by cash.

[30] For the Leland (1994) exogenous growth model, $\delta_B/\delta_0 = 39.73\%$ in the first-best parametrization, and $\delta_B/\delta_0 = 37.24\%$ in the unlevered firm parametrization.

unsatisfactory performances has a lower cash flow level, or smaller size. But given a smaller risk-compensation, shareholders find it cheaper to motivate the agent, which gives them more value to wait for future improvement. Because of the delayed default, my model produces similar, but slightly lower, credit spreads (Panel B in Table 2) than Leland (1994), conditional on comparable leverages.[31]

There are various theoretical models in which agency problems are countercyclical (e.g., Bernanke and Gertler, 1989; Eisfeldt and Rampini, 2008, etc.). Acknowledging that the agency issue becomes more severe in recessions (for instance, a higher cash flow volatility or a higher constant absolute risk aversion when the agent's wealth is lower), it is interesting to further explore the impact of agency problems on credit spreads. Chen, Collin-Dufresne and Goldstein (2009) argue that the key mechanism in explaining the high credit spread puzzle is that firms default more frequently in those higher risk premium states. This countercyclical default policy might be related to exacerbated agency issues in these bad states, because severe agency frictions can lead to a great reduction in shareholder' value of keeping the firm under their control. For instance, consider small firms $\delta_0 = 50$. By raising the agent's constant absolute risk aversion coefficient γ from 5 to 10, while fixing coupon level $C = 24.50$, the default boundary increases from 7.97 to 8.92, and as a result the credit spread goes up substantially (49 bps versus 85 bps). I leave this for future research.

3.4.5. Asset volatility and leverage effect

Various empirical studies find that equity returns become more volatile as the firm approaches bankruptcy. This phenomenon can be explained by the so-called leverage effect, even holding the volatility of the underlying asset constant.

In this model, the state variable for a firm's financial status is its cash flow level, and I also assume a constant instantaneous (return) volatility in the continuous-time setup. However, when the sampling frequency cannot be arbitrarily high, the estimated variance differs from the instantaneous volatility. Because of the hump-shaped endogenous effort (see Fig. 2), this model generates higher conditional variances (based on infrequent sampling) when firms are in financial distress.

The mechanism is as follows. Due to debt-overhang, the firm's endogenous growth is positively correlated with underlying performance shocks in financial distress. To see this, consider the implemented effort policy in the left panel in Fig. 2. When δ is close to $\delta_B = 7.97$, $a^*(\delta)$ is increasing in δ. Then, a positive shock to δ, by reducing debt-overhang, gives rise to a higher effort $a^*(\delta)$. This further leads to a higher cash flow growth rate $a^*(\delta) + \phi$ and, in turn, magnifies the positive shock. Therefore, when sampling is infrequent, the observed return volatility is higher than the constant volatility $\sigma = 25\%$. For

instance, when $\delta = 15$, over one year the annualized volatility based on monthly observed data is about 25.12%. When δ is far away from the bankruptcy threshold, $a^*(\delta)$ is decreasing in δ, and the exact opposite force leads to a lower volatility estimate (when $\delta = 50$, the above mentioned annualized volatility is about 24.68%).

In sum, for a firm near bankruptcy, its financial status becomes more sensitive to underlying performance shocks. In fact, this general message does not depend on the discrete-sampling, as in this model the firm value (rather than the instantaneous cash flow rate) displays a higher instantaneous volatility during financial distress.[32]

3.4.6. Debt covenants

A commonly observed debt covenant is that debt holders can force shareholders to go bankrupt when the firm's cash flow δ hits a prespecified level δ_B^b. This covenant is along the same line as "positive net-worth covenants for protected debt" in Leland (1994) which stipulates that the firm defaults whenever the asset value drops to the debt face value. Under the current cash flow framework, this can also be interpreted as a hard covenant on the interest coverage ratio, which, combined with coupon C, gives the bankruptcy threshold.[33]

In the standard trade-off model as in Leland (1994), a forced earlier default always hurts firm value, because an earlier default reduces the tax benefit and raises the bankruptcy cost. In fact, if equity holders can fully commit in Leland (1994), then no default is the first-best outcome.

In contrast, due to the debt-overhang problem, in my model specifying $\delta_B^b > \delta_B$ might be welfare improving. The reason is that, now around δ_B^b, there is no longer smooth-pasting condition as in (31), and shareholders benefit from improving the firm performance. Then, specifying $\delta_B^b > \delta_B$ is equivalent to committing to provide incentives (to the agent) even in the deep financial distress, and it might be socially optimal to do so. For instance, for small firms $\delta_0 = 50$, given the coupon $C = 24.50$, the optimal $\delta_B^b = 8.05 > \delta_B = 7.97$, and the firm value becomes $1133.81 > 1133.67$. Though small magnitude, this interesting finding distinguishes this model from Leland (1994) and its variations where the firm growth is exogenously specified.

4. Concluding remarks

By generalizing the optimal contracting result to widely used cash flow setups in finance, this paper offers a more tractable framework to investigate the impact of agency problems in various economic contexts. The absence of wealth effect due to CARA preference simplifies the optimal contracting problem with private savings, and

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[31] For instance, based on the growth and the volatility estimates (Panel C) of medium-size firms ($\delta_0 = 150$), under Leland (1994) a 53.57% of leverage ratio translates into a credit spread of 175 basis points (bps), which is higher than 146 bps in my model.

[32] The firm's asset value, generated by future cash flows, can be defined as $X(\delta_t) = \mathbb{E}_t[\int_t^{\tau(\delta = \delta_B)} e^{-r(s-t)}\delta_s\, ds + f(\delta_B)]$, where $\tau(\delta = \delta_B)$ is the first passage time of δ hitting δ_B. Then, the instantaneous return volatility of $X(\delta_t)$ is larger when δ_t is close to δ_B. The reason is that $X(\delta_t)$ takes into account the impact on the stochastic growth μ, which positively correlates with δ_t during distress.

[33] Because part of the coupon C can be interpreted as constant operating cost (in other words, the leverage derived here includes operating leverage, too), the effective interest coverage ratio can be much higher than implied by δ_B^b/C.

I characterize the optimal contract by an ODE. When applying my results to Leland (1994), I find that the underinvestment of effort due to debt-overhang produces a lower optimal leverage ratio, and the interesting interaction between agency issue and debt-overhang leads smaller firms to take less debt in their leverage decisions.

The relatively simple structure in this paper leaves several directions for future research. For instance, incorporating investment decisions would be desirable, as one can explore the investment distortion and its interaction with financing decisions under agency problems. Also, incorporating time-varying risk premia that is correlated with agency frictions (e.g., time-varying risk-aversion) is valuable to investigate the agency impacts on credit spreads.

Appendix A

A.1. Proof of Lemma 2

Consider any contract $\Pi = \{c; a\}$ that induces an optimal policy $\{\hat{c}; \hat{a}\}$ from the agent with a value $\hat{V}_0$. The agent's budget equation yields

$$S_t = \int_0^t e^{r(t-s)}(c_s - \hat{c}_s) \, ds,$$

which gives the agent's optimal savings path, with the transversality condition $\lim_{T \to \infty} \mathbb{E}[e^{-rT} S_T] = 0$. The transversality condition holds for all measures induced by any feasible effort policies.

By invoking the replication argument similar to revelation principle, I consider giving the agent a direct contract $\hat{\Pi} = \{\hat{c}; \hat{a}\}$, and shareholders have the same cost to deliver this contract as $\Pi = \{c; a\}$. Clearly taking consumption-effort policy $\{\hat{c}; \hat{a}\}$ is feasible for the agent without private savings. I need to show that $\{\hat{c}; \hat{a}\}$ is optimal for the agent. Suppose that given the contract $\hat{\Pi}$, the agent finds that $\{c'; a'\}$ yields a strictly higher payoff $V_0' > \hat{V}_0$ in problem (2), with associated savings path

$$S_t' = \int_0^t e^{r(t-s)}(\hat{c}_s - c_s') \, ds,$$

which satisfies transversality condition. However, consider the following saving path

$$S_t'' = S_t + S_t' = \int_0^t e^{r(t-s)}(c_s - c_s') ds,$$

which also satisfies the transversality condition. But this implies that, given the original contract Π, the saving rule S_t'' supports $\{c'; a'\}$ but delivering a strictly higher payoff V_0'. This contradicts with the optimality of $\{\hat{c}; \hat{a}\}$ under the contract Π.

A.2. Proof of Lemma 3

Given any savings $S_t = S$ and a contract $\Pi = \{c; a\}$, from time-t on the agent's problem is [recall Eq. (2)]

$$\max_{\{\hat{c}_s\}, \{\hat{a}_s\}} \quad \mathbb{E}\left[\int_t^\infty -\frac{1}{\gamma} e^{-\gamma(\hat{c}_s - g(\delta_s, \hat{a}_s)) - r(s-t)} \, ds\right] \qquad (33)$$

$$\text{s.t.} \quad dS_s = rS_s \, ds + c_s \, ds - \hat{c}_s \, ds, \quad S_t = S, \; s > t$$

$$d\delta_s = \mu(\delta_s, \hat{a}_s) \, ds + \sigma(\delta_s) \, dZ_s.$$

Denote by $\{\hat{c}_s^*, \hat{a}_s^*\}$ the solution to the above problem, and by $V_t(\Pi, S)$ the resulting agent's value.

Now consider the problem with $S = 0$, which is the continuation payoff along the equilibrium path:

$$\max_{\{\hat{c}_s\}, \{\hat{a}_s\}} \quad \mathbb{E}\left[\int_t^\infty -\frac{1}{\gamma} e^{-\gamma(\hat{c}_s - g(\delta_s, \hat{a}_s)) - r(s-t)} \, ds\right]$$

$$\text{s.t.} \quad dS_s = rS_s \, ds + c_s \, ds - \hat{c}_s \, ds, \quad S_t = 0, \; s > t$$

$$d\delta_s = \mu(\delta_s, \hat{a}_s) \, ds + \sigma(\delta_s) \, dZ_s.$$

I claim that the solution to this problem is $\{\hat{c}_s^* - rS, \hat{a}_s^*\}$, and therefore the value is $V_t(\Pi, 0) = e^{\gamma rS} V_t(\Pi, S)$. There are two steps to show this. First, this solution is feasible. Second, suppose that there exists another policy $\{\hat{c}_s', \hat{a}_s'\}$ that is superior to $\{\hat{c}_s^* - rS, \hat{a}_s^*\}$, so that the associated value $V_t'(\Pi, 0) > e^{-\gamma rS} V_t(\Pi, S)$. Consider $\{\hat{c}_s' + rS, \hat{a}_s'\}$, which is feasible to the problem in (33). But under this plan the agent's objective is

$$e^{-\gamma rS} \cdot \max \mathbb{E}_t\left[\int_t^\infty -\frac{1}{\gamma} e^{-\gamma(\hat{c}_s' - g(\delta_s, \hat{a}_s')) - r(s-t)} \, ds\right]$$

$$= e^{-\gamma rS} V_t'(\Pi, 0) > V_t(\Pi, S),$$

which contradicts with the optimality of $\{\hat{c}_s^*, \hat{a}_s^*\}$. As a result, $V_t(\Pi, S) = e^{-\gamma rS} V_t(\Pi, 0)$.

A.3. Subsection 3.1

Note that $rA^{FB} = \max_{a \in [0, \bar{a}]} \{1 + A^{FB}(\phi + a) - (\theta/2)a^2\}$, and I am interested in characterizing the condition for the first-best effort a^{FB} to bind at $\bar{a}$. When a takes an interior solution, then $a^* = A^{FB}/\theta$, and $A^{FB} = 2/(r - \phi + \sqrt{(r-\phi)^2 - 2/\theta})$. First, I need to ensure that $(r-\phi)^2 > 2/\theta$ so that A^{FB} is real. Otherwise, the firm value is unbounded, resulting in an unbounded a^{FB}. Therefore, when $(r-\phi)^2 \leq 2/\theta$, $a_t^{FB} = \bar{a}$. Second, even when $(r-\phi)^2 > 2/\theta$, it is possible that the implied first-best effort $(1/\theta)2/(r - \phi + \sqrt{(r-\phi)^2 - 2/\theta}) > \bar{a}$. In this case, the first-best effort also binds at $a_t^{FB} = \bar{a}$. Under both scenarios, $A^{FB} = (1 - (\theta/2)\bar{a}^2)/(r - \phi - \bar{a})$.

A.4. Subsection 3.2

Under the boundary conditions specified in Section 3.2, f' is always positive in Eq. (25), therefore a^* never binds at zero in this problem. To see this, clearly $f'(0) > 0$ [notice $f(0) = 0$, and for $\delta > 0$, even with zero effort (so without the agent) the value is positive]. When $\delta \to \infty$, using Eqs. (25) and (26) one can check that

$$f(\delta) \simeq \bar{f}(\delta) \equiv \frac{1}{r - \phi} \delta + \frac{1}{2(r-\phi)^2 \theta^2 \gamma r \sigma^2}. \qquad (34)$$

which is increasing in δ. Now suppose that there exists $\delta_1 > 0$ such that $f'(\delta_1) = 0$. Take the smallest one so that $f''(\delta_1) < 0$ (i.e., f has to be concave on δ_1). Therefore, $rf(\delta_1) = \delta_1 + \frac{1}{2} f''(\delta_1) \sigma^2 \delta_1^2 < \delta_1$. But there must exist another point $\delta_2 > \delta_1$, such that f is deceasing in $[\delta_1, \delta_2]$, and f becomes increasing again after δ_2 (as f is increasing when δ is large enough). This implies $f(\delta_2) < f(\delta_1)$,

$f'(\delta_2) = 0$ and $f''(\delta_2) > 0$. However, Eq. (25) implies that $rf(\delta_2) > \delta_2 > \delta_1 > rf(\delta_1)$, a contradiction with $f(\delta_2) < f(\delta_1)$.

A.5. Subsection 3.3.2

When $\delta \to \infty$, the probability of bankruptcy is negligible, and the firm value can be viewed as the unlevered firm value minus the present value of after-tax coupon payment. Eq. (34) gives the unlevered firm value f when $\delta \to \infty$, and f' approaches to $1/(r-\phi)$ similar to the standard Gordon growth model. Therefore $f^E(\delta) \simeq \bar{f}(\delta) - (1-\tau)C/r$ when δ is sufficiently large.

I use *Matlab* built-in solver *bvp4c* to solve the model, setting the tolerance level to be 10^{-8}. I set $\bar{\delta} = 500$. In solving for the bankruptcy boundary δ_B, I first choose one candidate δ_B^1 and solve f^E based on Eqs. (30)–(32) using *bvp4c*. Then I evaluate $f^{E'}(\delta_B^1)$. If $f^{E'}(\delta_B^1) > (<)0$ which means that δ_B^1 is too low (high), I adjust δ_B^1 upward (downward) to, say, δ_B^2. I use a bisection method to search for δ_B, which converges quickly.

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